

ALTERNATING CURRENT BRIDGE METHODS

FOR THE MEASUREMENT OF INDUCTANCE,
CAPACITANCE, AND EFFECTIVE RESISTANCE
AT LOW AND TELEPHONIC FREQUENCIES

*A Theoretical and Practical Handbook
for the use of Advanced Students*

BY

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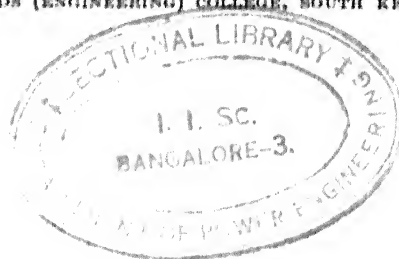
LECTURER IN ELECTRICAL THEORY AND MEASUREMENTS IN THE
ELECTRICAL ENGINEERING DEPT. OF THE CITY AND GUILDS
(ENGINEERING) COLLEGE, SOUTH KENSINGTON

WITH A FOREWORD

BY

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FOREWORD

FOR many years past the need of a book on Alternating Current Bridge Measurements for the use of students has been keenly felt in the Electrical Engineering Department of the City and Guilds (Engineering) College, South Kensington. When Mr. Hague joined the staff of the Department in May of 1920, and took charge of the delicate testing laboratories, I suggested that such a book would be a material help to the more advanced students. As a result of this suggestion Mr. Hague undertook the production of the present volume, on which he is, in my opinion, to be warmly congratulated. Students and others who have occasion to make bridge measurements by alternating currents, will find the book of great assistance in their more advanced work, especially as all the most important bridges are fully discussed by the use of symbolic methods, the vector diagrams drawn and explained, and the conditions for maximum sensitiveness worked out in the principal cases. Further, numerical results obtained in actual tests are incorporated in the work; these show the arrangements required in practice to get reliable results, and also indicate the accuracy attainable in given cases.

Numerous references to original papers on the subject of a.c. bridge measurement form a valuable feature of Mr. Hague's book, and these will, I feel sure, be of great service to all interested in research work on this important branch of electrical testing.

T. MATHER.

LONDON,
June, 1923.

PREFACE

THE object of the author in preparing the present volume is to deal with the subject of Alternating Current Bridge Measurements of Inductance, Capacitance, and Effective Resistance at low and telephonic frequencies in a manner suited to the needs of the advanced student. The importance of such measurements in modern laboratory and test-room practice, in research work and in the training of students, would seem to be sufficient reason for the publication of a handbook dealing fairly completely with all the matters involved. As the book is intended primarily for practical use, every endeavour has been made to make clear the experimental side of the subject. At the same time an attempt has been made to provide a logical treatment of the theory underlying the use of a.c. bridge networks, since this is a matter which falls outside the scope of text-books dealing with the theory of alternating currents.

The book is based on a course of lectures given for the past three years to third-year students of the City and Guilds (Engineering) College, amplified by the addition of material intended to make the volume useful to post-graduate workers and to others engaged on original research or accurate testing. The subject-matter is divided into five chapters, each dealing with some aspect of the theme. The object of Chapter I is to define the various quantities which are to be dealt with in the rest of the book; considerable attention has been paid to the discussion of electrostatic phenomena. In Chapter II the theory of alternating currents is developed from the standpoint of the symbolic method, and an attempt is made to show the true relationship between the symbolic method and the more usual vector diagram and mathematical treatment of a.c. problems.

The apparatus required for bridge measurements is considered at some length in Chapter III, attention being chiefly directed to the explanation of the principles underlying the action of the various instruments rather than to a catalogue-like description of constructional details. The various bridge networks are classified in Chapter IV, the theory, uses, and full

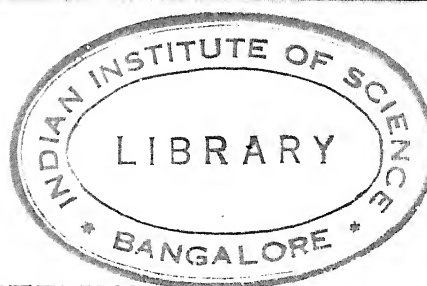
details of laboratory procedure being given in each case. With few exceptions, typical measurements have been made by all the methods described in this chapter, the results being included in the text as a guide to the student in carrying out his own experiments. Finally, the choice of the method suitable for a given measurement and the general precautions to be observed in laboratory practice are dealt with in Chapter V.

In preparing the book, full advantage has been taken of the information contained in original papers widely scattered throughout technical literature, complete references being given in the footnotes. In particular, the writings of Mr. A. Campbell and his associates at the National Physical Laboratory, of the papers published by the Washington Bureau of Standards, and by the Physikalische Technische Reichsanstalt have been drawn upon to a considerable extent. The manuscript of the book was in the hands of the printer before Mr. Campbell's informative articles in the *Dictionary of Applied Physics* were published, but reference has been made to them in revising the proofs for press.

In conclusion, thanks are due to many friends for help, advice and criticism during the preparation of the manuscript and proofs. In particular, the author wishes especially to thank Professor T. Mather, F.R.S., who not only suggested the preparation of the book, but who gave his unstinted help throughout. He has read the entire text in proof, and has provided the foreword. Mr. G. W. Sutton, B.Sc., has helped with the revision of the final proofs. Mr. S. Butterworth, M.Sc., of the Admiralty Research Laboratory, has kindly read the sections of the book on which he is an acknowledged authority.

The diagrams were all specially prepared for the book by the author with the skilled assistance of Mr. M. G. Say, M.Sc., to whom thanks are accorded. The author also wishes to record his indebtedness to the various firms, at home and abroad, who have kindly supplied information during the preparation of Chapter III.

LONDON.
July, 1923.



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ABBREVIATED TITLES FOR REFERENCES

<i>Amer. J. Sc.</i>	The American Journal of Science and Arts
<i>Annales de Ch. et Ph.</i>	Annales de Chimie et de Physique
<i>Ann. de l'Ecole normale</i>	Annales scientifiques de l'Ecole normale supérieure de Paris
<i>Ann. der Phys.</i>	Annalen der Physik und Chemie (1824-99), continued as <i>Annalen der Physik</i>
<i>Arch. f. Elekt.</i>	Archiv für Elektrotechnik
<i>Assoc. Française p. l'avance- ment d. Sc.</i>	Association Française pour l'avancement des Sciences
<i>Bull. Bur. Stds.</i>	Bulletin of the Bureau of Standards
<i>Comptes Rendus</i>	Comptes rendus hebdomadaires des séances de l'Académie des Sciences
<i>Ecl. Elect.</i>	L'Eclairage Electrique
<i>Electr.</i>	The Electrician
<i>Elec. Rev.</i>	The Electrical Review (continuation of <i>Tel. J</i> and <i>Elec. Rev.</i>)
<i>Elec. World</i>	The Electrical World and Engineer
<i>Elekt. Zeits.</i>	Elektrotechnische Zeitschrift
<i>Gen. Elec. Rev.</i>	General Electric Review
<i>Jahrb. d. D. Tel.</i>	Jahrbuch der Drahtlosen Telegraphie
<i>Journal Amer. I. E. E.</i>	Journal of the American Institute of Electrical Engineers
<i>Journal F. Inst.</i>	Journal of the Franklin Institute
<i>Journal I. E. E.</i>	Journal of the Institution of Electrical Engineers (continuation of <i>Journal S. T. E.</i>)
<i>Journal I. E. E. Japan</i>	Journal of the Institution of Electrical Engineers of Japan
<i>Journal de Phys.</i>	Journal de Physique Théorique et Appliquée
<i>Journal P. O. E. E.</i>	The Post Office Electrical Engineers Journal
<i>Journal S. T. E.</i>	Journal of the Society of Telegraph Engineers (continued as <i>Journal I. E. E.</i>)
<i>Journal Tel.</i>	Journal Télégraphique
<i>Mem. Acc. Tor.</i>	Memoria della Reale Accademia delle Scienze di Torino
<i>N. P. L. Researches</i>	Collected Researches of the National Physical Laboratory
<i>Nuova Cimento</i>	Il Nuovo Cimento, Giornale di Fisica, Chimica, e Storia Naturale
<i>Phil. Mag.</i>	The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science
<i>Phil. Trans. Roy. Soc.</i>	Philosophical Transactions of the Royal Society of London
<i>Phys. Rev.</i>	The Physical Review
<i>Phys. Zeits.</i>	Physikalisches Zeitschrift
<i>Proc. Phys. Soc.</i>	Proceedings of the Physical Society of London
<i>Proc. Roy. Soc.</i>	Proceedings of the Royal Society of London
<i>Rad. Rev.</i>	The Radio Review
<i>Tel. J. and Elec. Rev.</i>	The Telegraphic Journal and Electrical Review (after 1891 continued as <i>Elec. Rev.</i>)
<i>Trans. Amer. I. E. E.</i>	Transactions of the American Institute of Electrical Engineers
<i>Trans. Far. Soc.</i>	Transactions of the Faraday Society
<i>Verh. d. Deutsch. Phys. Ges.</i>	Vorhandlungen der Deutschen Physikalischen Gesellschaft
<i>Zeits. f. Inst.</i>	Zeitschrift für Instrumentenkunde

ALTERNATING CURRENT BRIDGE METHODS

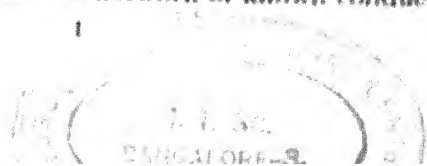
CHAPTER I

FUNDAMENTAL PRINCIPLES

1. Introductory. Among the applications of physics to modern electrical engineering practice, the recent developments in land and submarine telegraphy, in the telephonic transmission of speech, and in radio-telegraphy and telephony must be counted as of extreme importance. The enormous advances which have been made in these branches of electrical technology are largely the result of careful experimental researches, guided by a fairly complete mathematical and physical theory, in which the various electric and magnetic quantities involved have been accurately measured. It is to be expected, therefore, that further developments will be largely dependent upon the possession of methods of measurement which can be used in precise investigations to test the predictions or conclusions of theory.

In telegraphy, telephony, and radio-technology more or less complicated circuits are used, in which electric currents of a transient or of a periodic nature are caused to flow. It becomes, therefore, a matter of extreme importance to be able to measure not merely the currents and potential differences in the circuits, but also the various quantities, such as resistance, inductance, capacitance, etc., of which the circuits are composed. For this reason a large amount of work has been done with the object of devising methods by means of which these quantities may be accurately measured under actual working conditions, and it is the aim of the following chapters to discuss the theory and practical working of a most important class of methods—known as *Alternating Current Bridge Methods*—which have a very wide practical application.

In general, the principle underlying an alternating current bridge method may be briefly summarized in the following way. Let a piece of apparatus, of which the constants are to be measured, be inserted in a network of known conductors,



e.g. a Wheatstone bridge. Two points in the network are connected to a source of alternating current; while two other points are "bridged" by some instrument capable of detecting alternating potential differences, such as a telephone. The constants of the known conductors are then adjusted until the two bridged points are at the same potential at every instant, this condition being indicated by a zero reading of the detecting instrument, e.g. silence in a telephone. From the known constants of the network the constants of the apparatus under test can then be calculated.

While it is theoretically possible to make use of the bridge principle with alternating current of any frequency, certain important experimental difficulties are encountered at high frequencies which impose limitations to the range of frequency over which the principle is easily applicable. It is quite easy to devise alternating current bridge methods which, without any extraordinary precautions being taken, will give accurate results at frequencies up to 500 or 600 cycles per second. With higher frequencies the experimental difficulties—due principally to the importance assumed by electrostatic capacity effects between the bridge network and its surroundings, and to skin-effects, etc., in the conductors as the frequency is raised—increase until, at the upper telephonic range of 2,000 to 3,000 cycles per second, extreme care and very special precautions become necessary to secure precise results. At frequencies above this limit the experimental troubles become very great indeed until, at the values used in radio-telegraphic measurements, the application of the bridge principle is a matter of extreme difficulty, and special modifications in the technique becomes necessary.

It is to be clearly understood, therefore, that the theories and methods to be developed in this book are intended primarily to apply to measurements carried out at frequencies not exceeding the higher acoustic values of 2,000 to 3,000 cycles per second, such as will be found in ordinary laboratory practice and in telephonic research. Measurements at radio-frequencies are not contemplated in this volume owing to their very special character, but indication will be given, where possible, to show how the methods applicable at low frequencies require to be modified in order that they may eventually be applied in radio-research. For the present, it may be stated that bridge measurements* at radio-frequencies present very great difficulties and that the subject is only in an early state of development.

* See *Dictionary of Applied Physics*, Vol. 2, "Radio-Frequency Measurements."

2. History of the Bridge Principle. The bridge principle was first introduced by S. H. Christie* in 1833, but was neglected until 1843, when Sir Charles Wheatstone† drew attention to Christie's idea and applied it to the comparison of resistances. In the well-known arrangement, shown in

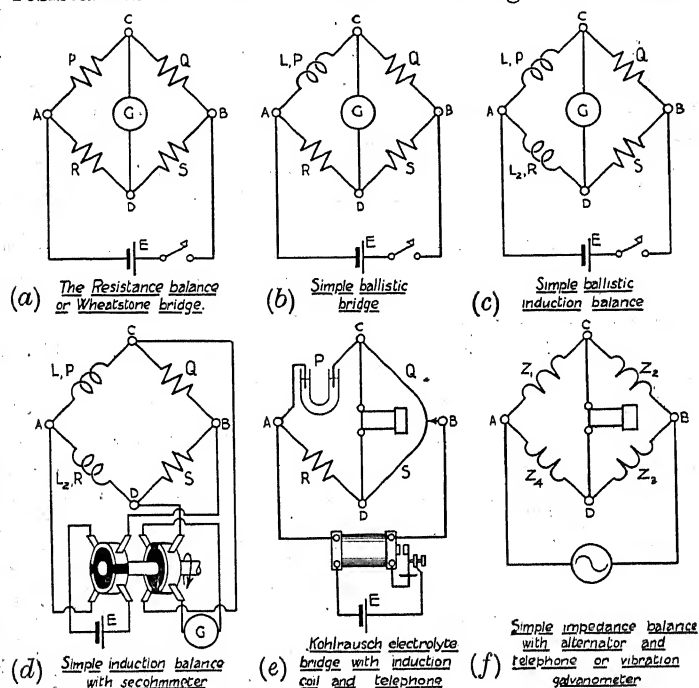


FIG. 1.—ILLUSTRATING THE EVOLUTION OF THE SIMPLE IMPEDANCE BALANCE

Fig. 1 (a), P is the resistance to be measured, Q and S are equal resistances, while R is an adjustable resistance or rheostat. A battery is connected across the points A and B , a galvanometer joining the points C and D . Clearly, the points C , D will be at the same potential and no current will flow through G if R be adjusted to be equal to P . The value of P is thus known in terms of the standard R .

* S. H. Christie, "Experimental determination of the laws of magneto-electric induction," *Phil. Trans. Roy. Soc.*, Vol. 123, pp. 95-142, (1833).

† C. Wheatstone, "An account of several new instruments and processes for determining the constants of a voltaic circuit," *Phil. Trans. Roy. Soc.*, vol. 133, pp. 303-327 (1843).

THE WHEATSTONE BRIDGE NETWORK. The arrangement was called by Wheatstone a *resistance balance*, because the resistances P, R are balanced against one another; the resistances Q, S are then referred to as the *arms* of the balance, by analogy with the process of weighing. With Q equal to S it will be seen that the greatest value of P which can be measured is limited by the maximum attainable value of R , and that the smallest measurable P is limited to the least value of R which is accurately readable. Werner von Siemens*, in 1848, increased the range of the balance for a given rheostat R by arranging Q and S to have values in any desired ratio. P and R are then compared on a balance which has unequal arms, so that $P = \frac{Q}{S}R$ by simple application of Ohm's law.

The process of balancing is then analogous to the comparison of masses by means of a steelyard or by a chemical balance with unequal arms. In this case, Q and S are called the *ratio arms*, Q/S having any convenient value.

There is another simple way of regarding the resistance balance. The points A, C, B, D, A are joined successively by the resistances P, Q, S, R . The diagonal AB consists of a conductor containing the battery E , while CD includes a galvanometer G . The whole arrangement constitutes a network of conductors joining the four points in all possible ways. The points C, D , which are to be at the same potential when balance occurs, are *bridged* by the detecting instrument; CD is therefore referred to as the *bridge*. The whole network is then called a *bridge network*, of which P, Q, R, S, G, E are the *branches*. By a process of abbreviation, the network is often simply referred to as the *Wheatstone bridge*.†

The Wheatstone bridge is, then, a null method for the comparison of resistances, and, when suitably arranged, is

* See C. W. Siemens, *Collected Scientific Works*, Vol. 2, pp. 126-127.

† In practice this nomenclature is frequently confused with that given in the preceding paragraph. By abbreviation the network is called a "bridge," confusion then arising from reference to P, Q, R, S as the "arms" of the bridge. The late Lord Rayleigh (*Journal I.E.E.*, Vol. 15, p. 29 (1886)), and Oliver Heaviside (*Electrical Papers*, Vol. 2, pp. 33 and 102 (1892)) have pointed out the absurdities arising from this mixing of notations. The confusion has entered the writings of French electricians, who refer to "le pont de Wheatstone." The Germans are more consistent, using "die Wheatstonesche Brücke" for the entire network, "die Brücke" for CD , and "der Zweig" (branch) for the conductors comprising the network. In this book any network in which the bridge principle is used will be called a *bridge network*,

capable of considerable sensitiveness and precision. Attempts were very soon made, therefore, to adapt the bridge principle to the measurement of other quantities, notably inductances and capacitances.

MAXWELL'S INDUCTION BALANCE. Maxwell,* in 1865, introduced the arrangement shown in Fig. 1 (b). Let the branch P of a Wheatstone bridge network be inductive, the bridge being balanced with steady current so that $SP = QR$. Then, on breaking or making the battery circuit, an electromotive force $L \frac{di}{dt}$ will be induced in the branch AC as the current i therein is diminishing or increasing. A quantity of electricity will thus be discharged through the galvanometer, since for transient currents the balance is not retained, producing a throw of the spot of light reflected from a mirror on the moving system. If the ballistic calibration of G be known, L is at once calculable; or, alternatively, the deflection of the galvanometer may be noted when the resistance of one of the branches is slightly increased, this deflection serving the purpose of a calibration when the period and damping of the instrument are known.

It is a simple step to convert this network into a null inductance balance. Let an adjustable inductive coil L_2 be put in the branch AD . Then, since the transient discharge from L_2 on making or breaking the battery circuit passes through the galvanometer in the reverse direction to that from L , a null condition can be arrived at by adjustment of L_2 until no ballistic kick is observed. The arrangement then forms a *ballistic bridge* or *induction balance*. Maxwell,† in his famous *Treatise*, describes several types of induction balance, many of them being in use with modern improvements at the present time. (see Fig. 1(c)).

the various conductors of which the network is composed being its *branches*. Those branches which are adjusted to attain balance are called *balancing branches*; balance being attained when no current flows in the conductor, called the *bridge*, containing the detecting instrument which is bridged across two points in the network.

* J. C. Maxwell, "A dynamical theory of the electromagnetic field," *Phil. Trans. Roy. Soc.*, Vol. 155, pp. 459-512 (1865). For the practical application of the method, see Lord Rayleigh and A. Schuster, *Proc. Roy. Soc.*, Vol. 32, pp. 104-141 (1881), and Lord Rayleigh, *Phil. Trans. Roy. Soc.*, Vol. 173, pp. 661-697 (1882).

† J. C. Maxwell, *A Treatise on Electricity and Magnetism*, see Sects. 756, 757, 778 of Vol. 2, 1st. edn., (1873).

In a similar way, condensers can be balanced by the use of a bridge, or a condenser may be compared with an inductance, and so on. In all cases the principle is the same: the network is first adjusted with steady current until the ordinary Wheatstone balance of resistances is satisfied. Then, without disturbing this condition, adjustments are made until the transient ballistic throw on make or break of the battery circuit is reduced to zero. For absolute balance it is necessary, therefore, to fulfil *two* balance conditions, one providing for the steady current null condition—called the *resistance balance*—while the other accounts for the vanishing of the transient effects—called the *ballistic* or *induction balance*.

Attempts were then made to increase the sensitiveness of the ballistic balance so as to be comparable with that obtainable for the resistance balance. Consider a bridge network containing inductances or capacitances, supposing the bridge to be balanced for steady currents and very nearly balanced ballistically. Then, on making or breaking the battery circuit, the galvanometer will receive impulses in opposite directions, the resulting deflections being small. Suppose the battery to be reversed; then twice the swing will be obtained, but successive reversals of the battery will again give opposite impulses to the galvanometer. If, however, the connections of the galvanometer to the bridge be reversed after each reversal of the battery, the impulses will all be in the same direction. If the rate of reversal be sufficiently high, a steady deflection will be produced by the summation of the impulses given to the galvanometer moving system.

THE SECORHMMETER. This principle is embodied in the Secorhmmeter of Ayrton and Perry,* illustrated in Fig. 1 (d), a similar device having been used by Brillouin in 1882. Two commutators are arranged on a shaft driven by hand gearing or by a small electric motor at a suitable speed. One commutator successively reverses and insulates the battery connections, while the other repeatedly changes the galvanometer leads. Resistance balance having been obtained with the commutators at rest, transient balance is secured by adjustment of the inductances or capacitances of the network with the commutators running. The apparatus can be applied

* W. E. Ayrton and J. Perry, "Modes of measuring the coefficients of self and mutual induction," *Journal, S.T.E.*, Vol. 16, pp. 292-343 (1888); W. E. Sumpner, "The measurement of self-induction, mutual induction, and capacity," *Journal, S.T.E.*, Vol. 16, pp. 344-379 (1888).

to increase the sensitiveness of any type of induction balance.* The use of the secohmmeter with an induction balance really amounts to the employment of an alternating current in the network. The voltage wave applied to the terminals *AB* is rectangular, since the battery is reversed at each half-revolution, and produces a corresponding current; in order to detect the impulses due to successive half-waves, an ordinary ballistic galvanometer combined with a rectifying commutator is used. The secohmmeter bridge may, therefore, be looked upon as an early type of alternating current bridge.†

USE OF THE TELEPHONE IN INDUCTION BALANCES. Measurements with the induction balance received a great impetus from the invention of the telephone by A. Graham Bell, in 1875, the high sensitivity of the instrument to small alternating currents leading experimenters to attempt to increase the sensitiveness of bridge methods by its use. The early work of Lord Rayleigh, Heaviside, Hughes, Kohlrausch, and others established the use of the telephone in bridge networks with alternating currents. The source of current in these early methods was frequently a small induction coil, providing a secondary current of somewhat irregular waveform and indefinite frequency; the detector was an ordinary Bell receiver. This arrangement is in use at the present day in a large amount of routine laboratory work where moderate accuracy is sufficient (see Fig. 1 (e)).

WIEN'S A.C. BRIDGE NETWORK. It is to Max Wien that the modern alternating current bridge is due. Following the lead of Oberbeck‡ he, in 1891, laid down the principles which are practically those used to-day. He supplied the network with alternating current of a definite steady frequency derived from an induction coil whose primary current was made and broken by a steadily vibrating wire. At a later date he used a small alternator and endeavoured to make the applied wave form approximately sinusoidal, so that tests could be carried out under precisely-known conditions. To increase still more the sensitiveness of the detector, he designed his "optical

* See, for example, S. R. Milner, "The use of the secohmmeter for the measurement of combined resistances and capacities," *Phil. Mag.*, 6th series Vol. 12, pp. 297-317 (1906).

† For a modern application of this principle, see p. 145.

‡ A. Oberbeck, *Ann. der Phys.*, Bd. 17, pp. 816-841 (1882); Max Wien, *Ann. der Phys.*, Bd. 42, pp. 593-621, Bd. 44, pp. 681-688, and pp. 689-712 (1891).

telephone," which was nothing more than a magnetic telephone of which the diaphragm could be tuned to be in resonance with the alternating current flowing in the telephone windings. By making use of the principle of resonance the amplitude of motion was greatly magnified, and was observed by reflecting a beam of light from a mirror operated by the moving diaphragm. This instrument is the forerunner of the "vibration galvanometers" used in modern practice. Using his apparatus Wien adapted a large number of old ballistic methods to work with alternating current, and introduced a number of new methods which are now standard practice (see Fig. 1 (f)).

MODERN DEVELOPMENTS. From the time of Wien to the present day steady progress has been made in the development of the technique of alternating bridge measurements. In general, the modern methods make use of networks of impedances arranged in the manner of the Wheatstone bridge or otherwise, as will be shown in Chapter IV; the source of current is chosen to have a steady frequency and a pure wave-form; the balance detector may be a highly sensitive telephone or some type of vibration galvanometer.

Since alternating current is used, the quantities measured in the bridge network are determined under the conditions in which they occur in practice, i.e. at the same frequency and with a known wave form. The quantities which occur in alternating current circuits, and which may be measured by a bridge method are capacitance, inductance, and effective resistance under the conditions of the test. It will be well at this point briefly to lay down what is meant by each of these terms—the remainder of this chapter being devoted to this purpose, though there will be no attempt at a complete account of the electrical theory involved. The reader is assumed to be familiar with the principles of alternating currents and to have clear ideas of the facts of electricity and magnetism.

Again, in dealing with the subject of this book it will be assumed that the sine-wave theory is adequate. This assumption is justified on two grounds: (i) that even if the currents in the network are not sinusoidal the sine-wave theory is sufficient since, by the application of Fourier's theorem, complex wave shapes can be treated by the summation of the results for a number of sine waves having frequencies in the proportions 1, 2, 3, etc.; (ii) since, in modern methods,

the tuned detector is so widely used, the sine theory is sufficient, because the sensitiveness of such instruments is extremely small to currents of frequencies even slightly removed from that to which they are tuned.

3. Capacitance. One of the principal uses of alternating current bridges is for the measurement of electrostatic capacity; and one of the most important experimental troubles encountered in making bridge measurements is due to the electrostatic effects between the bridge network and its surroundings. It becomes essential, therefore, to lay down exact definitions and clear ideas of electrostatic phenomena. The reader is assumed to be acquainted with the simpler qualitative and quantitative laws of the electrostatic field.

4. Elementary Ideas. Consider a single conductor placed so that it is at a very great distance from all other conductors, and let it receive a charge. Assuming the potential of the conductor to be initially zero, let the electrostatic charge necessary to raise the potential to unity be C ; the quantity C is then called the *capacity of the conductor*, i.e. the charge required to raise its potential to one electrostatic unit. To be consistent with the prevailing use of the terms "resistance," "inductance," etc., it is becoming the custom to refer to C as the *capacitance* of the conductor.

If the potential of the conductor be v electrostatic units, the charge upon it will be

$$q = Cv,$$

since the total charge is proportional to v . Now, in practice the electrostatic field is, in general, due to charges distributed over a system of several conductors and the earth. It is important, therefore, to define the meaning of the term capacitance when more than one conductor is involved. This can be readily done by the aid of the theory of Maxwell* in its modified form due to Orlich.†

5. Theory of Charged Conductors. Consider a system of n conductors arranged in proximity to the earth as shown in Fig. 2. A clear idea of the meaning of capacitance as applied to such a system is best obtained by an examination of the distribution of the electrostatic tubes of force. A tube of

* J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 1, 3rd edition, Sects. 87-90b (1892).

† E. Orlich, *Kapazität und Induktivität*, § 10, pp. 20-24 (1909).

force is bounded by lines of force, a unit tube starting from a unit positive charge and terminating on an equal negative charge. The total number of unit tubes emitted from a given charged area is called the electrostatic flux from the area.

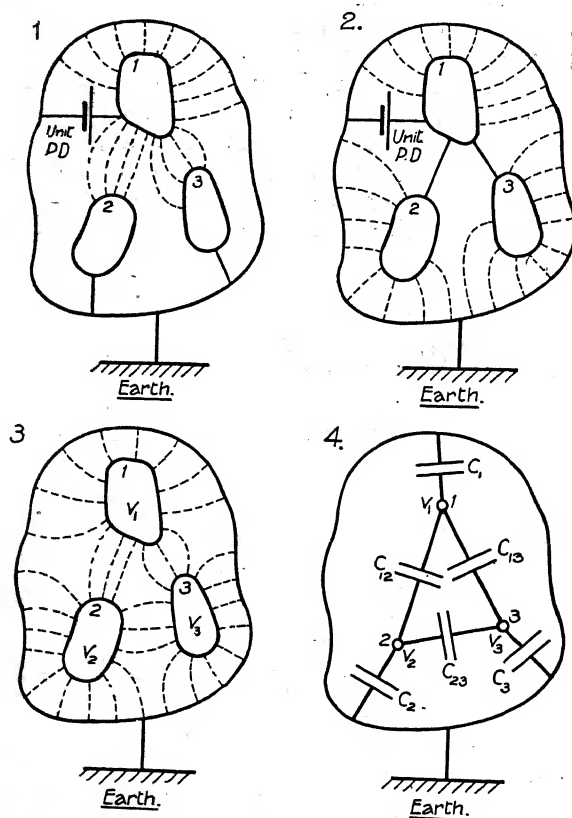


FIG. 2.—ILLUSTRATING THE PARTIAL CAPACITIES OF A SYSTEM OF CONDUCTORS.

Examining the third diagram in Fig. 2, the conductors 1, 2, 3 . . . n are shown raised to potentials $v_1, v_2, v_3 . . . v_n$, the approximate paths of the tubes of force being indicated by dotted lines. Consider such a conductor as 1, in the figure. The total flux leaving it may be divided into several portions represented by distinct groups of tubes of force passing in

certain directions. Firstly, there is the flux passing directly from the conductor to earth, the amount of it being proportional to the potential of the conductor. There remain $n-1$ groups of tubes of force passing between 1 and each of the other conductors, the number of unit tubes in each group being proportional to the difference of potential between 1 and the conductor upon which the tubes terminate. A similar analysis can be made of the flux from the surface of each conductor in the field. It therefore remains to express the earth-fluxes and inter-fluxes mathematically.

Referring to the second diagram in Fig. 2, let all the conductors be connected together and raised to unit potential. Each conductor will be positively charged so that tubes of force extend from each of them to earth. Let $c_1, c_2, c_3 \dots c_n$ be the charges on each under these conditions. The quantities $c_1, c_2, c_3 \dots c_n$ are called the *earth-capacities* of the n conductors, and represent the flux passing from each conductor to earth when all are raised to unit potential. The earth-capacities depend on the size and shape of the conductors, on their positions relative to earth and on the nature of the medium in which the conductors are situated.

Now let all the conductors be earthed, except the r th, which is raised to unit potential, as in the first diagram of Fig. 2. This conductor is, therefore, positively charged, and tubes of force setting out from it will terminate on a negative charge distributed over the remaining earthed conductors and the earth. The flux will be made up of a part to earth starting from a charge c_r , together with parts passing from r to 1, r to 2, \dots r to s , \dots r to n . Denoting these fluxes by $c_{r1}, c_{r2}, \dots, c_{rs}, \dots, c_{rn}$, the charges from which they spring, the total charge on r is $c_r + c_{r1} + c_{r2} + \dots c_{rs} \dots + c_{rn}$. The quantity c_{rs} is called the *intercapacity* between r and s ; it represents the flux passing between r and s , when r is raised to unit potential, s and all other conductors being earthed. The intercapacities are functions of the size, shapes, and relative positions of the conductors and the medium in which they are placed.

Now let the conductors be raised to potentials $v_1, v_2, v_3 \dots v_r, v_s, \dots v_n$, as in the third diagram of Fig. 2. Then the flux from r may be divided into n parts, viz., a part $c_r v_r$ passing from the conductor to earth, and parts $c_{r1}(v_r - v_1), c_{r2}(v_r - v_2) \dots c_{rs}(v_r - v_s) \dots c_{rn}(v_r - v_n)$ passing between the conductor r and

1, 2, . . . s, . . . n respectively. These fluxes are called the partial fluxes from r , the coefficients $c_r, c_{r1}, c_{r2},$ etc., being called the *partial or component capacities* of the conductor r (*Teilkapazitäten*). Clearly the flux leaving r and received upon s is equal in amount and opposite in direction to the flux which passes between s and r ; i.e. $c_{rs}(v_r - v_s) = -c_{sr}(v_s - v_r)$ or $c_{sr} = c_{rs}$. The charges $q_1, q_2 \dots q_r \dots q_n$ upon the n conductors are then

$$q_1 = c_1 v_1 + c_{12}(v_1 - v_2) + c_{13}(v_1 - v_3) + \dots c_{1n}(v_1 - v_n)$$

$$q_2 = c_2 v_2 + c_{12}(v_2 - v_1) + c_{23}(v_2 - v_3) + \dots c_{2n}(v_2 - v_n)$$

$$q_n = c_n v_n + c_{1n}(v_n - v_1) + c_{2n}(v_n - v_2) + \dots \text{etc.}$$

For n conductors there are clearly n earth-capacities and $\frac{1}{2}n(n-1)$ intercapacities, making a total of $\frac{1}{2}n(n+1)$ partial capacities, as the student can easily verify.

SIMPLE CONDENSER. In practice a most important example is that of two conductors arranged so that their intercapacity is very great compared with their earth-capacities or their capacities to other conductors. Such an arrangement is called a *simple condenser*, since all the electrostatic field is condensed within the space between the conductors and none exists outside. $C = c_{12}$ is then referred to as the *capacitance of the condenser*, and is understood to mean the charge which appears on one of the two conductors per unit potential difference between them.

Referring again to the general case of n conductors, if attention is directed merely to their potentials and charges, it will be obvious that the conductors may be replaced by n points maintained at the potentials $v_1, v_2, \dots v_n$, with simple condensers of capacitance equal to the respective partial capacities joining the points to one another and to earth. Thus, in the case of the three conductors illustrated in the third diagram of Fig. 2, the system of charges thereon can be replaced by six simple condensers, three having capacitances c_{12}, c_{23}, c_{13} joining the points 1, 2, 3 cyclically to represent the intercapacities; while three others, of capacitances c_1, c_2, c_3 , represent the earth-capacities. The points 1, 2, 3 have potentials v_1, v_2, v_3 ; the equivalent simple condensers being shown in the fourth diagram in Fig. 2.

In practice, condensers in which the intercapacity is large and the earth-capacities are small can be treated as simple

condensers with $C = c_{12}$; for example, an ordinary mica or paper condenser above about 0.1μ farad is very nearly a simple condenser. However, there are important practical instances when the earth-capacities must be taken into account; for example, in the case of an air-condenser or other condenser where the inter-capacity is small and the bulk large, the earth-capacities may have a large effect on the capacitance measured across the condenser terminals.

In an air-condenser there is usually a pair of electrodes insulated from and surrounded by a metallic screen, which is usually earthed. Let c_{12} be the intercapacity between the electrodes, c_1 and c_2 being the earth-capacities between the electrodes and the screen. The potentials and charges of the electrodes being v_1, v_2 , and q_1, q_2 , the above theory gives

$$q_1 = c_1 v_1 + c_{12}(v_1 - v_2),$$

and

$$q_2 = c_2 v_2 + c_{12}(v_2 - v_1).$$

Suppose now that the potentials and dimensions of the conductors are such that the charges upon them are equal in magnitude but opposite in sign, i.e. $q_1 = -q_2$. Then from above $c_1 v_1 = -c_2 v_2$, so that

$$q_1 = -q_2 = (v_1 - v_2) \left(c_{12} + \frac{c_1 c_2}{c_1 + c_2} \right);$$

from which

$$C = \frac{q_1}{(v_1 - v_2)} = c_{12} + \frac{c_1 c_2}{c_1 + c_2}.$$

The quantity C is the charge on one conductor per unit difference of potential between them when earth-capacities are not negligible, and is called the *working capacitance* (*Betriebskapazität*) of the condenser. It differs from the intercapacity by the term $c_1 c_2 / (c_1 + c_2)$, which is the combination of the earth-capacities in series. The theory of working capacitance just outlined has many other applications in practice, e.g. in the case of a multicore cable with earthed sheath, and is developed in connection with air condensers on page 112.

From these remarks it will be realized that the term "capacity" is one which must be very carefully used, since, if it is to have a precise meaning, the dimensions, positions, and symmetry of the conductors concerned must be exactly specified.

DISTRIBUTED CAPACITY. In all the above discussion the surface of each conductor has been assumed to be an equipotential; the earth and inter-capacities are then definitely localized quantities. There are important practical cases where this condition is not fulfilled and where the ideas of capacitance just developed require some amplification. For

example, consider the case of a resistance composed of two parallel wires connected at one end. Then the capacitance between the wires and between each wire and earth is uniformly distributed along their length, and cannot be accurately represented by localized simple condensers. Again, in a coil there is distributed capacity from turn to turn and from the turns to earth, the exact analytical treatment of which is a matter of very great difficulty.

In alternating current bridge networks some of the branches are composed of coils in which the distributed inter-turn and earth-capacities are usually small quantities. Hence, except with alternating currents of very high frequency, it becomes allowable to use approximations to these distributed capacities. The distributed inter-turn capacity of a coil can be represented, to a first approximation, by a simple condenser connected across its terminals, this condenser being referred to as the *self-capacity* of the coil. When an alternating potential difference is applied to the parallel combination of condenser and capacity-free coil the current entering the terminals and its phase relationship to the applied p.d., will be approximately the same as in the original coil. In a similar way, distributed earth-capacity of a coil can be approximately represented by a suitable system of simple condensers joining portions of the coil to earth.

6. Condensers with Alternating Current. The system of equations relating the charges, the potentials, and the partial capacitances are immediately applicable when the potentials are periodic functions of time. The principle involved can be illustrated by consideration of a simple case.

Consider a simple condenser of capacitance C to the electrodes of which a potential difference $e = e_1 \cos \omega t$ is applied. The charge is then

$$q = Ce = Ce_1 \cos \omega t$$

at any instant t . The rate at which the charge is supplied to the condenser is the current in the leads connected thereto. That is

$$i = dq/dt = Ce_1 \omega \cos\left(\omega t + \frac{\pi}{2}\right),$$

so that the current is a quarter-period in advance of the applied potential difference. Any condenser in which the current and p.d. are in exact quadrature is called a *perfect*

condenser, represented very closely in practice by condensers with gaseous dielectrics.

When a condenser has a charge q upon one of its plates and therefore a charge $-q$ upon the other, and these charges change with time, the entry of an additional charge dq at the one terminal is accompanied by the entry of $-dq$ at the other terminal. This is equivalent to the transference of a quantity dq from one terminal to the other; the rate of passage of the quantity dq/dt is called the *displacement current* within the dielectric. Hence, when used in alternating current circuits a condenser is said to carry a displacement current and behaves as a conductor, though the current is in quadrature with the p.d. and not in phase therewith, as it would be in a simple resistance.

In all practical condensers with solid or liquid dielectrics the passage of the displacement current is accompanied by a dissipation of energy in the dielectric, the current and p.d. not being exactly in quadrature. Such condensers are referred to as *imperfect condensers*, and are considered at some length in Chapter III.

7. Dimensions and Units. It has been pointed out above that the partial capacities of a system of conductors are a function of their sizes, shapes, and relative positions, and of the medium, called the dielectric, in which they are placed. Consider a simple condenser having a vacuum maintained between its electrodes, and let C_v be its capacitance. Now let some substance be introduced to occupy the space between the electrodes, C_s being the new capacitance. Faraday then defined the ratio $\epsilon = C_s/C_v$ as the *specific inductive capacity* of the medium, the modern term being *dielectric constant* or *dielectric ratio*. Thus ϵ is conventionally unity for a vacuum, and C_v can be calculated from the form and dimensions of the electrodes.

Electrostatic charges exert forces of attraction or repulsion on one another proportionate to the magnitudes of the charges and inversely as the square of the distance between them. The force is also inversely proportional to ϵ . Unit charge at a point repels a similar charge 1 cm. distant with a force of 1 dyne. If the work done in moving unit positive charge between two points in an electric field is 1 erg, unit electrostatic potential-difference exists between the points. By definition, the capacitance of a conductor (or of a condenser)

is the charge per unit potential difference. It remains now to consider the units in which capacitance is measured (i) in the absolute electrostatic system of units just defined, (ii) in the absolute and the practical electromagnetic units defined below.

Examining first the electrostatic system of units, let $[M]$, $[L]$, $[T]$ be the dimensional symbols for mass, length, and time, the units being the gramme, the centimetre, and the second. Then, since *Capacitance* = *charge/potential* and *potential* = *work/charge*, $[Capacitance] = [charge^2/work]$. Now from the law of inverse squares for electrostatic charges $[charge^2] = [force \times length^2 \times \epsilon]$, so that $[C] = [force \times L^2 \times \epsilon / (force \times L)] = [L\epsilon]$. The dimensions of capacitance in the absolute electrostatic system of units are those of length; a capacitance is, therefore, specified in centimetres.

In most practical work the Practical electromagnetic system of units is used* for all measurements. In this system the unit of potential difference is the volt and that of electric charge the coulomb. The unit of capacitance is the *farad*, and is the capacitance of a condenser in which a p.d. of 1 volt produces a charge of 1 coulomb. In all practical work the farad is an inconveniently large unit, a working unit of a *microfarad* = $\mu F. = 10^{-6}$ farad being generally adopted. In dealing with many quantities arising in bridge measurements a still smaller unit is used, the micro-microfarad = $\mu\mu F. = 10^{-12}$ farad.

The volt is 10^8 absolute electromagnetic units of p.d. and the coulomb 10^{-1} absolute electromagnetic units of quantity. Hence the farad is 10^{-9} , the $\mu F.$ 10^{-15} and the $\mu\mu F.$ 10^{-21} absolute e.m. units of capacitance.

The practical unit and the electrostatic unit of capacitance are simply related. One centimetre capacitance is equivalent to $1/v^2$ absolute electromagnetic units, v being the velocity of light, 3×10^{10} cm. per sec. Hence 1 cm. capacitance = $1/(9 \times 10^{20})$ e.m. units = $1/(9 \times 10^{11})$ farad. A convenient rule is 0.9 cm. = $1 \mu\mu F.$, or, conversely, 1 cm. = $1.11 \mu\mu F.$

8. Inductance. The most important quantities measured

* More correctly, the units used in technical work are those of the International system wherein the current and resistance units are defined in terms of actual standards intended to represent the units of the Practical system to a high order of accuracy. Except in work of the highest precision, the differences between the International and the Practical units may be neglected.

in a.c. bridges are self and mutual inductances. The properties of a.c. circuits in which inductances are present are dealt with in Chapter II; it is proposed here to set down the simpler physical ideas underlying electromagnetic induction so that the later work may be adequately understood. At the same time, definitions and units applying to the various quantities involved will be given. The student is supposed to have a knowledge of the properties of the magnetic field of a current and of the general principles of electromagnetic induction.

9. Elementary Ideas. The principles and laws of electromagnetism are based upon Faraday's famous experiments on electromagnetic induction. Consider two closed circuits, one containing a battery and switch while the other includes a galvanometer, and suppose the circuits are placed so that they can influence one another. So long as the current in the first circuit remains steady the galvanometer in the second circuit is unaffected; if, however, the current be changed, an immediate indication is given by the galvanometer. For example, if the first circuit be broken the galvanometer gives a momentary deflection, indicating that a transient current has flowed in the second circuit in the same direction as the current just removed from the first. On the other hand, if the first circuit be closed again, so that a current is established in it, the galvanometer gives a transient deflection in the opposite direction, showing that the second circuit has been traversed momentarily by a current in the reverse direction to that which has been established in the first. The transient current induced in the second circuit by the changes of the current flowing in the first was said by Faraday to be due to *mutual induction*. He showed that, for the same strength of inducing current, the magnitude of the transient effect was the same no matter which of the circuits carried the current or which contained the galvanometer. Moreover, if one of the circuits be removed, Faraday showed that transient induction effects, formerly referred to as "extra-current," occur in the remaining circuit when a current is started, stopped, or otherwise changed in it. The effect is then said to be due to *self-induction*.

Induction phenomena are best approached from a consideration of the magnetic field produced by a current-carrying circuit. When a steady current flows in a circuit, a magnetic

field is produced in surrounding space; this field can be mapped out by tubes of magnetic flux which are linked through the circuit producing them. A second circuit put into the field of the first will be linked by a certain number of tubes of flux, the total number of linkages being dependent on the size, shape, and position of the circuit. If now the strength of the current in the first circuit be changed, the strength of the magnetic field at every point will be altered, i.e. the linkages of flux with the circuits will be varied. Hence, when the flux linking the circuits is changed, Faraday's experiments show that electromagnetic induction takes place, transient currents appearing during the time that the linkages are changing. Observation shows that the induced currents are in such a direction as will tend to maintain the magnetic field in the same state as before the changes began, i.e. the induced currents tend to set up a magnetic field in such a direction as to oppose the changes. This statement is referred to as Lenz's law.

Considering now the inducing circuit, any attempt to alter the current in it, and thereby to modify its magnetic field, is immediately accompanied by transient induction effects tending to maintain the field in its original state. Hence the magnetic field of a circuit acts as a kind of electro-magnetic inertia to the current-producing effects which tend to delay the establishment of the current in the circuit and generally to retard any change in its strength. The appearance of these induction effects can be thought of as due to an electromotive force introduced into the circuit when the linkages of flux with it are changed; the direction of this induced e.m.f. is such as to oppose the growth of the current, and it is proportional in magnitude to the rate at which the current, and therefore its linkages, is changed. When the linkages, N , with the circuit are increasing (dN/dt positive), the induced e.m.f. opposes the increasing current; when N diminishes (dN/dt negative) the induced e.m.f. tends to keep the current flowing in its original direction. The e.m.f. of the battery is thus transiently increased by an e.m.f. of induction proportional to $-dN/dt$.

In a precisely similar way, if the current in a circuit be changing, the e.m.f. of induction appearing in a neighbouring circuit is proportional to the rate of diminution of the magnetic linkages threading it from the first circuit. This e.m.f. acts

in addition to the e.m.f. of any battery or other source of current in the circuit.

By a suitable choice of units the induced e.m.f. can be expressed numerically. If a rate of change of one linkage per second produces one electromagnetic unit of e.m.f. (see p. 22), then, in general, $e = -dN/dt$.

10. Self and Mutual Inductance. If the circuits considered on page 18 were placed in air or other magnetically indifferent medium, the flux, and consequently the linkages, will be proportional to the strength of the current; this would not be the case were the circuits linked by magnetic material such as iron.

Consider now the case of a number of circuits, 1, 2, . . . m , . . . n , in a magnetically indifferent medium such as air. Let all the circuits be opened except n , in which one electromagnetic unit of current is caused to flow. The total number of linkages of flux with the circuit itself is then L_n and, in addition, the remaining $n - 1$ circuits have linkages M_{1n} , M_{2n} , . . . M_{mn} , . . . etc., due to flux passing through them from circuit n . Now let n be opened and unit current flow in m . The linkages of flux in m are L_m and in each of the others M_{1m} , M_{2m} , . . . $M_{(m-1)m}$, . . . M_{nm} . Applying this process in turn to all the circuits, a number of linkage coefficients of these types are found, these having important properties.

Coefficients such as L_n are called *coefficients of self-induction*, or simply *self-inductances*. L_n denotes the number of linkages of flux with the circuit n when it is carrying unit current.

Coefficients such as M_{mn} are called *coefficients of mutual induction*, or *mutual inductances*. It follows from Faraday's experiments, or by deduction from energy considerations (Rayleigh's reciprocal theorem) that it is immaterial which of two circuits is treated as the inducing circuit and which as the induced; hence $M_{mn} = M_{nm}$ numerically. M_{mn} denotes the number of linkages of flux with the circuit m when unit current flows in n ; reciprocally, it is equal to the linkages with n due to the field of unit current in m .

The self and mutual inductances are functions merely of the size, shape, and relative positions of the circuits when the medium in which they are placed has a constant permeability. When iron is present the inductances depend also on the strength of the current in the circuits, and are not constants

dependent only on their geometrical properties. There are clearly n self-inductances and $\frac{1}{2}n(n-1)$ mutual inductances, making a total of $\frac{1}{2}n(n+1)$ coefficients.

Suppose now that the circuits are all closed and contain e.m.f.'s $e_1, e_2, \dots, e_m, \dots, e_n$ the resulting currents at any instant being $i_1, i_2, \dots, i_m, \dots, i_n$. Then the total linkages with each circuit will be the sum of its own self linkages and those due to all the other circuits, that is,

$$N_1 = L_1 i_1 + M_{12} i_2 + M_{13} i_3 + \dots + M_{1n} i_n,$$

$$N_2 = M_{12} i_1 + L_2 i_2 + M_{23} i_3 + \dots + M_{2n} i_n,$$

$$N_n = M_{1n} i_1 + M_{2n} i_2 + M_{3n} i_3 + \dots + L_n i_n.$$

The corresponding induced e.m.f.'s will be

$$-dN_1/dt, -dN_2/dt, \dots, -dN_n/dt.$$

By Ohm's law the total e.m.f. in each circuit must be equal to the resistance drop round it. If $R_1, R_2, \dots, R_m, \dots, R_n$ are the resistances of the circuits, the equations of equilibrium are

$$e_1 - \frac{dN_1}{dt} = R_1 i_1$$

$$e_2 - \frac{dN_2}{dt} = R_2 i_2$$

$$e_n - \frac{dN_n}{dt} = R_n i_n$$

from which the currents can readily be found when the applied electromotive forces are given. (For an example when there are six inducing circuits, see p. 51.) The equations apply no matter how the e.m.f.'s, and consequently the currents, may vary with time and hence apply directly to alternating quantities.

11. Dimensions and Units. Consider a current flowing in a wire bent into the form of a circle of 1 cm. radius, and let a free unit magnetic pole be imagined brought up to the centre of the circle. If the pole experiences a force of 2π dynes the current in the circle is said to be *one absolute electromagnetic unit*. When the current is of this strength, 1 absolute electromagnetic unit of quantity or electric charge passes any section of the circuit per second. If two points exist in the

circuit such that 1 erg of work is necessary to transfer 1 e.m. unit of charge between them, the points are said to have a difference of potential of 1 e.m. unit.

If the free unit pole is allowed to move from any point in the magnetic field of a current it will trace out a closed curve, called a line of force, the tangent to which is in the direction of the field strength at every point on it. The lines of force passing through the boundary of any small closed area are said to bound a tube of force; the field can be thought of as mapped out by tubes of force. Although this conception is useful it leads to some trouble, chiefly due to the hypothetical nature of the exploring pole, when applied to the magnetic field within a magnetic material. As it is with the inductive effect of a magnetic field that one is concerned, a definition based on the inductive property is more appropriate.

Accepting the definitions of current and p.d. in the absolute system, a circuit in which absolute unit of current is produced by unit p.d. is said to have 1 absolute unit of resistance. Let a single loop having an area of 1 sq. cm. be placed at any point in the field; let the loop be connected to a ballistic galvanometer by twisted leads so that a circuit of 1 e.m. unit of resistance results. If the plane of the loop be turned about until, on making or breaking the current producing the magnetic field, the ballistic galvanometer gives a maximum deflection, the normal to the plane of the loop lies in the direction of the magnetic induction at the point. If 1 e.m. unit of quantity passes through the galvanometer when exciting circuit is broken, the loop is traversed by unit induction. A line drawn through the field to show the direction of the induction by its tangent is called a line of induction. A tube bounded by lines of induction and containing unit induction, is called a unit tube of induction; the number of unit tubes crossing normally unit area is called the normal induction or flux density. In air, the tubes of induction and tubes of force form identical systems; in iron, they have distinct significance.*

Each tube of induction may be regarded as enclosing a kind of circuital stream of magnetic flux linked with some part of the exciting circuit, and perhaps at some part of its path linking some portion of a neighbouring circuit. The number of linkages of flux with a circuit per unit current in it is the self-inductance of the circuit. Alternatively, when

* See C. C. Hawkins, *The Dynamo*, 6th edition, Vol. 1, p. 4 *et seq.*

the linkages with a circuit change at the rate of 1 tube per second and 1 e.m. unit of e.m.f. is induced, the circuit has unit inductance. Similar definitions clearly apply to mutual inductance.

From these definitions the dimensions of the absolute unit of inductance can easily be obtained. Remembering that free unit poles follow the inverse square law of attraction $[pole^2] = [force \times length^2]$. The field strength or force per unit pole at the centre of a circle carrying a current measured in absolute units is inversely proportional to the radius of the circle and directly proportional to the current, i.e. $[current] = [force \times length/pole] = [\sqrt{force}]$. Now the quantity of electricity induced in a circuit is proportional to the linkages of the tubes of induction and inversely as the resistance of the circuit, i.e. $[quantity] = [current \times time] = [linkages/resistance]$; thus $[Inductance] = [linkages/current] = [resistance \times time]$

$$= [e.m.f. \times time/current] = \left[\frac{work}{quantity} \times \frac{time}{current} \right]$$

$$= \left[\frac{force \times distance}{current^2} \right] = [distance].$$

Hence in the absolute electromagnetic system of units inductance has the dimensions of length, and is measured in centimetres.

Owing to the inconvenient size of the absolute units, decimal multiples and sub-multiples of them are chosen for use in practical work. Thus—

1 volt	=	10 ⁸	absolute e.m. units of p.d.
1 ampère	=	10 ⁻¹	current
1 coulomb	=	10 ⁻¹	quantity
1 ohm	=	10 ⁹	resistance
1 henry	=	10 ⁹	inductance

Hence 1 henry = 10⁹ cm. Many inductances are expressed in millihenrys, mH = 10⁶ cm.; very small inductances are often measured in microhenrys, 10³ cm.

Thus, a circuit has one henry inductance when there are 10⁸ linkages of induction with the circuit per ampère. Alternatively, if the current in a circuit changes at the rate of 1 ampère per second, and an induced e.m.f. of 1 volt results, the circuit has an inductance of 1 henry.

12. Effective Resistance. If a direct current I ampères is

passed through a circuit, heat is generated at a rate I^2R watts, where R is the resistance of the circuit in ohms as measured in a d.c. Wheatstone bridge. R is sometimes called the "*ohmic*" resistance of the circuit.

If now an alternating current of r.m.s. value I ampères flows in the circuit, heat is produced at a greater rate I^2R' , where R' is the *effective resistance* of the circuit as measured in an a.c. bridge. The sources of additional loss in the circuit are numerous, and all arise from the pulsating nature of the magnetic and the electrostatic fields around the circuit. Briefly may be enumerated (i) the so-called "*skin-effect*" in the conductor; (ii) losses due to eddy currents induced by transformer action in neighbouring masses of metal which lie in the magnetic field of the circuit; (iii) conduction and dielectric losses in the insulation of the circuit, and the effects of self-capacity.

SKIN EFFECT. Of these sources of additional loss, one of the most important is the "*skin-effect*," the nature of which can be understood from a simple example. Consider a long straight conductor of circular section in which a current can be made to flow by application of a p.d. between its extremities. If the conductor carry a direct current, the density of which is uniformly distributed over the cross-section, then the strength of the magnetic field at a radius r from the centre of the wire is proportional to r within the conductor and to $1/r$ outside it.

Now let the conductor carry an alternating current and assume at first that the current density is uniform, so that the distribution of the magnetic field is the same as with a direct current. Examine two equal filaments, one at the centre and one at the circumference of the conductor. The former is linked by the entire alternating flux produced by the wire; the latter, on the other hand, is linked only by the flux outside the wire. A similar filament at a radius r is linked solely by the flux outside that radius. Hence, under the supposition of a uniform current density, i.e. equal currents in each similar filament of which the conductor is composed, the inner filaments have a greater inductance than the outer. It follows, therefore, that, since all filaments are subjected to the same p.d., the current density over the inner portion of the conductor must be less than over the outer. Apparently then, the current cannot be uniformly distributed and is crowded towards the

periphery of the wire. At high frequencies it flows chiefly a thin circumferential layer or skin, thus increasing the apparent resistance of the wire and the heating loss in it. The "skin effect" is clearly greater at high frequencies and also with the conductor of which the circuit is composed is of appreciable cross-section. Hence standard coils are often wound with a conductor made up of a large number of separately insulated strands connected in parallel; the skin-effect in each will be much reduced by making the strand sufficiently fine.

The skin-effect will modify not only the effective resistance of a circuit but will also change its effective inductance, since the distribution of the magnetic field with a.c. is different from that obtained with d.c. owing to the current being no longer uniformly distributed over the cross-section of the conductor. Again, the insulation conductance and the self-capacitance of the circuit, acting as shunts across its terminals, will cause the effective resistance and inductance to change with frequency (see pp. 90 and 91). Hence, due to these secondary effects the effective resistance and inductance of a circuit are functions of frequency, the change with frequency of the former being usually greater than that of the latter.

In general, whenever an alternating current flows in a circuit secondary effects come into play and modify the resistance of the circuit from its d.c. value, at the same time producing an alteration of the inductance from the value which would be found by a ballistic measurement. The effects depend on the frequency of the current, and are of great importance at the higher values. The field of utility of a.c. bridge methods therefore, lies in the measurement of effective resistance, inductance, and capacitance under exactly known conditions of frequency and wave-form similar to those obtaining in actual practical a.c. working.

CHAPTER II

THE SYMBOLIC THEORY OF ALTERNATING CURRENTS AND THE APPLICATION OF IT TO ALTERNATING CURRENT BRIDGE NETWORKS

1. Introduction. In Chapter I it has been shown that measurements by means of an alternating current bridge method are made by arranging a network of conductors,* one of which is the piece of apparatus which is to be tested, while the others are suitable standards of resistance, inductance, capacitance, or combinations of these quantities; these conductors are referred to as the *branches* of the network. Two points in the network are then connected to a source of alternating potential difference, while a second pair of points are "bridged" by a conductor in which is contained a suitable detecting instrument. The constants of the various branches are then adjusted until the detector indicates that no current flows in the bridge conductor. The network is then said to be *balanced* and certain relations, called the *balance conditions*, will exist between the constants of the branches. From these balance conditions it is then possible to calculate the unknown constants of the apparatus under test.

The purpose of this chapter is to show, in a general way, how these conditions may be most easily determined. To this end, a brief account of the elements of the theory of alternating currents is first given, treated according to the symbolic vector method. Maxwell's theory of networks is then explained and combined with the symbolic method to demonstrate those properties of alternating current circuits which are of service in the present problem. Typical bridge networks can then be considered and the general conditions of balance found.

2. Elements of Vector Algebra.† A *vector quantity*, that

* The term "conductor" is here employed in a general way to include not only ordinary metallic conductors but also condensers when used in alternating current circuits, see page 15.

† The author wishes to acknowledge helpful suggestions from Mr. F. M. Colebrook, B.Sc., during the preparation of this and subsequent sections. The student requiring further details concerning the general use of vector

is a quantity having magnitude, direction, and sense, can be represented graphically by a line, called a *vector*. The length of the line represents to scale the magnitude of the vector quantity; the inclination of the vector to some datum line gives the direction of the quantity; and an arrow-head drawn

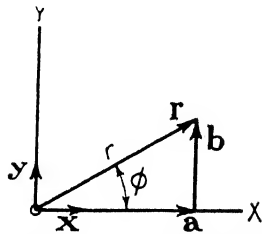


FIG. 3. — GRAPHICAL REPRESENTATION OF A VECTOR \mathbf{r} IN TERMS OF UNIT VECTORS \mathbf{x} AND \mathbf{y}

ϕ with the axis OX ; then, resolving \mathbf{r} in the directions of the two axes, let \mathbf{a} and \mathbf{b} be the component vectors. Then if a be the number of times \mathbf{x} is contained in \mathbf{a} and b the number of times \mathbf{b} contains \mathbf{y} ,

$$\mathbf{a} = ax,$$

$$\mathbf{b} = by,$$

are the components; and

$$\mathbf{r} = a\mathbf{x} + b\mathbf{y} (1)$$

is the *vector equation* determining \mathbf{r} . The sign “+” is understood to mean vector addition. This equation represents a vector of magnitude $r = \sqrt{a^2 + b^2}$ and direction $\phi = \tan^{-1}b/a$ relative to the axis OX , the sense being indicated by the arrow-head.

It will be apparent that the joint effect or sum of any number of vectors in a plane can be obtained by adding their component vectors parallel to OX and OY . The resultant

analysis should refer to standard treatises on the subject, e.g. J. G. Coffin, *Vector Analysis*, 2nd edition (1911).

For a treatment of vector algebra from the point of view of the electrical engineer, see W. G. Rhodes, *An Elementary Treatise on Alternating Currents*. Chaps. VII and VIII (1902). Also Oliver Heaviside, *Electrical Papers*, Vol. 2, pp. 4-5, for accurate definitions.

vector \mathbf{r} obtained by adding n vectors of the type given by Equation (1) will be

$$\mathbf{r} = (\sum_1^n a_n)\mathbf{x} + (\sum_1^n b_n)\mathbf{y} = a\mathbf{x} + b\mathbf{y},$$

an equation which has an obvious geometrical interpretation illustrated in Fig. 4. The extension of this process to the subtraction of vectors is obvious; the sense of the vector to be subtracted is reversed and the rule for addition applied.

When the product of two vectors is desired, it will be clear that, as the vectors have not only magnitude but also direction the nature of the product becomes a matter for definition. It is found that there are two types of product to which definite physical meaning can be given, called the *scalar product* and the *vector product* respectively, these being defined in the following way—

(i) The *scalar* or *dot product* of two vectors \mathbf{r} and \mathbf{r}' is defined as the product of their magnitudes and the cosine of the angle α between their directions, i.e.

$$\mathbf{r} \cdot \mathbf{r}' = rr' \cos \alpha.$$

The result is a scalar quantity. Note that $\mathbf{r} \cdot \mathbf{r}' = \mathbf{r}' \cdot \mathbf{r}$.

(ii) The *vector* or *cross product* of two vectors \mathbf{r} and \mathbf{r}' is defined as the product of their magnitudes and the sine of the angle α between their directions, i.e.

$$\mathbf{r} \times \mathbf{r}' = (rr' \sin \alpha)\mathbf{z}.$$

The result is a vector perpendicular both to \mathbf{r} and \mathbf{r}' , \mathbf{z} being unit vector in that direction. The sense of \mathbf{z} relative to that of the other vectors is such that when \mathbf{r} is turned to coincide with \mathbf{r}' the positive direction of \mathbf{z} is related to the rotation in the same way as the translation and rotation of an ordinary right-handed screw. Note that $\mathbf{r} \times \mathbf{r}' = -\mathbf{r}' \times \mathbf{r}$.

In order to appreciate the meaning of these definitions, consider the two products of a vector of force \mathbf{F} , with a vector of linear displacement \mathbf{d} . The dot product clearly represents mechanical work expended by the action of the force \mathbf{F} through distance \mathbf{d} . It is a scalar

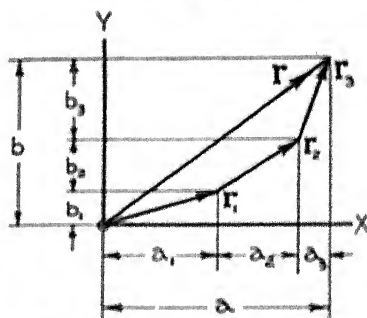
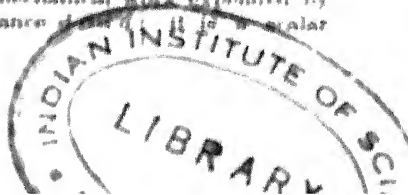


FIG. 4.—THE ADDITION OF VECTORS



quantity. On the other hand, the cross product is the couple produced by the force F acting at a lever arm $d \sin \alpha$; this is clearly a vector quantity having the direction of the axis of the couple, i.e. perpendicular both to \mathbf{F} and to \mathbf{d} .

The theory of alternating currents to be developed in a later section is directly deduced from the idea of scalar product of two vectors. In particular the scalar product of a vector \mathbf{r} with the unit vector \mathbf{x} in Fig. 3 is

$$\mathbf{r} \cdot \mathbf{x} = r \cos \phi, \quad (2)$$

a theorem of considerable utility in later work.

3. The Operator j . It is now necessary to introduce the idea of an operation into the preceding vector algebra. As a preliminary, consider the simpler conception of a scalar operator embodied, for example, in the symbol 2. According to the method of interpretation the symbol 2 can denote either (i) a magnitude of two units; or (ii) if written in the form $2(1)$ may be read to signify the arithmetic operation of doubling unit magnitude. In the latter interpretation the symbol is used as a notation for scalar operation.

When this idea is applied to a vector quantity an extension of the principle is necessary, since direction as well as magnitude must be taken into account. The result of applying a scalar operation to a vector is simply to multiply the magnitude of the vector without affecting the direction of it. Now let the symbol $j()$ denote the operation of rotating a vector through an angle $\pi/2$ without alteration to its magnitude. Then, according to this definition,*

$$\begin{aligned} j\mathbf{x} &= \mathbf{y}, \\ j(j\mathbf{x}) &= j\mathbf{y} = -\mathbf{x}, \\ j[j(j\mathbf{x})] &= j(-\mathbf{x}) = -j\mathbf{x}, \end{aligned}$$

and so on, as illustrated by Fig. 5. Hence denoting successive operations with j upon any vector operand by powers of j

$$\begin{aligned} j() &= j(), \\ j^2() &= -(), \\ j^3() &= -j(), \end{aligned}$$

and so on. From this it follows that the arithmetic relation existing between the results of successive operations with j

* The bracket following j , in which the vector operand is contained, may be omitted in cases where there can be no ambiguity concerning the operand to which j is applied.

is, when expressed by powers of j , the same as that connecting the number $\sqrt{-1}$ with its powers.* Hence in the evaluation of expressions, the arithmetic effect of operation with j can be treated as equivalent to multiplication by $\sqrt{-1}$.

It is now possible to re-write Equation (1) in the operational notation just described. Since $j\mathbf{x} = \mathbf{y}$, Equation (1) becomes

$$\mathbf{r} = (a + jb)\mathbf{x}, \quad (3)$$

illustrated by Fig. 6. The expression $(a + jb)$ is called a *complex operator*, and has the following properties when it acts on a vector operand. Referring to the above equation, and to Fig. 6, the interpretation of the equation is seen to

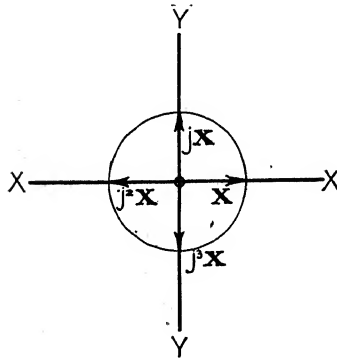


FIG. 5.—THE OPERATOR j

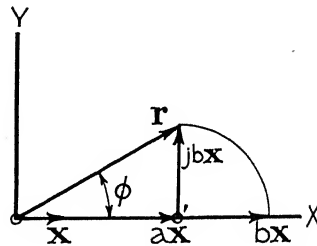


FIG. 6.—GRAPHICAL REPRESENTATION OF A VECTOR \mathbf{r} IN TERMS OF UNIT VECTOR \mathbf{x} AND THE OPERATOR j

be this; take a vector $a\mathbf{x}$ along OX and to the extremity of it attach a vector $b\mathbf{x}$. Operate with j upon the latter, i.e. turn it through $\pi/2$; then the joint effect of $a\mathbf{x}$ and jbx is a vector \mathbf{r} .

This operation may be looked at in another way: The

* This is by no means the same as the statement, so often made in text-books, that $j \equiv \sqrt{-1}$. The symbol j denotes the geometric operation of rotating a vector through $\pi/2$ radians; the quantity $\sqrt{-1}$ is an imaginary numeric. It is looseness of statement to describe these very different conceptions as identical; the real meaning of j and its relationship to $\sqrt{-1}$ is that given in the text. See T. R. Lyle, "Currents in branched and mutually influencing circuits produced by harmonically varying electromotive forces," *Electron.*, Vol. 41, pp. 816-818 (1898), and Vol. 42, pp. 72-74, 148-151 (1899).

vector \mathbf{r} has a magnitude $r = \sqrt{a^2 + b^2}$ and a direction $\phi = \tan^{-1} b/a$ relative to \mathbf{x} . Hence the effect of $(a + jb)$ is to multiply the magnitude of it by r and to rotate it through an angle ϕ . The operation consists, therefore, partly of extending or *tensor* element and partly of a rotatory or *quaternion* effect. The two equivalent ways of regarding a complex operation may be symbolically represented by

$$(a + jb) = \sqrt{a^2 + b^2} / \tan^{-1} b/a = r / \phi$$

Moreover, it is clear that the arguments just applied to operations on unit vector apply with equal force to any complex vector operand.

4. Alternating Currents as Vectors.* The current in a circuit is a physical quantity which has at every instant a definite magnitude and flows in a certain sense round the circuit; it is possible in virtue of these properties to represent it by a vector, and a similar argument will apply to the electromotive force to which the current is due.

In particular, if the current is a sinusoidal function of time, it is referred to as an *alternating current*. The simplest type of alternating current with which it is proposed exclusively to deal, can be represented as

$$i = i_1 \cos \omega t,$$

where i is the instantaneous value of the current at time t , i_1 is its maximum value, and ω is 2π times the frequency of alternation.

Now consider a vector \mathbf{i} , of magnitude i_1 rotating about

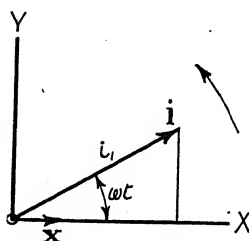


FIG. 7.—REPRESENTATION OF AN ALTERNATING CURRENT BY MEANS OF A ROTATING VECTOR

* See T. R. Lyle, *loc. cit.*; W. G. Rhodes, *loc. cit.*; W. E. Sumpner, "vector properties of alternating currents and other periodic quantities," *Proc. Roy. Soc.*, Vol. 61, pp. 465-478 (1897); C. V. Drysdale, *The Foundations of Alternate Current Theory*, Chap. XIII (1910). The symbolic treatment of alternating currents is exclusively adopted in the writings of Steinmetz; for example, *Theory and Calculation of Alternating Current Phenomena*, 4th edition (1908).

Also E. Orlich, *Kapazität und Induktivität*, Sec. 35, pp. 98-107 (1909).

point O with angular velocity ω , as shown in Fig. 7. Take the scalar product of \mathbf{i} with \mathbf{x} , as in Equation (2), then

$$\mathbf{i} \cdot \mathbf{x} = i_1 \cos \omega t = i. \quad (5)$$

Hence the harmonic vector \mathbf{i} will represent completely the current i , since the result of taking its scalar product with \mathbf{x} gives the magnitude of its OX component, i.e. the instantaneous value of the current. This provides at once a means of transforming the well-known scalar equations of the theory of alternating currents into vector notation, a proceeding which results in considerable analytical simplification and in greater clarity of statement.

In many alternating current problems the differential coefficients of i with respect to t are involved; it will now be shown how these may be expressed in symbolic notation. From Equation (5),

$$\frac{di}{dt} = \omega i_1 \cos \left(\omega t + \frac{\pi}{2} \right) = \frac{d}{dt} (\mathbf{i} \cdot \mathbf{x}) = \mathbf{x} \cdot \frac{d\mathbf{i}}{dt},$$

since \mathbf{x} is not a function of time. Now operate on Equation (5) with j , thus,

$$\mathbf{x} \cdot j\mathbf{i} = i_1 \cos \left(\omega t + \frac{\pi}{2} \right);$$

multiply by ω ,

$$\mathbf{x} \cdot j\omega\mathbf{i} = \omega i_1 \cos \left(\omega t + \frac{\pi}{2} \right).$$

From this and the above result of direct differentiation

$$\frac{di}{dt} = j\omega i.$$

In general,

$$\frac{d^n i}{dt^n} = (j\omega)^n i, \quad (5a)$$

by an obvious extension of the above process. Hence the result of successive differentiations of a harmonic vector is the same as that obtained by successive operations with $j\omega$.

As an example of the use of these principles consider the case of a coil of inductance L and resistance R to which an

alternating potential difference is applied. If e be the instantaneous value of the potential difference and i that of the current, the scalar equation for the coil is

$$e = L \frac{di}{dt} + Ri.$$

Now let \mathbf{e} be the vector of potential difference and \mathbf{i} that of current, then $e = \mathbf{e} \cdot \mathbf{x}$ and $i = \mathbf{i} \cdot \mathbf{x}$. Making use of Equation 5 (a) the scalar equation becomes

$$\mathbf{e} \cdot \mathbf{x} = j\omega L \mathbf{i} \cdot \mathbf{x} + R \mathbf{i} \cdot \mathbf{x}$$

or simply

$$\mathbf{e} = (R + j\omega L)\mathbf{i}$$

is the vector equation.

From this example it is apparent that the rule to convert a given scalar equation, which involves harmonic functions of time, into vector notation is to replace the scalar symbols by the vector symbols and to substitute successive operations with $j\omega$ for the differential coefficients.

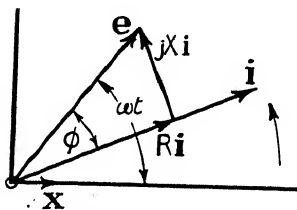


FIG. 8.—GRAPHICAL REPRESENTATION OF OHM'S LAW FOR HARMONIC VECTORS

5. The Vector Ohm's Law. The particular problem worked out above is capable of important generalization. Take the case of a circuit

having resistance R and reactance X , \mathbf{e} being the vector representing the potential difference applied to it and \mathbf{i} that denoting the current flowing into it. Then, as illustrated in Fig. 8, the vector \mathbf{e} is composed of two components, one, $R\mathbf{i}$, in phase with \mathbf{i} , and a second, $jX\mathbf{i}$, perpendicular to \mathbf{i} . The vector equation is

$$\mathbf{e} = (R + jX)\mathbf{i} = z\mathbf{i},$$

the symbol z denoting the *impedance operator* of the circuit.

Examining this equation, it will be seen that the form is the same as that representing Ohm's law for pure resistances, $e = Ri$. It may be said, therefore, to express *Ohm's law for harmonic vectors*. In virtue of this fact it appears that any result proved for direct currents will hold for alternating

currents if vector symbols are used in place of scalar symbols and impedance operators* are substituted for resistances.

6. The Operator z . The impedance operator z is a complex operator which can be expressed in another way by means of Equation (4), that is,

$$z = R + jX = \sqrt{R^2 + X^2} / \tan^{-1} \frac{X}{R} = Z / \phi$$

The quantity $Z = \sqrt{R^2 + X^2}$ is called the *impedance* of the circuit.

Making use of this result in the preceding paragraph gives

$$e = (R + jX)i = zi = (Z / \phi)i, \quad (6)$$

which shows that the vector of potential difference is obtained by multiplying the current vector by Z and advancing its phase by ϕ . Hence the vector e is Z times the size of the vector i and, when ϕ is positive, is ahead of the latter in phase; a result in agreement with the vector diagram of the circuit and concisely represented by Equation (6).

In order to carry out calculations on electric circuits in which alternating currents are flowing, it is necessary to determine z . A few simple examples will now be considered and applied to circuits commonly met with in bridge networks.

1. Pure Resistance. In this case the current and potential difference are obviously in phase, so that if R be the resistance

$$z = R = Z, \text{ and } \phi = 0.$$

2. Inductive Coil. Let L be the inductance and R the resistance of the coil, then the scalar equation is

$$e = L \frac{di}{dt} + Ri,$$

which in vector form is

$$e = (R + j\omega L)i$$

by the principles on page 32.

Thus,

$$z = R + j\omega L$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1} \omega L / R.$$

* For the general discussion of operators applied to the theory of electric circuits, see O. Heaviside, "On resistance and conductance operators, and their derivatives, inductance, and permittance, especially in connection with electric and magnetic energy," *Electrical Papers*, Vol. 2, pp. 355-374 (1892). The operators considered in this and the next paragraphs are special forms of the resistance and conductance operators of Heaviside when sinusoidal operands are assumed.

3. *Condenser.* Suppose q to be the charge in the condenser at an instant, then if C is the capacitance,

$$e = \frac{q}{C},$$

is the scalar equation.

$$\text{Differentiating, } \frac{de}{dt} = \frac{1}{C} \frac{dq}{dt} = \frac{1}{C} i.$$

In vector notation

$$j\omega e = \frac{i}{C}$$

$$e = \frac{i}{j\omega C} = \frac{-j}{\omega C} i.$$

For this case, therefore,

$$z = -\frac{j}{\omega C},$$

$$Z = \frac{1}{\omega C},$$

and

$$\phi = \tan^{-1}(-\infty) = -\frac{\pi}{2},$$

so that the potential difference lags behind the current by a quart of a period.

4. *Impedances in Series.* Consider the case of a number of impedances in series, e being the applied harmonic potential difference and the corresponding vector of current. Then

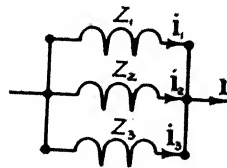


FIG. 9.—IMPEDANCES IN SERIES AND IN PARALLEL

$e = z_1 i + z_2 i + z_3 i + \dots$ etc. = $z i$ if z is the impedance operator of the circuit. From this

$z = z_1 + z_2 + z_3 + \dots$ etc., so that the operators of impedances in series are added in the same way as resistances are added in direct current circuits. (See Fig. 9.)

5. *Impedances in Parallel.* When impedances are connected in parallel, as in Fig. 9,

$$e = z_1 i_1 = z_2 i_2 = z_3 i_3 \dots \text{etc.},$$

so that $i = i_1 + i_2 + i_3 + \dots = e \left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots \right) = \frac{e}{z}$. Then,

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots$$

so that the operators of impedances in parallel are combined by the rule which applies to resistances in parallel.

By the use of the five examples worked out above, it is easy to calculate the impedance operator, and thence the impedance and phase angle, of any given circuit. In Fig. 10* a number of circuits of common occurrence in alternating

IMPEDANCE OPERATORS.

$$e = z i \quad \text{and} \quad i = e/Z,$$

$$\text{where } z = R + jX = \sqrt{R^2 + X^2} \angle \tan^{-1} \frac{X}{R} = Z \angle \phi.$$

$$\frac{1}{z} = \frac{1}{R + jX} = \frac{1}{\sqrt{R^2 + X^2}} \angle -\tan^{-1} \frac{X}{R} = \frac{1}{Z} \angle -\phi.$$

$$\text{If } e = e_m \cos \omega t, \quad i = (e_m/Z) \cos(\omega t - \phi).$$

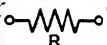

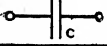


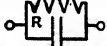
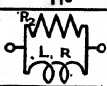

Circuit	z	$\frac{1}{z}$	Z	ϕ
	R	$\frac{1}{R}$	R	0
	$R + j\omega L$	$\frac{R - j\omega L}{R^2 + \omega^2 L^2}$	$\sqrt{R^2 + \omega^2 L^2}$	$\tan^{-1} \frac{\omega L}{R}$
	$-\frac{j}{\omega C}$	$j\omega C$	$\frac{1}{\omega C}$	$-\frac{\pi}{2}$
	$R + j(\omega L - \frac{1}{\omega C})$	$\frac{R - j(\omega L - \frac{1}{\omega C})}{R^2 + (\omega L - \frac{1}{\omega C})^2}$	$\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$	$\tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$
	$R - \frac{j}{\omega C}$	$\frac{R + \frac{j}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}}$	$\frac{1}{\omega C} \sqrt{1 + \omega^2 C^2 R^2}$	$-\tan^{-1} \frac{1}{\omega C R}$
	$\frac{R(1 - j\omega CR)}{1 + \omega^2 C^2 R^2}$	$\frac{1}{R} + j\omega C$	$\frac{R}{\sqrt{1 + \omega^2 C^2 R^2}}$	$-\tan^{-1} \omega C R$
	$\frac{R_2 \{ [R(R + R_2) + \omega^2 L^2] + j\omega L R_2 \}}{(R + R_2)^2 + \omega^2 L^2}$	$\frac{[R(R + R_2) + \omega^2 L^2] - j\omega L R_2}{R_2 (R^2 + \omega^2 L^2)}$	$R_2 \sqrt{\frac{R^2 + \omega^2 L^2}{(R + R_2)^2 + \omega^2 L^2}}$	$\tan^{-1} \frac{\omega L R_2}{R(R + R_2) + \omega^2 L^2}$
	$\frac{R + j\omega L [(1 - \omega^2 CL) - CR^2]}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}$	$\frac{R - j\omega L [(1 - \omega^2 CL) - CR^2]}{(R^2 + \omega^2 L^2)}$	$\sqrt{\frac{R^2 + \omega^2 L^2}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}}$	$\tan^{-1} \frac{\omega L [(1 - \omega^2 CL) - CR^2]}{R}$

FIG. 10.—TABLE OF IMPEDANCE AND ADMITTANCE OPERATORS

current bridge networks are shown, together with the appropriate values of z , Z , and ϕ . Frequent reference will be made in subsequent chapters to the results tabulated in this diagram.

7. The Operator $\frac{1}{z}$ In alternating current calculations it is usually required to find the current in terms of the

* The student to whom the subject is new is advised to verify as an exercise the results tabulated in this diagram.

applied potential difference. Transposing the vector equation gives

$$\mathbf{i} = \frac{1}{z} \mathbf{e},$$

so that it is necessary to interpret the operation $\frac{1}{z}$, called *admittance operator*.

Now,

$$\frac{1}{z} = \frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 + X^2}.$$

Further, $R - jX = \sqrt{R^2 + X^2} \angle \tan^{-1} \frac{X}{R} = Z \angle -\phi,$

so that $\frac{1}{z} = \frac{1}{Z} \angle -\phi,$

and $\mathbf{i} = \frac{1}{z} \mathbf{e} = \left(\frac{1}{Z} \angle -\phi \right) \mathbf{e} \quad . \quad . \quad .$

The quantity $\frac{1}{Z}$ is called the *admittance* of the circuit.

Equation (6a) shows that the current vector is obtained from the potential difference vector by multiplying the latter by the admittance and retarding its phase by an angle ϕ . For example, suppose $e = e_1 \cos \omega t = \mathbf{e} \cdot \mathbf{x}$ as shown in Fig. 6a, then, from Equation (6a),

$$\mathbf{i} \cdot \mathbf{x} = \left(\frac{1}{Z} \angle -\phi \right) \mathbf{e} \cdot \mathbf{x}$$

which is $i = \left(\frac{1}{Z} \angle -\phi \right) e_1 \cos \omega t = \frac{e_1}{Z} \cos(\omega t - \phi),$

in agreement with the geometry of the diagram.

8. The Impedance of Parallel Wires. In the circuit contemplated on page 35 the resistances, inductances, and capacitances involved are supposed to be localized between definite points in the networks. However, there are many cases in practice where this supposition is not strictly applicable, a most important example being a circuit composed of two similar parallel wires joined together at one end. It will be clear that the inductance and capacitance of such a pair of wires is uniformly distributed along their length, and can be correctly represented as localized between any pair of points chosen along the wires. Such an arrangement of the

wires is very often used in alternating bridge practice as a standard resistance (see p. 81), and the results now to be obtained can also be taken as a first approximation to the impedance operator of an ordinary "bifilar" resistance coil (see p. 67).

Two similar, parallel wires of equal length l , connected together at one end and supplied with a sinusoidal voltage e_0 at the other, can be treated analytically in the same way as a transmission line* short-circuited at the distant end. Let ρ , λ , and κ be the resistance, inductance, and capacitance of the wires per unit length. Then, at a distance x from the terminals (Fig. 11), where the voltage between the wires is e and the current i , the equations of equilibrium are

$$-(\rho + j\omega\lambda)dx \cdot i = de \text{ and } -j\omega\kappa dx \cdot e = di;$$

$$\text{or } \frac{de}{dx} = -(\rho + j\omega\lambda)i \text{ and } \frac{di}{dx} = -j\omega\kappa e.$$

Eliminating i ,

$$\frac{d^2e}{dx^2} = j\omega\kappa(\rho + j\omega\lambda)e = a^2e,$$

whence

$$e = A \sinh ax + B \cosh ax,$$

where A and B are constant vectors to be determined as follows—

Now when $x = l$, $e = 0$, so that $B = -A \tanh al$ and

$$e = A(\sinh ax - \tanh al \cosh ax).$$

Also when $x = 0$, $e = e_0$ and so $e_0 = -A \tanh al$.

From the first equilibrium equation,

$$i = -\frac{1}{(\rho + j\omega\lambda)} \frac{de}{dx} = -\frac{Aa}{(\rho + j\omega\lambda)} (\cosh ax - \tanh al \sinh ax).$$

At the terminals $x = 0$ and $i = i_0$, or $i_0 = -\frac{Aa}{(\rho + j\omega\lambda)}$. The impedance operator is then

$$z = \frac{(\rho + j\omega\lambda)}{a} \tanh al = \sqrt{\frac{\rho + j\omega\lambda}{j\omega\kappa}} \cdot \tanh \sqrt{j\omega\kappa(\rho + j\omega\lambda)} l.$$

* For a full treatment of the transmission problem, see J. A. Fleming, *The Propagation of Electric Currents in Telephone and Telegraph Conductors*, p. 68, pp. 84–85 (1911). See also E. Orlich, *Kapazität und Induktivität*, Sec. 41, pp. 126–133 (1909).

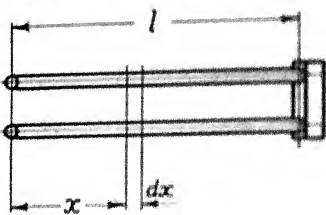


FIG. 11. ILLUSTRATING THE IMPEDANCE OF PARALLEL WIRES

Let R , L , C_1 be the total distributed resistance, inductance, and capacitance of the wires, then

$$z = \sqrt{\frac{R + j\omega L}{j\omega C_1}} \cdot \tanh \sqrt{j\omega C_1(R + j\omega L)}.$$

In resistances constructed in this manner, L and C_1 are both small quantities, such that their powers and products can be neglected when compared with unity. Expanding the hyperbolic tangent in series* gives

$$z = (R + j\omega L) \left[1 - \frac{1}{3} \{j\omega C_1(R + j\omega L)\} + \frac{2}{15} \{j\omega C_1(R + j\omega L)\}^2 \dots \right],$$

or
$$z = R + j\omega \left(L - \frac{C_1 R^2}{3} \right),$$

a result whose practical importance is discussed on pages 67 and 82.

9. Maxwell's Theory of Networks. In the preceding sections attention has been confined to the consideration of the current flowing in a single conductor. It is now necessary to develop some general principles which will enable the currents in a network of conductors to be calculated. For this purpose let an examination be made of a network composed of resistances carrying direct currents; the results obtained can then be readily generalized, so that they are applicable to a network of impedances in which alternating currents are flowing.

Let Fig. 12 represent a network of resistances supplied with current from a battery of electromotive force E . In this particular example 6 conductors join 4 points in such a way that 3 closed meshes are formed, 3 conductors meeting at each point. In general, if there are c conductors arranged to form a network by joining p points, the number of meshes will be $c - (p - 1)$. Each of the conductors will carry a definite current, as indicated by the arrows drawn on the resistances in the diagram, the magnitude of which can be found by the following procedure. Apply Ohm's law to each mesh in turn,

* See G. Greenhill, *Differential and Integral Calculus*, 2nd edition, pp. 238-39, p. 233, ex. (i), and p. 234, ex. (viii) (1891).

writing down $c - p + 1$ equations connecting the total resistance drop round each mesh with the electromotive force of any battery included in it. Then to $p - 1$ of the points of junction apply Kirchhoff's first rule, making the current reaching a point equal in magnitude and opposition in sign to

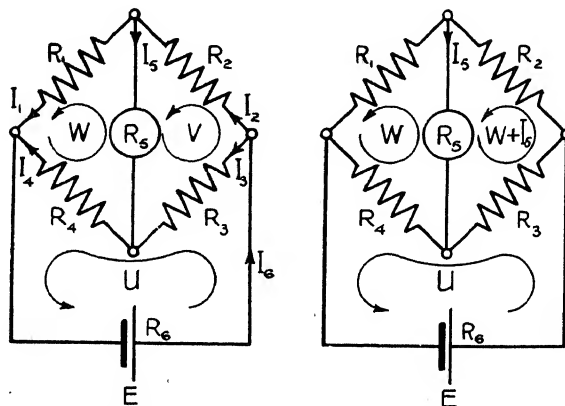


FIG. 12.—MAXWELL'S THEORY OF CYCLIC CURRENTS IN NETWORKS

the sum of all currents leaving the point; an additional $p - 1$ equations will thus be obtained. From the total c equations the current in any conductor can be found by elimination of all currents except the one desired, making use of the ordinary algebraic processes. In the example illustrated in Fig. 12, application of Ohm's law to the three meshes gives

$$E = R_6 I_6 + R_3 I_3 + R_4 I_4,$$

$$0 = R_2 I_2 + R_5 I_5 - R_3 I_3,$$

$$0 = R_1 I_1 - R_4 I_4 - R_5 I_5;$$

using Kirchhoff's first rule at three branch-points gives

$$0 = I_6 - I_3 - I_2,$$

$$0 = I_2 - I_5 - I_1,$$

$$0 = I_1 - I_6 + I_4.$$

Hence, sufficient equations to determine any one of the c currents can be found by taking the $c - p + 1$ mesh equations and only $p - 1$ equations from the points of junction.

The reader will appreciate that, although the method just

described is direct and obvious, the algebra involved in solving the c equations for any given current will be long and tedious, especially if the network be composed of a large number of conductors. Much of the complexity may be avoided by making use of Maxwell's* artifice of hypothetical *cyclic currents*. In this, each mesh is supposed to carry a current of different strength, these cyclic currents being independent and having the same direction of circulation. The current in any conductor will then be equal to the difference between the cyclic currents in the two meshes, of which the conductor forms the common side. For example, in Fig. 12 the circulating currents in the meshes are u , v , and w ; the current in a conductor such as R_5 will be $I_5 = v - w$, by Maxwell's artifice. All that is necessary is now to make a circuit of each mesh in turn and to apply Ohm's law to it. There will then be as many equations as there are meshes or cyclic currents, namely, $c - (p - 1)$, which will be sufficient to determine the cyclic currents completely. Then, by taking the difference between appropriate pairs of mesh currents the true current in any conductor will be found. It is clear that the algebra involved in this method of calculation is of a more modest nature, since the number of equations to be solved is reduced from c to $c - (p - 1)$.

In the example shown in Fig. 12, the application of Maxwell's artifice reduces the number of independent equations from six to three, as follows—

$$\begin{aligned}(R_3 + R_4 + R_6)u - R_3v - R_4w &= E, \\ -R_3u + (R_2 + R_3 + R_5)v - R_5w &= 0, \\ -R_4u - R_5v + (R_1 + R_4 + R_5)w &= 0.\end{aligned}$$

It is now easy to solve for u , v , and w and thence to find any desired current from the differences between appropriate pairs of them.

It is usually desired to find the current in one particular conductor of the network, such as R_5 in Fig. 12. The algebra can be further simplified by writing for the two cyclic currents bounded by R_5 , w and $w + I_5$ respectively; I_5 is the true current in the resistance R_5 . The necessity for solving the above equations for w and v and the ultimate subtraction of

* J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 1, 3rd edition; Sec. 282b, pp. 406-7; Sec. 347, pp. 475-7 (1892).

them is avoided, I_5 being found directly. Applying this simplification gives, on substitution of $v = w + I_5$,

$$\begin{aligned} -R_3 I_5 - (R_3 + R_4)w + (R_3 + R_4 + R_6)u &= E, \\ (R_2 + R_3 + R_5)I_5 + (R_2 + R_3)w - R_3 u &= 0, \\ -R_5 I_5 + (R_1 + R_4)w - R_4 u &= 0; \end{aligned}$$

solving these equations for I_5 by the usual method of determinants,*

$$I_5 = \frac{\begin{vmatrix} E & -(R_3 + R_4) & (R_3 + R_4 + R_6) \\ 0 & (R_2 + R_3) & -R_3 \\ 0 & (R_1 + R_4) & -R_4 \end{vmatrix}}{\begin{vmatrix} -R_3 & -(R_3 + R_4) & (R_3 + R_4 + R_6) \\ (R_2 + R_3 + R_5) & (R_2 + R_3) & -R_3 \\ -R_5 & (R_1 + R_4) & -R_4 \end{vmatrix}}$$

Evaluating the numerator, and writing the symbol Δ for the denominator gives

$$I_5 = \frac{(R_1 R_3 - R_2 R_4)}{\Delta} E. \quad (7)$$

The network of resistances drawn in Fig. 12 is the much used Wheatstone bridge, R_5 including a galvanometer. When the resistances of the four balancing branches, R_1 , R_2 , R_3 , and R_4 are arranged to make $I_5 = 0$, the galvanometer remains undeflected and the bridge is said to be *balanced*. From Equation (7) the condition for balance is seen to be

$$R_1 R_3 = R_2 R_4.$$

The principles which have been deduced in this section and applied to a typical network were, as pointed out earlier, originally due to Maxwell. Professor J. A. Fleming† has developed them in some detail in an important paper to which the reader is referred for further information; sufficient has been said, however, to enable the balance conditions of a network to be deduced and to allow of the determination of its sensitivity as a method of measurement.

10. Application of Maxwell's Theory of Networks to Alternating Currents.

It has been shown on page 32 that Ohm's

* See H. S. Hall and S. R. Knight, *Higher Algebra*, 4th edition, Chap. 33, pp. 409-428 (1903).

† J. A. Fleming, "Problems on the distribution of electric currents in networks of conductors treated by the method of Maxwell," *Proc. Phys. Soc.*, Vol. 7, pp. 215-255 (1886). See also O. Heaviside, *Electrical Papers*, Vol. 1, pp. 412-415 (1892).

law for direct currents has an exact analogue in the theory of harmonic alternating currents. It follows, therefore, that Maxwell's theory of networks outlined for direct currents in a previous paragraph may be applied to networks of impedances carrying alternating currents by putting symbols for harmonic vectors in place of the symbols for direct currents, and replacing resistances by impedance operators.

Professor Fleming, in the paper mentioned above, has treated certain cases where the current is interrupted; the application of the principle to harmonic vectors has been made by many writers, notably by Professor Lyle* and R. Appleyard.† In the succeeding sections of this chapter Maxwell's theory will be combined with the symbolic vector notation and applied to deduce the properties of various alternating current bridge networks.

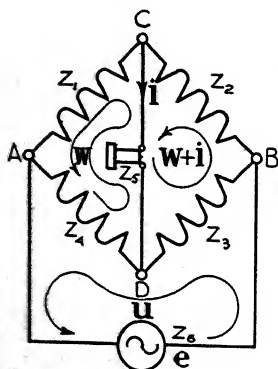


FIG. 13.—THE WHEATSTONE OR FOUR-BRANCH IMPEDANCE NETWORK

and z_4 . Let an alternator, having a sinusoidal electromotive force represented by the vector e , be applied to the points AB ; and across the points CD connect a suitable detector. Then, if z_5 be the operator for the detector, z_6 for the alternator, and u , $w + i$, w be the three mesh currents, Fig. 13 is the alternating current analogue of Fig. 12. Hence from Equation (7) the current i in the detector is given by

$$i = \frac{z_1 z_3 - z_2 z_4}{\Delta} \cdot e,$$

$$\text{where } \Delta = \begin{vmatrix} -z_3 & -(z_3 + z_4) & (z_3 + z_4 + z_6) \\ (z_2 + z_3 + z_5) & (z_2 + z_3) & -z_3 \\ -z_5 & (z_1 + z_4) & -z_4 \end{vmatrix}. \quad (8)$$

* T. R. Lyle, *loc. cit.*

† R. Appleyard, "The solution of network problems by determinants," *Proc. Phys. Soc.*, Vol. 24, pp. 201-209 (1912).

‡ T. R. Lyle, *loc. cit.*

O. Heaviside, *Electrical Papers*, Vol. 2, pp. 102-106, pp. 256-277 (1892).

If the bridge is to be balanced there must be no current in the detector at any time, i.e. $i = 0$, whence

$$z_1 z_3 = z_2 z_4,$$

is the symbolic condition for balance which must be satisfied by the branch impedance operators.

This condition can be written in a different form; remembering that $z = R + jX$ gives

$$(R_1 + jX_1)(R_3 + jX_3)\mathbf{r} = (R_2 + jX_2)(R_4 + jX_4)\mathbf{r}, \dots (8a)$$

allowing each side to operate on an arbitrary vector \mathbf{r} . Again, since $z = R + jX = Z/\phi$ the vector equation becomes

$$(Z_1/\phi_1)(Z_3/\phi_3)\mathbf{r} = (Z_2/\phi_2)(Z_4/\phi_4)\mathbf{r}.$$

Now the result of the operation Z_1/ϕ_1 on a vector is to multiply it by Z_1 and to advance its phase by ϕ_1 ; a further operation Z_3/ϕ_3 therefore multiplies the result of the first operation by Z_3 and advances the phase by a further angle ϕ_3 . The total effect is equivalent to an operation $Z_1 Z_3 / \phi_1 + \phi_3$ on the original vector. A similar argument applies to the operations on the right-hand side, so that

$$(Z_1 Z_3 / \phi_1 + \phi_3)\mathbf{r} = (Z_2 Z_4 / \phi_2 + \phi_4)\mathbf{r}. \quad (8b)$$

Now when two vectors are equal they are of equal magnitude and are coincident in phase, hence

$$Z_1 Z_3 = Z_2 Z_4$$

and

$$\phi_1 + \phi_3 = \phi_2 + \phi_4,$$

a result which has an important graphical meaning, to be noticed in the following section. The results given in these equations show that there will be *two* adjustments of the branch impedances necessary to satisfy the balance conditions, which is sufficiently obvious when it is remembered that, in dealing with the alternating quantities, phase as well as magnitude must be taken into account. The relations existing at balance between the branch constants are simply found by treating the operations in Equation (8a) algebraically, thus

$$\begin{aligned} & [(R_1 R_3 - X_1 X_3) + j(X_1 R_3 + X_3 R_1)] \mathbf{r} \\ &= [(R_2 R_4 - X_2 X_4) + j(X_2 R_4 + X_4 R_2)] \mathbf{r} \end{aligned}$$

and phase by coincident points C, D in Fig. 14. Then the remainder of the diagram is constructed by remembering that

$$\begin{aligned} e_{AC} &= e_{AD} = z_1 i_C = z_4 i_D, \\ e_{CB} &= e_{DB} = z_2 i_C = z_3 i_D, \\ e &= e_{AC} + e_{CB} = e_{AD} + e_{DB}. \end{aligned}$$

The currents are found in magnitude and phase from

$$\begin{aligned} i_C &= e/(z_1 + z_2) \\ i_D &= e/(z_3 + z_4) \end{aligned}$$

Since the vectors AC and AD are equal in magnitude and coincident in phase, and since also $CB = DB$ it follows from the geometry of the diagram that

$$\frac{Z_1}{Z_4} = \frac{Z_2}{Z_3},$$

and also $\phi_1 - \phi_4 = \phi_2 - \phi_3$;

hence the balance conditions are

$$Z_1 Z_3 = Z_2 Z_4$$

and

$$\phi_1 + \phi_3 = \phi_2 + \phi_4,$$

in agreement with page 43.

13. General Method for Deriving the Balance Condition in Bridge Networks having any number of Branches. The four-branch impedance bridge, the general theory of which has been discussed in previous paragraphs, includes a very large number of the bridge networks used in practice. There are, however, a number of bridges in common use wherein more than four balancing branches are employed. It is proposed to show in this section the simplest way in which to find the balance conditions for such networks, the process being illustrated by the case of a network with six balancing branches. The general theory of this network was first given by S. Butterworth,* and it contains as special cases many common bridges, including Anderson's well-known method. It will be referred to as the Anderson network, on this account.

When the balance condition of a bridge is required, it is not necessary to find the absolute value of i , the current in the detector, in terms of e ; it will be sufficient if its value in terms of some other current in the network be determined. Accordingly, let i be expressed in terms of the current in the alternator, u . Suppose that there are m meshes in the network. Assume

* S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75-94 (1912).

a cyclic current in each mesh, and then write down $m - 1$ equations for the potential drop round each mesh. The equation omitted is that for the mesh which contains the alternator. Transfer the terms involving \mathbf{u} to the right-hand side and solve the equations for \mathbf{i} in terms of \mathbf{u} . The solution will consist of the ratio of two determinants, so that if \mathbf{i} is to be zero, the determinant in the numerator must vanish. Now this determinant is formed of the coefficients of \mathbf{u} and those of all other cyclic currents except the coefficients of \mathbf{i} ; hence, to find the balance condition, write down this determinant and equate it to zero.

Consider first a simple case, that of the four-branch network of Fig. 13. The mesh equations for the meshes ACD and CBD are

$$\begin{aligned}(z_2 + z_3 + z_5)\mathbf{i} + (z_2 + z_3)\mathbf{w} &= z_3\mathbf{u} \\ -z_5\mathbf{i} + (z_1 + z_4)\mathbf{w} &= z_4\mathbf{u}.\end{aligned}$$

The numerator determinant in the solution for \mathbf{i} in terms of \mathbf{u} is

$$\begin{vmatrix} z_3 & z_2 + z_3 \\ z_4 & z_1 + z_4 \end{vmatrix},$$

which must vanish if \mathbf{i} is to be zero. Writing equal to zero and evaluating gives

$$z_1 z_3 = z_2 z_4$$

for balance, as proved above.

Now examine the Anderson network of Fig. 15. Omitting the equation for the alternator mesh, the equations for the three remaining meshes are—

$$\begin{aligned}-z_7\mathbf{i} + (z_3 + z_6 + z_7)\mathbf{v} - (z_6 + z_7)\mathbf{w} &= z_3\mathbf{u}, \\ -z_5\mathbf{i} - z_6\mathbf{v} + (z_1 + z_4 + z_6)\mathbf{w} &= z_4\mathbf{u}, \\ (z_2 + z_5 + z_7)\mathbf{i} - z_7\mathbf{v} + (z_2 + z_7)\mathbf{w} &= 0;\end{aligned}$$

the numerator determinant, equated to zero, is

$$\begin{vmatrix} z_3 & (z_3 + z_6 + z_7) & -(z_6 + z_7) \\ z_4 & -z_6 & (z_1 + z_4 + z_6) \\ 0 & -z_7 & (z_2 + z_7) \end{vmatrix} = 0.$$

Evaluating and collecting terms

$$z_7(z_1 z_3 - z_2 z_4) = z_2 \{ z_6(z_3 + z_4) + z_3 z_4 \}$$

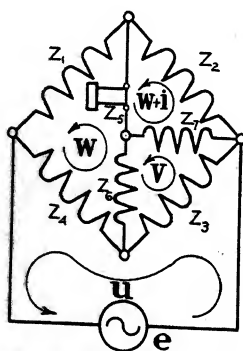


FIG. 15.—THE ANDERSON OR SIX-BRANCH IMPEDANCE NETWORK

is the condition that the Anderson network be balanced. It should be observed that when $z_6 = 0$ and $z_7 = \infty$ the network reduces to the four-branch type, the above equation becoming $z_1 z_3 = z_2 z_4$, the usual balance relation.

The Anderson network discussed above represents the most complex type of impedance bridge for which it is worth while to deduce the balance condition in general terms. There are in practical use a few bridge networks in which there are more than six balancing branches, but the balance condition for these can be readily deduced when the occasion arises by simple application of the rule described above.

Without entering into further general theory, the reader will appreciate from the preceding analysis that the impedance operators of the alternator and of the detector do not appear in the balance condition determinant, as a consideration of the symmetry of the mesh equations will show. Accordingly, the positions of the alternator and of the detector in an impedance bridge may be interchanged without affecting the condition of balance. If two networks differ only in the relative positions occupied by their alternators and detectors, the networks are said to be *conjugate*. The two networks will have the same balance conditions, but their sensitivities, i.e. the current caused to flow in the detector by a given alteration of one of the branch impedances from the balance value, may be very different. As is shown in a later section, advantage is taken of this conjugate property in arranging a network for the greatest sensitivity, as on page 54.

14. Mutual Inductance in Bridge Networks. In all the bridge networks which have been considered above, the balancing branches have consisted of independent impedances. There is, however, an important class of networks in which certain pairs of branches are arranged to react mutually on one another; an examination of the effects of mutual inductance is, therefore, necessary.

If a circuit carries a current i and M is its coefficient of mutual induction with respect to a second circuit, the electromotive force which must be applied to the latter to balance the effect of mutual inductance is $M \frac{di}{dt}$, or in vector notation, $j\omega Mi$ for sinusoidally varying quantities.

Now, although this equation represents the effect of mutual inductance in a general way, it is not sufficiently explicit as

a statement of all the physical facts, since, as is obvious, M can be a positive or a negative quantity. A convention will now be adopted to make determinate the sign of mutual induction effects.

Consider first a simple inductive coil carrying a current i , as shown in Fig. 16, an electromotive force e driving the current through the coil. Then e may be imagined to consist of two components, maintaining equilibrium with the potential drop in the resistance and with the effects of self-induction respectively. By Ohm's law, the current

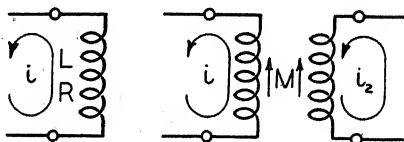


FIG. 16.—ILLUSTRATING THE RELATIVE SIGNS OF SELF AND MUTUAL INDUCTION EFFECTS

flowing in the resistance will produce a potential drop of magnitude Ri tending to oppose i ; that is, the reaction of the resistance against the current flow is $-Ri$, to balance which e must contain a term Ri in order that i be maintained. Now suppose i to be increasing, so that di/dt is positive; then by Lenz's law an electromotive force of magnitude $L di/dt$ will be induced in the circuit tending to oppose the changing current; that is, the electromotive force of self-induction is $-L di/dt$, to balance which e must provide a term $L di/dt$ in order that the current may change at the prescribed rate. Hence, for a simple inductance, the equation for e is

$$e = L \frac{di}{dt} + Ri.$$

Consider next two mutually influencing circuits, the coils of which are wound in the same direction, as in Fig. 16. Let the current in the second coil, i_2 , be increasing; then an electromotive force will be induced in it opposing i_2 , as shown by the straight arrow. Now since the first coil is wound in the same direction as the second, and as the linkages with it are increasing with i_2 , there will be an electromotive force induced in the first coil by mutual induction having precisely the same direction as the electromotive force of self-induction in the second. The magnitude of the mutual induced electromotive force is $M di_2/dt$ and if its direction coincide with that of the current i it is taken as of positive sign. Thus for the two coils illustrated the equation is

$$e = Ri + L \frac{di}{dt} + M \frac{di_2}{dt}.$$

Hence the convention* is this : (i) imagine all coils to be wound in the same direction and let them carry cyclic currents flowing in the same sense ; (ii) let the current in the inducing circuit be supposed to increase ; (iii) take the direction of the induced electromotive force in the neighbouring circuit to be the same as that of the self-induced electromotive force in the inducing circuit. Then, if the electromotive force of mutual induction in a circuit coincide in direction with the cyclic current in the circuit, M is given a positive sign.

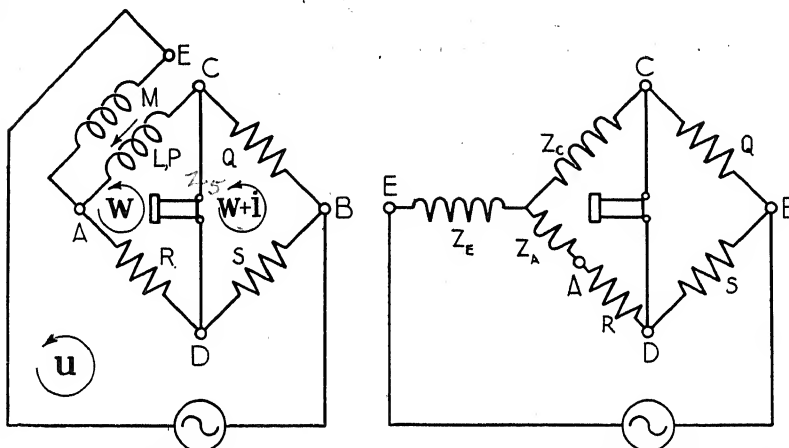


FIG. 17.—EXAMPLE OF MUTUAL INDUCTANCE IN A BRIDGE NETWORK

To show the application of this convention, consider the case of Maxwell's method for comparison of a mutual inductance and a self-inductance. The network is shown in Fig. 17, the cyclic currents being u , $w + i$, and w , i being the detector current vector ; it is required to find the condition for i to be zero. To find the mutual inductance effect in the mesh containing w due to the current u , imagine the latter to be increased ; then the direction of the mutual induced electromotive force opposes u , as shown by the straight arrow, and therefore assists w . Thus in the mesh ACD the sign of M is positive. The mesh equations for ACD and CBD are, if z_s be the detector impedance,

$$(P + R + z_s + j\omega L)w - z_s(w + i) + (j\omega M - R)u = 0,$$

$$(Q + S + z_s)(w + i) - z_s w - Su = 0 ;$$

* This convention will be found to be in agreement with Maxwell's expressions. It is necessary to adopt some such arrangement in order to secure analytical consistency ; the above is convenient in practice. See R. Appleyard *loc. cit.*

re-arranging, gives

$$\begin{aligned} -z_s i + (P + R + j\omega L)w &= (R - j\omega M)u, \\ (Q + S + z_s)i + (Q + S)w &= Su. \end{aligned}$$

By the principles on page 46, i will be zero when

$$\begin{vmatrix} R - j\omega M & P + R + j\omega L \\ S & Q + S \end{vmatrix} = 0,$$

that is, when

$$SP = QR$$

$$L = -\left(1 + \frac{Q}{S}\right)M,$$

which are the required balance conditions.

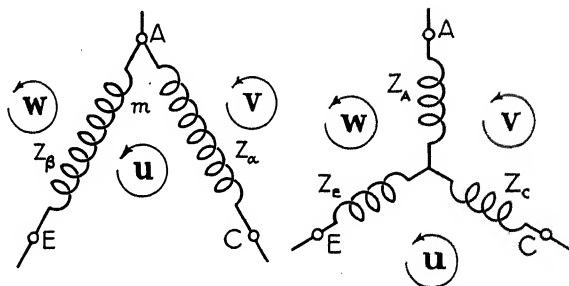


FIG. 18.—TRANSFORMATION OF TWO COILS WITH MUTUAL INDUCTANCE INTO A STAR-CONNECTED ARRANGEMENT OF THREE IMPEDANCES

15. Transformation of Networks containing Mutual Inductance. Network problems involving mutual inductance can often be much simplified by the use of certain transformations due to Professor G. A. Campbell,* and developed in England by S. Butterworth,† one of these being noted here.

Let two coils, having operators z_α and z_β form branches of a network, $m = j\omega M$ being the mutual operator between them, as shown in Fig. 18. Then, making use of the convention of the preceding paragraph, the equations for the potential differences between the points CA and EA will be

$$\begin{aligned} e_{CA} &= (z_\alpha + m)u - z_\alpha v - mw, \\ e_{EA} &= -(z_\beta + m)u + mv + z_\beta w. \end{aligned}$$

Now suppose that the two coils be removed and replaced by a

* G. A. Campbell, "Cisoidal Oscillations," *Trans. Amer. I.E.E.*, Vol. 30, Part 2, pp. 873-913 (1911).

† S. Butterworth, "Capacity and eddy current effects in inductometers," *Proc. Phys. Soc.*, Vol. 33, p. 314 (1921).

star-connected system of three impedances z_A, z_C, z_E , such that the potential differences and the currents remain unaltered. The equations now are

$$e_{CA} = z_C u - (z_A + z_C)v + z_A w,$$

$$e_{EA} = -z_E u - z_A v + (z_A + z_E)w.$$

Comparing the coefficients,

$$z_A = -m,$$

$$z_C = m + z_a,$$

$$z_E = m + z_\beta.$$

Hence a pair of mutually influencing coils can be replaced by a star-connected system of three impedances without mutual inductance.

Apply this device to the example worked out in the last section. The transformed circuit is shown in Fig. 17; $z_a = P + j\omega L$, $z_\beta =$ operator of mutual inductance secondary, $m = j\omega M$; then, from above,

$$z_A = -j\omega M.$$

$$z_C = P + j\omega(L + M).$$

$$z_E = j\omega M + z_\beta.$$

The impedance z_E is in the alternator branch; hence the transformed network is a four-branch impedance bridge with $z_1 = z_C$, $z_2 = Q$, $z_3 = S$, $z_4 = R + z_A$. For balance $z_1 z_3 = z_2 z_4$, so that

$$S\{P + j\omega(L + M)\} = Q(R - j\omega M),$$

giving

$$SP = QR$$

and

$$L = -\left(1 + \frac{Q}{S}\right)M$$

as the balance conditions, in agreement with the direct method.

16. The Generalized Wheatstone Network. By the use of the principles established above, it is possible to generalize the theory of the Wheatstone network discussed on page 42 to include the effect of mutual inductance between any pair of branches, and thereby to find the general condition for balance of a large class of bridge networks in which mutual inductance is used to attain the null condition. This generalization was first made by Heaviside* in a series of papers published in 1886-7 and will be given here with such modification as is necessary to adapt it to the methods and notation of this book.

Consider the Wheatstone network shown in Fig. 13 and assume that there is, in addition to the impedance operators

* Oliver Heaviside, *Electrical Papers*, Vol. 2, pp. 33-38, pp. 106-115, and particularly pp. 284-286; (1892).

shown in the six branches, mutual inductance between every pair of branches in the network. Let the mutual inductances be denoted by double subscripts; thus M_{12} is the mutual inductance between branches 1 and 2, M_{56} between 5 and 6, and so on. It should be remembered that $M_{12} = M_{21}$, and in general $M_{AB} = M_{BA}$. Let the symbol m_{AB} denote the mutual inductance operator $j\omega M_{AB}$; then the equations for the meshes ACD and BCD are

$$\begin{aligned} (z_2 + z_3 + z_5)\mathbf{i} + (z_2 + z_3)\mathbf{w} - z_3\mathbf{u} + [2(m_{23} + m_{25} + m_{35})\mathbf{i} \\ + (m_{12} + m_{13} - m_{15} + 2m_{23} + m_{24} + m_{25} + m_{34} + m_{35} - m_{45})\mathbf{w} \\ - (m_{23} + m_{24} - m_{26} + m_{34} + m_{35} + m_{36} - m_{45} - m_{56})\mathbf{u}] = 0, \\ -z_5\mathbf{i} + (z_1 + z_4)\mathbf{w} - z_4\mathbf{u} + [(m_{12} + m_{13} - m_{15} + m_{24} - m_{25} + m_{34} \\ - m_{35} - m_{45})\mathbf{i} \\ + (m_{12} + m_{13} + 2m_{14} + m_{15} + m_{24} - m_{25} + m_{34} - m_{35} + m_{45})\mathbf{w} \\ - (m_{13} + m_{14} - m_{16} + m_{34} - m_{35} + m_{45} + m_{46} + m_{56})\mathbf{u}] = 0. \end{aligned}$$

Collecting coefficients, the balance determinant can be written as

$$\begin{vmatrix} z_3 + \alpha & z_2 + z_3 + \beta \\ z_4 + \gamma & z_1 + z_4 + \delta \end{vmatrix} = 0,$$

where

$$\begin{aligned} \alpha &= m_{23} + m_{24} - m_{26} + m_{34} + m_{35} + m_{36} - m_{45} - m_{56} \\ \beta &= m_{12} + m_{13} - m_{15} + 2m_{23} + m_{24} + m_{25} + m_{34} + m_{35} - m_{45} \\ \gamma &= m_{13} + m_{14} - m_{16} + m_{34} - m_{35} + m_{45} + m_{46} + m_{56} \\ \delta &= m_{12} + m_{13} + 2m_{14} + m_{15} + m_{24} - m_{25} + m_{34} - m_{35} + m_{45} \end{aligned}$$

so that no current flows in the detector when

$$(z_1 z_3 - z_2 z_4) + \alpha(z_1 + z_4 + \delta) - \gamma(z_2 + z_3 + \beta) + z_3 \delta - z_4 \beta = 0.$$

As an example, consider again the network shown in Fig. 17. Put $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$; make all mutual inductance operators zero except $m_{16} = j\omega M$.

Then, $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$; so that

$$S(P + j\omega L) - QR + j\omega M(Q + S) = 0.$$

Separating the components, gives for balance,

$$SP = QR,$$

$$L = -\left(1 + \frac{Q}{S}\right)M,$$

as found in the previous paragraphs.

This theory is of extreme generality since it includes all possible bridges of the Wheatstone type in which mutual inductance effects are also present. It also covers a variety of cases in which the bridge is not, at first sight, in the Wheatstone four-branch form, but can be transformed into that form by some slight change. A large variety of examples will be found in Chapter IV to illustrate the straightforward use of the theory; it is proposed here to consider two important special applications of a less simple character.

Consider first the Campbell frequency bridge described on page 264 and illustrated in Fig. 71(a), the balance condition being there deduced from first principles. It is easy to show that this network can be readily re-drawn in such a way as to become a Wheatstone network; the reader can verify that $z_1 = R_2 + j\omega L_2$, $z_2 = \infty$, $z_3 = 0$, $z_4 = -j/\omega C$ are the branch impedance operators, and that there is then mutual inductance between z_1 and the alternator, i.e. $m_{1s} = j\omega M$. Then $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$, giving

$$\infty \left\{ \frac{j}{\omega C} + j\omega M \right\} = 0, \text{ i.e. } \omega^2 = -1/MC.$$

Fig 63(c) Karapetoff* has recently given an artifice by means of which the theory can be applied to bridges in which the sole connection with the alternator is via mutual inductance. An important case is that of Maxwell's method for the comparison of mutual inductances, p. 236, Fig. 64(e). Suppose a connection to be taken from the terminal of each primary where it joins the alternator, these connections passing to points infinitesimally close to the upper point where the telephone is attached to the network. In the limit, when the points of attachment of these leads and that of the telephone coincide, the extra connections take no current into the bridge, and mutual inductance is the only connection between bridge and alternator. In this case the network becomes a four-branch Wheatstone arrangement in which $z_1 = 0$, $z_2 = 0$, $z_3 = R_2 + j\omega L_2$, $z_4 = R_1 + j\omega L_1$, as the reader can verify. Mutual inductances $m_{s3} = j\omega M_2$ and $m_{s4} = j\omega M_1$ exist between the alternator and z_3 , z_4 , so that $\alpha = j\omega M_2$, $\beta = 0$, $\gamma = j\omega M_1$, $\delta = 0$. Then from above, $j\omega M_2(R_1 + j\omega L_1) - j\omega M_1(R_2 + j\omega L_2) = 0$ for balance, or $M_1/M_2 = L_1/L_2 = R_1/R_2$.

17. The Sensitivity of Bridge Networks. It is the object of a bridge method to measure a given quantity with the greatest precision, so that the network should be arranged for the greatest sensitiveness. With a given method, that arrangement of the various branches will be most sensitive in which, for a given deviation of adjustment from balance, the current through the detector is greatest.

* V. Karapetoff, "General equations of a balanced alternating current bridge," *Phil. Mag.*, 6th series, Vol. 44, pp. 1024-1032 (1922).

The theory of sensitiveness of bridges of the Wheatstone type used with direct current is well-known and was worked out by Schwendler* and by Heaviside.† Lord Rayleigh‡ generalized the results by the symbolic vector method to apply to Wheatstone impedance networks. From the equations on page 42 the following results can be deduced.

With a given alternator z_6 and detector z_5 to measure an impedance z_1 the symbolic conditions for sensitiveness are

$$z_3 = \sqrt{z_5 z_6}, \quad z_2 = \sqrt{\frac{z_5 z_1}{z_5 + z_1}} (z_5 + z_6), \quad z_4 = \sqrt{\frac{z_6 z_1}{z_6 + z_1}} (z_5 + z_1).$$

Hence z_3 should be chosen to have the value stated and z_1 measured approximately; then setting z_2 to the sensitivity value, final balance is made by alteration of z_4 .

When a certain network has been set up interchanging the alternator and detector may increase the sensitivity; the rule for their position is the same as for the d.c. bridge: Of the two z_5 and z_6 that which has the larger impedance should connect the junction of the two largest consecutive impedances in the bridge with the junction of the two smallest consecutive impedances.

Lord Rayleigh has pointed out that adjustment of the impedance of the alternator and detector branches may produce further increase in sensitiveness. The best alternator is that which has internal impedance equal to the external impedance across its terminals; the best detector has an impedance equal to the impedance external to its terminals. Assuming these to be adjusted by inclusion of suitable coils or condensers in series with the alternator and detector, or, alternatively, by joining source and detector to the bridge through suitable transformers (p. 144), the greatest sensitiveness occurs when

$$z_1 = z_2 = z_3 = z_4 = z_5 = z_6;$$

i.e. in an equal ratio bridge in which the source and detector have each an impedance equal to that to be measured. Thus,

* L. Schwendler "On the galvanometer resistance to be employed in testing with Wheatstone's diagram," *Phil. Mag.* 4th series, Vol. 31, pp. 364-368 (1866).

† O. Heaviside, *Electrical Papers*, Vol. 1, pp. 3-8, 8-12 (1892).

‡ Lord Rayleigh, "On the sensitiveness of the bridge method in its application to periodic electric currents," *Proc. Roy. Soc.*, Vol. 49, pp. 203-217 (1891).

when measuring low impedances, e.g. a small coil, low impedance source and detector are necessary; while with high impedances, e.g. small condensers at low frequencies, they must have high values.

In the modern bridge, certain other conditions must be fulfilled in order that the arrangement may be the most favourable to use with a tuned vibration galvanometer. The reader is referred to the papers of Butterworth* and Jaeger† for further information, and to the various remarks and references in Chapters III (pp. 161–175) and IV.

The detectors, whether vibration galvanometers or telephones, used in modern laboratory practice are usually of very high sensitiveness so that the bridge network can, in most instances, be set up with quite large deviations from the "best" conditions contemplated above. These sensitivity conditions then serve in practice to provide a criterion by which a bridge may be set up *ab initio*; rigorous observance of these requirements is generally less necessary in modern experimental work since the detectors usually employed provide a sufficient margin of sensitiveness for most purposes. For this reason the subject of sensitivity of bridge networks is not treated here in further analytical detail; special cases are considered in Chapter IV as occasion arises.

* S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75–94 (1912).

† W. Jaeger, "Günstigste Schaltung der Vibrationsgalvanometer," *Arch.f. Elekt.*, Bd. 4, pp. 262–268 (1916).

CHAPTER III

APPARATUS

1. Introduction. The reader will have gathered from the remarks which have been made in the preceding chapters that the apparatus required for the construction and use of a bridge network will consist of suitable standards with which to construct the branches, of a means of supplying the network with alternating current at a definite frequency, and of a detecting instrument to indicate when balance is attained. More specifically, the apparatus required for the purpose of making alternating current bridge measurements will be classified under the following three headings—

1. Standards of Resistance, Self and Mutual Inductance, and Capacitance.
2. Sources of Alternating Current.
3. Detectors.

STANDARDS OF RESISTANCE

2. General Considerations. A perfect resistance for use with alternating current should be absolutely non-reactive and should offer the same resistance at all frequencies. This ideal is never attained in practice, owing to the effects of inductance and capacitance in the windings, and to the influence of certain secondary phenomena which accompany the use of alternating currents. It is possible, however, to construct a resistance in which these disturbing effects can be reduced to a minimum, so that the imperfections to which they give rise can be made of negligible importance and the ideal very nearly approached.

The various factors which have to be considered in the design of a non-reactive resistance winding are : (i) the inductance and capacitance of the coils ; (ii) eddy currents ; (iii) permanence of value and temperature influences ; (iv) effects arising from the grouping of resistance coils to form " plug " or " dial " boxes. Throughout the following discussion the word " coil " does not necessarily mean that the resistance wire is wound in a spiral upon a circular bobbin, but is

intended to signify any convenient way in which the wire is arranged in a compact form.

3. Residual Inductance and Capacitance in Resistances.

The inductance of a coil can be reduced to a minimum by arranging the windings so that they have the least possible magnetic effect. This can be attained by winding the coil in such a way that portions of it which lie close to one another carry equal currents in opposite directions; the resultant magnetic field of the coil is thereby made very small.

The electrostatic capacity of the coil, and the effects due to it, can be minimized by subdivision of the winding in such a way that the neighbouring portions of the coil possess only a very small capacity and have small differences of potential between them.

However the process of winding be carried out, there will always be a small residual magnetic effect, since the inductance can only be made zero by arranging the "go" and "return" portions of the winding to be in absolute coincidence. In a similar manner, since the portions of the coil must be very close in order to minimize the inductance, there will necessarily be a residue of capacitance. It is the object of a satisfactory method of winding resistance coils to reduce the residual inductance and capacitance as far as possible, and it will be clear that such a method will probably involve a compromise between the effects of residual inductance, on the one hand, and the influence of residual capacitance on the other.

The residuals of a resistance coil can be dealt with in three different ways, viz.: (i) the inductance and the capacitance can be independently reduced to be negligibly small; (ii) the effects of inductance can be balanced against those of capacitance; (iii) the resistance can be made in such a way that the residuals may be calculated from its dimensions. The first two methods are used in the construction of coils for use in resistance boxes; the third method is applied in work of the highest precision where a knowledge of the residuals is necessary.

4. Methods of Constructing Resistances with Small Residuals.

In the last section it has been shown that the principle underlying all methods for the reduction of the residual effects in a resistance, is to subdivide the winding in such a way that neighbouring parts of it have small electrostatic capacity and only slight differences of potential between them. At the

same time, the winding must be so distributed that its total magnetic effect is very small. It is generally easy to secure the latter result, but it is by no means so simple to remove the effects of capacity.

In order to appreciate the principle upon which all methods of reducing self-capacitance effects are based, consider the case of a "non-inductive" resistance composed of two parallel portions, OA , AB , of a wire folded back on itself at its mid-point A . Let a sinusoidal potential difference of E volts be maintained on the terminals O , B , Fig. 19. Assume first that

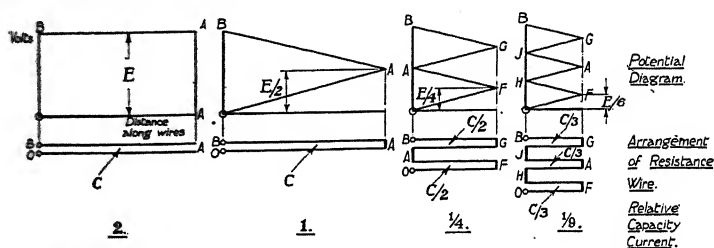


FIG. 19.—ILLUSTRATING THE REDUCTION OF CAPACITANCE EFFECTS IN NON-INDUCTIVE RESISTANCES

the wires are separated at A and that their distributed capacitance may be represented by C ; then the potential of OA being zero and of BA being E volts, a definite displacement current will flow in the intervening dielectric. Now let the wires be joined at A ; a current will flow between O and B along the wires, producing a uniform fall of potential down them. The average potential difference acting on the distributed capacitance is, therefore, halved; so that the capacity current is also halved.

Again, let each of the original portions OA , BA be folded back upon themselves, the two loops so formed being connected in series. Then, since the loops are half the length of the original loop, the capacitance of each will be approximately $C/2$. The average potential difference across one loop is now $E/4$, so that the capacity current is reduced to one-eighth of the value first obtained. By a similar process of reasoning, subdivision of the wire into three equal loops will reduce the capacity effect to one-eighteenth of the original value, and so on.

In general, subdividing a given length of wire into n non-inductive sections will give a capacitance effect approximately $1/n^2$ as great as that obtained if the wire were arranged in one non-inductive winding.

To a first approximation the capacitance effect in a coil of resistance R and inductance L can be represented by a condenser of capacitance C connected in parallel with the coil; this condenser is referred to as the self-capacitance of the coil. Then, as stated in Fig. 10, the impedance operator for such an arrangement is

$$z = \frac{R + j\omega [L(1 - \omega^2 CL) - CR^2]}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}.$$

Now in a resistance coil L and C are small quantities, so that to a first approximation the effective resistance and inductance of the coil become

$$R' \doteq R\{1 + \omega^2 C(2L - CR^2)\}$$

and

$$L' \doteq L - CR^2.$$

The phase-displacement between the voltage applied to the coil and the current flowing into it is

$$\phi \doteq \tan^{-1} \frac{\omega(L - CR^2)}{R} \doteq \tan^{-1}(\omega L'/R)$$

to the same order of approximation.

The quantity L' is the effective residual inductance of the coil, or, simply, its *residual*. If the coil be such that the effects of inductance preponderate over those of capacitance ($L > CR^2$), L' is positive. If, on the other hand, the capacitive residual is the greater, then CR^2 exceeds L and L' becomes negative. The time-constant of the coil is

$$T = \frac{L}{R} - CR = L'/R$$

which is positive or negative, according as $L \gtrless CR^2$, and is of the order 10^{-7} second or less in a well-designed resistance.

SIMPLE WINDINGS. A simple coil in which wire is wound upon a bobbin has, in general, a small self-capacity effect, since the potential difference between neighbouring turns is only a small fraction of that applied to the coil. The inductance of such a coil is, however, considerable. To reduce it, a coil can be wound as shown in Fig. 20 (a), in which the first few turns wound in one direction are followed by an equal number of turns wound in the opposite direction of rotation, and so on until the end of the coil is reached. Curtis and Grover* have shown that

* H. L. Curtis and F. W. Grover, "Resistance coils for alternating current work," *Bull. Bur. Stds.*, Vol. 8, pp. 495-517 (1913).

this method is very suitable for high resistance* coils, if the direction of winding be reversed sufficiently frequently to keep the residual inductance small. Curtis and Grover's winding is made in the following way: a cylinder of biscuit porcelain, i.e. baked, but unglazed, so that it can be cut by a steel tool, 2.7 cm. diameter and 15 cm. long, is prepared and slit through diametrically for about two-thirds of its length. Unslotted portions are left at each end for strength, and the cylinder is

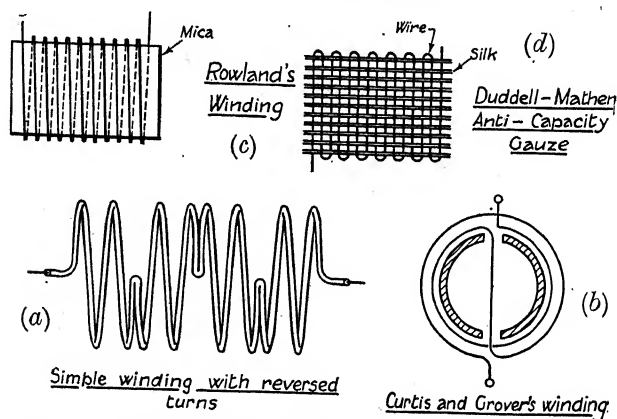


FIG. 20.—SIMPLE WINDINGS WITH LOW TIME-CONSTANTS

then re-baked to harden it. Winding is carried out by putting on one turn, passing the wire across the slit and then winding a turn in the reverse direction, and so on, as shown diagrammatically in Fig. 20 (b). The winding is thus reversed at every turn and it is found that a 10,000 ohm coil can be made in this manner to have a time-constant of about 10^{-8} second. The coil is a little difficult to wind, but the good result repays the trouble involved.

* It is convenient to refer here to two other ways of constructing very high resistances with negligible residuals which have had some application in bridge work. Kundt's resistances consist of a tube of porcelain, upon which a very thin layer of platinum glaze is deposited, the glaze being divided into two spiral strips joined at one end of the tube. Values of about 10^6 ohms can be secured, but the resistances have a large temperature coefficient. Recently, H. Schering and R. Schmidt, "Die Messung des Phasenwinkels grosser Drahtwiderstände durch Vergleich mit Widerständen aus Mannit-Borsäure-Lösung," *Arch. f. Elekt.*, Bd. 1, pp. 421-432 (1913), have used as a high resistance a tube filled with an aqueous solution of mannitol ($C_6H_8(OH)_6$) and other salts.

Another way to reduce inductance of a simple coil is to diminish the cross-sectional area of the bobbin upon which the wire is wound. This is, in effect, the method devised by Rowland and Penniman,* who have described a simple and effective winding, which has been much used for wattmeter potential circuit resistances and to some extent for bridge work. Fine resistance wire is wound upon a thin sheet of mica in the form of an inductive coil; then since the cross-section of the sheet is not great, the inductance of the winding is small. The inductance can be further reduced by the artifice of winding a few turns successively in opposite directions. The time-constant of such resistance cards lies between 10^{-6} and 10^{-7} second. Care must be exercised in arranging the cards to form resistance boxes, since the sheets have considerable area and the capacitance between adjacent cards may have an influence on the time-constant (see Fig. 20 (c)).

The same result is attained by the anti-capacity gauze of Duddell and Mather.† A fabric or ribbon is woven in which the warp consists of silk or cotton threads and the weft is the resistance wire. A piece of the material 90 mm. wide and 1 metre long, woven to have a resistance of 3,300 ohms, has an effective inductance of 0.08 millihenry and a time-constant of 2.4×10^{-8} second. Such material is very suitable for the construction of ratio boxes since the inductance and resistance of the ribbon are each proportional to its length‡ (see Fig. 20 (d)).

BIFILAR WINDINGS. One of the oldest and simplest ways of reducing the inductance of a coil is the method of bifilar winding. In this a length of wire is taken and folded back upon itself at its middle point. The bifilar conductor so formed is then wound upon a cylindrical bobbin to produce a spiral coil,§ as shown in Fig. 21 (a). Then, since two wires carrying current in opposite directions lie side by side, the inductance of the coil will be very small, and can be still further reduced by twisting the bifilar leads together before winding the coil.

A simple bifilar winding will, however, possess considerable capacity, since the "go" and "return" wires lie close together and have the full potential difference between their terminals. It will be clear that in a low resistance coil, in which there is a small quantity of wire, the capacity effect will be small

* H. A. Rowland and T. D. Penniman, "Electrical Measurements," *Amer. J. Sc.*, 4th series, Vol. 8, pp. 35-57 (1899).

† W. Duddell and T. Mather, "Improvements in non-inductive resistances," *British Patent*, No. 5,171 (1901). See also W. E. Ayrton, "An improved method of covering wire for electrical purposes, and in the orderly arrangement of multiple electrical conductors," *British Patent*, No. 785 (1881). For another variant see also "Induktion und Kapazitätsfreier Widerstand mit Kreuzwicklung," *Elekt. Zeits.*, 33 Jahrgang, p. 721 (1912).

‡ In connection with the importance of this fact, see p. 251.

§ The individual turns of this and other coils shown in Figs. 20, 21, and 22 are shown opened out so that the manner of winding may be displayed.

compared with that of the residual inductance. In a high resistance coil, on the other hand, the capacity will preponderate over the inductance since such a coil will contain a great length of fine wire. Indeed, so important does the capacity effect become in high resistance bifilar coils that it

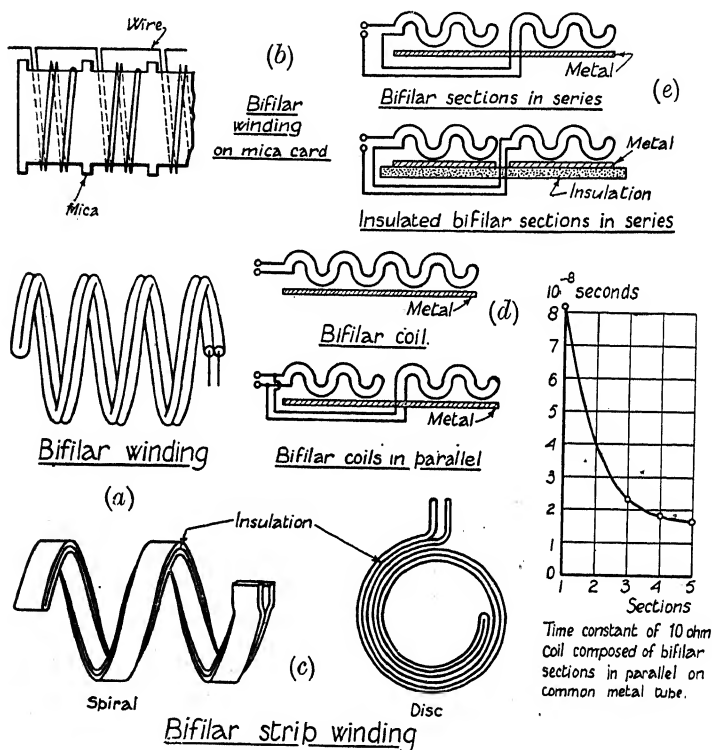


FIG. 21.—BIFILAR RESISTANCES

was noticed at a very early date by Kohlrausch* in the course of his work on the resistance of electrolytes. In his later work he avoided the use of coils having a resistance greater than 2,000 ohms. More recently, Taylor and Williams† have

* F. Kohlrausch, *Ann. der Phys.* Bd. 138, pp. 280-298, 370-390 (1869), and "Ueber das Leitungsvermögen einiger Electrolyte in äusserst verdünnter wässriger Lösung," *Ann. der Phys.*, Bd. 26, pp. 161-226 (1885).

† A. H. Taylor and E. H. Williams, "Distributed capacity in resistance boxes," *Phys. Rev.*, Vol. 26, pp. 417-423 (1908).

endeavoured to reduce the capacity by spacing out the turns of a 1,000 ohm coil wound with twisted bifilar strand; but the more precise experiments of Brown* have shown that such a coil is not so free from capacity as Taylor and Williams were led to suppose.

A type of bifilar winding very frequently employed is shown in Fig. 21 (b). A length of wire is taken and a number of bifilar loops formed from it. The loops are then wound upon a sheet of mica, the parts of the wire forming each loop being frequently twisted together before winding. The proximity of currents flowing in opposite directions in the loops, and the small section of the mica sheet, greatly reduces the inductance. Division of the wire into a number of loops in series diminishes the capacitance.

It has been pointed out above that in a low resistance coil, where the amount of wire is small, the capacity effect in a bifilar winding is negligible in comparison with the inductance effect. The bifilar winding is, therefore, much used in the construction of resistances of 10 ohms or less. In order to reduce the residual inductance as far as possible, the coils are wound with manganin strip so that the current-carrying conductors lie as close as is feasible. A suitable length of the material is prepared and folded upon itself at the middle of its length with a layer of silk tape or mica between the halves. The composite conductor is then wound upon a bobbin to form a spiral or a disc coil, Fig. 21 (c). The capacity introduced by putting the halves of the strip close together helps to compensate the inductance and to reduce the time-constant.

Low resistance coils may also be made by connecting in parallel a number of high resistance, bifilar wound sections. Fig. 21 (d)† shows the result of the application of this principle to a 10 ohm resistance. In this diagram the effect of the number of sections connected in parallel upon the value of the time-constant is clearly shown. The greater the number of sections in parallel, the smaller will the total inductance become; the self-capacities of the sections will be added, so

* S. L. Brown, "Distributed capacity in resistance boxes," *Phys. Rev.*, Vol. 27, pp. 511-514 (1908).

† From results given by K. W. Wagner and A. Wertheimer, "Über Präzisionswiderstände für hochfrequenten Wechselstrom," *Elekt. Zeits.*, 34 Jahrgang, pp. 613-616, 649-652 (1913). For further information on bifilar coils, see W. Hüter, "Kapazitätsmessungen an Spulen," *Ann. der Phys.*, Bd. 39, pp. 1350-1380 (1912).

that for the whole coil the capacity is increased. Hence, in the time-constant $T = \frac{L}{R} - CR$, L is reduced and C increased, so that T will diminish and may ultimately become negative. For example, the diagram shows that the time-constant of a 10 ohm coil wound in one bifilar section is reduced to one-fifth by making the coil of 5 sections of 50 ohms in parallel.

Coils of higher resistance, say up to 1,000 ohms, can be made by connecting bifilar sections in series (see Fig. 21 (e)). The capacity of the whole is thereby reduced, since the self capacities of the sections are in series. It is frequently of importance to wind each section upon a separate metal tube, so that capacity between the windings and the bobbin is made smaller than would be the case were the sections wound on a common tube.

CHAPERON WINDINGS. Perhaps the most important method of overcoming the defects of a bifilar coil is that introduced by Chaperon.* A coil is made by winding with single wire in an even number of layers. The first layer is wound to the end of the bobbin, the second layer being wound back to the starting point in the reversed direction, and so on. Thus, two wires carrying current in opposite directions lie over one another, thereby reducing the inductance while, at the same time, the capacity effect is small, since the potential difference between neighbouring wires is slight. The winding is shown diagrammatically in Fig. 22.

Wagner and Wertheimer† have made an important investigation of Chaperon's winding, and have introduced certain improvements in the design of resistances. They have shown that considerable control can be exercised over the time-constant of a coil by arranging the winding in the form of a number of Chaperon sections connected in series. It is usual

* G. Chaperon, "Sur l'enroulement des bobines de résistance destinées aux mesures par les courants alternatifs," *Comptes Rendus*, tome 108, pp. 799-801 (1889); also "Mesure des résistances polarisables par les courants alternatifs et le téléphone," *J. de Phys.*, tome 9, 2me. série, pp. 481-484 (1890). Also see J. Cauro, "Sur la capacité électrostatique des bobines, et son influence dans la mesure des coefficients d'induction par le pont de Wheatstone," *Comptes Rendus*, tome 120, pp. 308-311 (1895), for a slight improvement on the Chaperon winding; also see Fig. 22.

† See *loc. cit.*; also W. Hüter, "Der Phasenfaktor von Rheostatenwiderständen mittlerer Grösse," *Ann. der Phys.*, Bd. 40, pp. 381-386 (1913), for tests on Chaperon coils of 20 to 500 ohms.

to wind these sections upon a metal tube, in order that the heat liberated from the coil when it is carrying current may be conducted away. These experimenters have shown that the additional capacity effect between the windings and the common metal tube may have a considerable effect upon the time-constant, and have proposed a winding (Fig. 22) in which

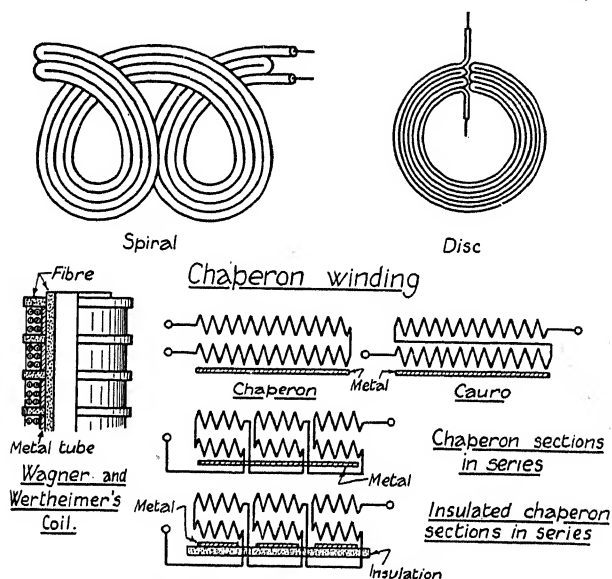


FIG. 22.—THE CHAPERON WINDING AND ITS MODIFICATIONS

the sections are wound upon separate insulated metal tubes. The effects of the subdivision of a coil into a number of Chaperon-wound sections, both on common and on separate tubes, is clearly shown in Fig. 23. The influence of the method of winding coils of various resistances is also illustrated in Fig. 23, Chaperon's, Wagner and Wertheimer's, and the bifilar method being compared.

5. Methods of Constructing Resistances with Balanced Residuals. In the methods just described the principle involved is the reduction of the residual inductance and capacitance so that both are very small quantities, the time-constant being thereby made very low. It is possible, however, to construct a coil of approximately zero time-constant by arranging the winding in such a way that the inductance

effect and the capacity effect balance one another. Thus, Mather and Sumpner* have described coils wound with doubled

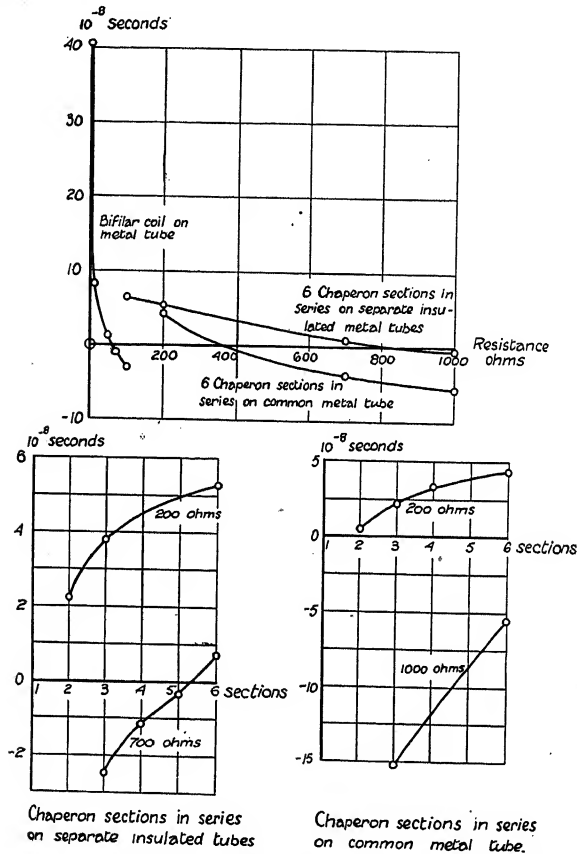


FIG. 23.—COMPARISON OF THE TIME-CONSTANTS OF RESISTANCES WOUND IN VARIOUS WAYS

strip between which a layer of insulating material of high dielectric constant is placed. By adjusting the thickness of the insulating layer, or by opening out the strips at the terminals, the necessary compensation can be effected. In

* T. Mather and W. E. Sumpner, "Improvements in constructing or winding coils for electrical apparatus," *British Patent*, No. 13,154 (1889).

many of the methods described in the preceding section, e.g. resistance gauze, compensation may also be arranged.

The theory underlying compensated resistances has been given by G. A. Campbell* and others. A resistance coil can be represented as a resistance R in series with inductance L , the capacitance being equivalent to a condenser C shunting the whole. Referring to page 59, it will be seen that it is not possible to make a resistance which shall be invariable with frequency and at the same time have zero phase-angle unless L and C be simultaneously zero. Assuming that ϕ is to be made zero, one must have

$$L = CR^2$$

which makes $R' = R[1 + \omega^2 C^2 R^2]$.

Since C is to be made small, $R' = R$, and the change of resistance with frequency will be negligible.

In the case of a bifilar coil or of a parallel wire resistance, the inductance and capacitance are distributed along the wires. Let R , L , and C_1 be the total distributed resistance, inductance, and capacitance; then the effective resistance and inductance of a bifilar resistance are

$$R' = R \left[1 + \omega^2 C_1 \left(\frac{1}{3} L - \frac{2}{15} C_1 R^2 \right) \right],$$

and
$$L' = L - \frac{1}{3} C_1 R^2,$$

to a high degree of approximation (see p. 38). From the second of these equations it is seen that the equivalent capacitance acting at the terminals of a bifilar resistance is $\frac{1}{3}$ of the total distributed capacitance, i.e. $C = \frac{1}{3} C_1$. The phase angle and time-constant will be zero if $L = \frac{2}{3} C_1 R^2$ and the change of resistance with frequency will then be very small.

One of the first attempts to make use of the compensation principle is that of Brown.† His resistance consisted of a pair of German silver wires stretched out parallel to one another upon a board at a distance apart such that the inductance and capacitance effects are balanced.

Fig. 24 (a) shows a winding devised by Orlich‡ for large resistances up to 25,000 ohms. A piece of slate 5 cm. broad, 12 cm. long, and 3 to 4 mm. thick has its edges well rounded,

* G. A. Campbell, "Resistance boxes for use in precise alternating current measurement," *Elec. World*, Vol. 44, pp. 728-729 (1904). See also Curtis and Grover, *loc. cit.*, pp. 499-500.

† S. L. Brown, *loc. cit.*; also "The residual of inductance and capacity in resistance coils. A standard resistance with balanced inductance and capacity," *Phys. Rev.*, Vol. 29, pp. 369-391 (1909).

‡ E. Orlich, "Über eine Kompensation der Kapazität in grossen Widerständen," *Verh. d. Deutsch. Phys. Ges.*, 12 Jahrgang, pp. 949-954 (1910).

and is then wound with half the required resistance wire in the form of an inductive spiral. Insulating caps are then put over the wound layer, and the remaining half of the wire is then wound over the caps with a direction opposite to that of the first layer. The caps are proportioned so that the two halves of the winding are separated by a distance d cm. By choosing d to have a suitable value, the capacitance between the two layers can be balanced against the inductance of the winding.

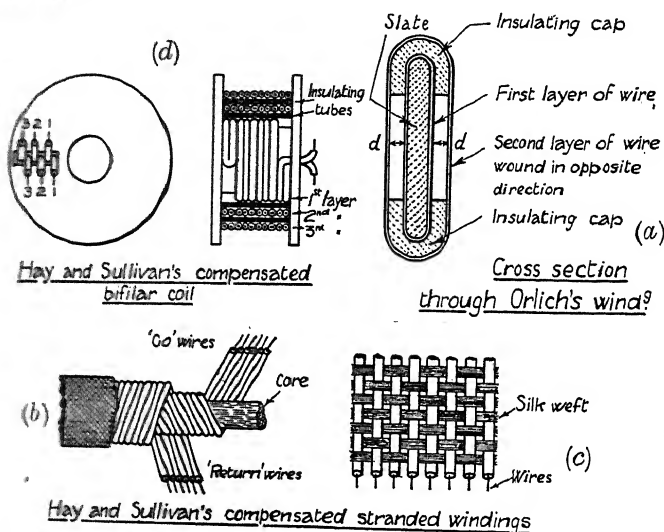


FIG. 24.—METHODS OF CONSTRUCTING RESISTANCES WITH BALANCED RESIDUALS

Hay and Sullivan* have described a variety of methods specially intended for the construction of resistances to be used at very high frequencies. A multiple conductor is used, consisting of a number of insulated strands connected in parallel. Two such conductors being taken, one is wound in a uniform spiral upon a core of insulating material. The

* C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal P.O.E.E.*, Vol. 5, pp. 451-454 (1913); also *Professional Papers*, No. 53, pp. 18-19. C. E. Hay and H. W. Sullivan, "Improvements in Electrical Resistances," *British Patent*, No. 22,375 (1912). Described and illustrated in this chapter by permission of H. W. Sullivan, Esq.

second is then wound over the first in a spiral of the opposite hand, the distance between the two layers being proportioned in such a way that the desired compensation of inductance by capacity is effected. The conductors are joined at one end, so that the core is covered by two concentric tubular conductors of which one forms the "go" and the other the "return" circuit (Fig. 24 (b)). An alternative to this construction is to fold a stranded conductor at the middle of its length, the distance apart of the two halves being regulated by means of an intervening insulating layer, so that when the composite conductor is wound into a disc coil (as in Fig. 21 (c)) compensation is attained. The multiple conductor in this case may very conveniently be made in the form of a ribbon in which silk or cotton weft is woven across a warp which is composed of the wire strands connected in parallel (Fig. 24 (c)).

The same inventors* have also designed a compensated bifilar winding, also shown in Fig. 24 (d). A multilayer coil is used, the distance apart of the layers being such that the capacity between the layers is small compared with that between the wires of each layer. A bifilar layer is wound so that it has a positive time-constant, i.e. residual inductance preponderates. Over this a second bifilar coil is wound which, having a greater length of wire, will have a smaller positive time-constant, and so on with additional layers until one is arrived at which has $T = 0$. Subsequent layers will then have a negative time-constant. Such a number of layers is wound that, when they are all connected in series, the time-constant of the coil is zero.

6. Secondary Effects in Resistance Coils. In the preparation of resistance standards there are a few matters of a secondary nature to which attention must be directed.

In order to reduce as far as possible any changes in the value of resistance coils with temperature, the material used for the windings is usually an alloy of very low resistance/temperature coefficient. For this purpose, *manganin*† is much used, this material being an alloy of copper (85 per cent), manganese (12 per cent), and nickel (3 per cent).

* C. E. Hay and H. W. Sullivan, "Improvements in Electrical Resistances," *British Patent*, No. 21,109, (1913).

† Discovered by E. Weston in 1889, and investigated in detail at the *Physikalische Technische Reichsanstalt*. Other high resistivity alloys, such as *Eureka*, are also used.

The resistivity is about 42×10^{-6} ohms per centimetre cube at 0°C ., the temperature coefficient being between zero and 0.0017 per cent per $^{\circ}\text{C}$. Except in work of the highest precision, it is usually unnecessary to take account of temperature corrections. The manganin is drawn into the form of wire or rolled into flat strip or ribbon; it is then insulated with silk.

Resistance coils are frequently wound upon insulated metal tubes, so that heat may easily pass out of them when they are carrying current, and for the same reason it is preferable to wind them in single layer form. When wound, the coil is dipped in shellac to protect the wire from oxidation, and baked for a number of hours at a temperature of about 120°C . so that the coil is freed of moisture and the shellac solvent. The coil may then be mounted for use. The heating also serves to anneal the manganin after the strain of winding, and a coil of greater permanence is produced.

It is found, however, that in the highest class of work such coils have small, erratic variations of resistance depending upon the humidity of the air. The moisture absorbed by the insulation causes a swelling of the latter, the resulting pressure exerted upon the wire producing the small variations of resistance. Rosa* has shown that the humidity trouble may be removed by dipping the coils into paraffin wax after the baking is complete, or by hermetically sealing the coils in brass cases. Such coils do not vary more than 1 part in 100,000 in fourteen months.

A possible cause of error in resistances is the so-called "skin-effect" in the wire. It is easy to show that at a frequency of 3,000 cycles per second a manganin wire must be as large as 2 mm. diameter before the skin-effect produces a change in the value of resistance so large as 1 in 10^5 . Now bridge coils are not usually intended to carry much current, and are, therefore, wound with much finer wire or with strip. Hence, the skin-effect is negligible up to the higher telephonic frequencies.

Other secondary effects act upon the residual capacity of the coil. The dielectric surrounding the wires is usually very poor and will alter its dielectric constant quite appreciably with temperature, changes of 1 per cent per $^{\circ}\text{C}$. being observed. For such changes to have little effect on the effective value of resistance, the residual capacity should

* E. B. Rosa and H. D. Babcock, "The variation of resistances with atmospheric humidity," *Bull. Bur. Stds.*, Vol. 4, pp. 121-140 (1908).

E. B. Rosa, "A new form of standard resistance," *Bull. Bur. Stds.*, Vol. 5, pp. 413-434 (1909).

The hygroscopic nature of shellac was noticed at an earlier date during the insulation of the coils for the standard current weigher at the N.P.L., see *Phil. Trans. Roy. Soc. A.*, Vol. 207, p. 497 (1908).

be very small. Dissipation of energy in the poor dielectric will also be reduced by diminution of the residual capacity. The absorption of moisture by the dielectric will change the capacity and can be avoided by coating the coil with paraffin wax or by hermetically sealing it away from the action of the air.

7. Application of Methods in Practice. In the preceding sections the student has been shown a number of ways in which resistances can be and have been constructed to be as nearly non-reactive as possible. Some of the methods are particularly suited to the production of high resistances, some to the production of low resistances, and others to the construction of coils of middle values. It will now be instructive to examine two typical sets of resistance coils, the first designed in America, and the second in Germany, in order to see the methods adopted and the degree of perfection attained.

Resistance of Coil.	Material of Spool.	Shape of Resistance Material.	Size of Resistance Material.			Method of Winding.	Effective Inductance of Sample Coil.	Time Constant.
			Width.	Thickness	Diameter			
Ohms.			mm.	mm.	mm.		Micro-henrys.	Seconds.
1	Brass or Porcelain	Strip	3 to 5	0.1		Bifilar	+ 0.05	5×10^{-8}
10	"	"	1	0.1		"	+ 0.2	2×10^{-8}
10	"	Wire			0.24	Three 30 ohm bifilar sections in parallel on same spool	+ 0.3	3×10^{-8}
100	"	"			0.24	Bifilar	- 1.6	$- 1.6 \times 10^{-8}$
1,000	Porcelain	"			0.10	Five 200 ohm bifilar sections in series on same spool	- 16	$- 1.6 \times 10^{-8}$
5,000	"	"			0.05	Inductive for 20 turns, then reversed for 20 turns, and so on	+ 210	$+ 4.2 \times 10^{-8}$
5,000	"	"			0.05	Inductive, reversed every turn	+ 30	$+ 0.6 \times 10^{-8}$
10,000	"	"			0.05	Inductive, reversed every turn	+ 100	$+ 1 \times 10^{-8}$

Curtis and Grover's Coils. By much experiment, Curtis and Grover* have devised a set of coils which have very small residuals. Single layer coils wound with silk-covered manganin upon spools 2.5 cm.

* H. L. Curtis and F. W. Grover, "Resistance coils for alternating current work," *Bull. Bur. Stds.*, Vol. 8, pp. 495-517 (1913).

diameter are used. The finished coils are varnished with shellac, baked at 120° C. for 10 hours, and then protected from the air by a coating of paraffin wax or by hermetical sealing.

Tests were carried out chiefly at frequencies between 1,200–1,500 cycles/second.

It will be seen that the time-constants of the coils are in all cases very small, in no case exceeding 5×10^{-8} second. At 3,000 cycles per second, this would correspond to a phase displacement between the voltage applied to such a coil and the current through it of only 3.5'. With coils below 100 ohms, efforts are directed to the reduction of residual inductance. A 100 ohm bifilar coil has a small time-constant and requires no further treatment. With coils of 1,000 ohms and higher values, attention is chiefly directed to the reduction of capacity effects. It becomes important to wind coils of such values upon insulating spools.

Wagner and Wertheimer's Coils. These investigators have also made a very thorough examination of the methods by which non-reactive coils are constructed, measurements on a large number of coils made

Resistance of Coil.	Material of Spool.	Arrangement of Winding.	Time Constant. Seconds.
0.1	Metal Tube	Bifilar wound manganin strip with mica between	$+ 9 \times 10^{-8}$
1	" "	" "	$+ 4.5 \times 10^{-8}$
10	" "	4 Bifilar sections of 40 ohms each, in parallel	$+ 1.25 \times 10^{-8}$
20	" "	4 " " 80 " " "	$+ 0.5 \times 10^{-8}$
30	" "	2 " " 60 " " "	$+ 0.7 \times 10^{-8}$
50	" "	Bifilar coil "	$+ 0.63 \times 10^{-8}$
70	Fibre core with separate metal tube for each section	2 Bifilar sections of 35 ohms in series	$- 0.55 \times 10^{-8}$
100	Metal Tube	2 " " 50 " " "	Very small
200	" "	2 Chaperon " 100 " " "	$+ 0.33 \times 10^{-8}$
300	" "	3 " " 100 " " "	Very small
500	Fibre core with separate metal tube for each section	3 " " 166 $\frac{2}{3}$ " " "	$+ 0.5 \times 10^{-8}$
700	" " "	5 " " 140 " " "	$- 0.29 \times 10^{-8}$
1,000	" " "	6 " " 166 $\frac{2}{3}$ " " "	$- 0.50 \times 10^{-8}$
1,000	Porcelain Tube "	6 " " 166 $\frac{2}{3}$ " " "	$+ 0.23 \times 10^{-8}$
1,000	" "	5 " " 200 " " "	$- 0.21 \times 10^{-8}$
3,000	" "	6 " " 500 " " "	$- 1.20 \times 10^{-8}$

A comparison of these figures with those in the preceding table will prove very instructive.

in a great variety of ways being recorded. It will be seen from the above table that Curtis and Grover, for coils below 5,000 ohms, favour the use of bifilar wound coils. Wagner and Wertheimer,* on the other hand, employ almost entirely the Chaperon winding in one or other of the forms illustrated in Fig. 22. Measurements were made at a frequency of 1,590 cycles/second for a variety of bridge currents. Photographs are given of typical coils of each class.

* K. W. Wagner and A. Wertheimer, "Über Präzisionswiderstände für hoch frequenten Wechselstrom," *Elekt. Zeits.*, 34 Jahrgang, pp. 613–616, 649–652 (1913). K. W. Wagner, *Elekt. Zeits.*, 36 Jahrgang, pp. 606–609, 621–624 (1915).

8. Resistance Boxes. When a suitable set of resistance coils has been prepared, it is necessary to mount them in some way so that they can be conveniently put into circuit in the bridge network. In work of the very highest class, the coils are usually mounted separately, each being fitted up in its own metal case. Massive terminals are provided and the coil is usually immersed in oil, so that its temperature may be accurately determined and maintained constant. In ordinary laboratory practice, it is more convenient to group sets of coils within a common case, suitable terminals or switches being provided so that any desired combination of coils may be connected in series. Such a piece of apparatus is called a "resistance box."

There are a few points connected with the arrangement of coils in resistance boxes which it is proposed now to mention quite briefly. These are (i) mounting of coils, (ii) methods of connecting the coils in circuit, (iii) effects of grouping of coils on residuals.

Mounting of Coils. Most coils for a.c. work are wound upon spools, usually of metal, and are mounted in the resistance box with the axis of the spool vertical. It is very common practice to fix the spools directly to the ebonite lid of the box, and to carry down from the contact blocks on top of the lid suitable rods to which the coils may be connected (Fig. 25).

In the simplest arrangement, as shown in the diagram, one rod is connected to each block and forms the common lead for two neighbouring coils. In resistance boxes intended for precision working each coil is frequently provided with its own pair of rods, two rods being attached to each contact block.

The Leeds and Northrup Company have improved on this construction by attaching the metal spool of the coil to the contact blocks; the radiating surface of these then serves for the dissipation of heat generated in the winding and increases the permissible number of watts which may safely be allowed in each coil.

Methods of Connecting the Coils in Circuit. The range of resistance commonly required in boxes designed for bridge work is from 10,000 ohms down to 1 ohm. It is desired to obtain any value of resistance between these limits in steps of 1 ohm. (For special work, boxes are frequently used in which the values are one-tenth of these figures.) For this purpose a certain number of resistance units are mounted in

the box and some arrangement of plug contacts or dial switches provided, so that the coils may be combined in series to any value within the desired limits and with the specified step.

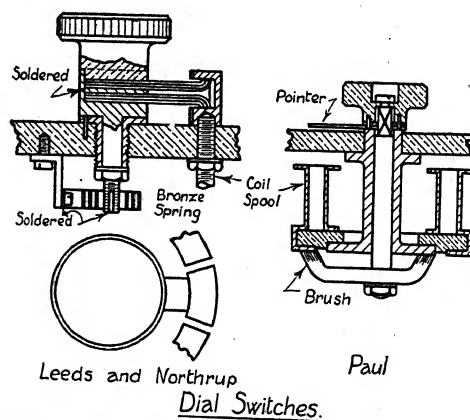
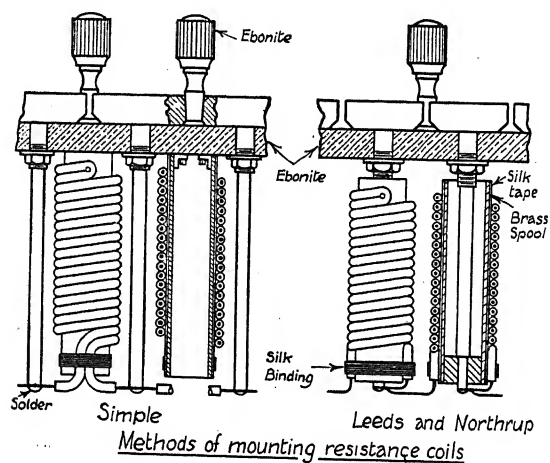


FIG. 25.—METHODS OF MOUNTING RESISTANCE COILS.
DIAL SWITCHES

The oldest way is to provide a number of coils connected as shown in Fig. 26 between contact blocks of brass mounted on the lid of the box. The insertion of a plug into the conical

hole between two blocks short-circuits the coil connected across them and cuts it out of circuit. Any coil is put in circuit by the *withdrawal* of a plug, and it will be clear that the number of plugs required will be equal to the number of coils. The

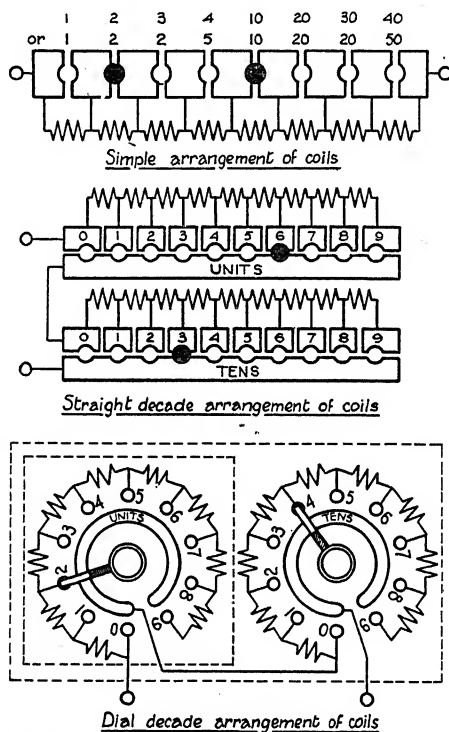


FIG. 26.—METHODS OF GROUPING COILS IN RESISTANCE BOXES

values of the coils may be 1, 2, 2, 5, with a similar grouping for the tens, hundreds, and thousands, two denominations only being shown in the diagram. Or the values may be 1, 2, 3, 4, etc. With either arrangement the desired range can be covered in steps of 1 ohm by removing the requisite plugs.

The disadvantages of this arrangement are numerous. Obviously, a large number of plugs, equal to the number of coils, will be required, and these must fit interchangeably in the holes, if errors due to plug contact resistances are to be

made small. Again, the largest number of plugs is inserted, and the consequent contact resistance is greatest, when low resistances are required in circuit. Moreover, when a setting has been made, the value of resistance must be found by adding the numbers corresponding to the plugs withdrawn.

To overcome these objections, the "decade" principle shown in Fig. 26 has been introduced, and is almost exclusively used in resistance boxes for precise work. In this method there are ten 1-ohm coils for the units, ten 10-ohm coils for the tens, and so on, each group being called a "decade." The 10 coils in series in any decade can then be compared with the first coil of the next higher group, thus providing a check among the various coils. If this advantage be not required, nine coils per group are sufficient for successful operation of the principle. Only one plug per decade is required, so that contact errors are very much smaller than in the old form of resistance box. This plug is *inserted* to put a desired number of coils in series. The value of resistance in circuit is at once read off from the positions of the plugs in the decades, no summation being required. It should be noted that the decade principle requires more coils than the old method; it is possible, however, to design decades with as few as four coils of suitably chosen values.*

The greatest advantage of the decade principle is that it readily allows of the use of dial switches, these being much more convenient to manipulate in practice than a number of loose plugs. All high-grade resistance boxes are constructed on this plan, the success of which depends entirely on the certainty of contact and lowness of contact resistance of the dial switches. Typical designs are shown in Fig. 25.†

In most bridge methods two of the branches of the network are very frequently non-reactive resistances which are required to stand in a definite ratio to one another, usually a simple integral multiplier. These resistances may be advantageously combined in a special type of plug resistance box, called a "ratio box," of which a variety of forms are in use.

Fig. 27 shows a simple type having two sets of coils of 10, 100, and 1,000 ohms, so that, by withdrawal of the

* See *Dictionary of Applied Physics*, Vol. 2, p. 693.

† R. W. Paul, "Improvements in Electrical Resistances," *British Patent*, No. 4,388 (1903). Also see the list of the Cambridge & Paul Instrument Company.

appropriate plugs, all decimal multiples and submultiples of 10 lying between 100 and $1/100$ may be obtained.

When inductances are to be measured by the Campbell method, ratios of 9, 99, etc., are required (*see* p. 247). Mr.

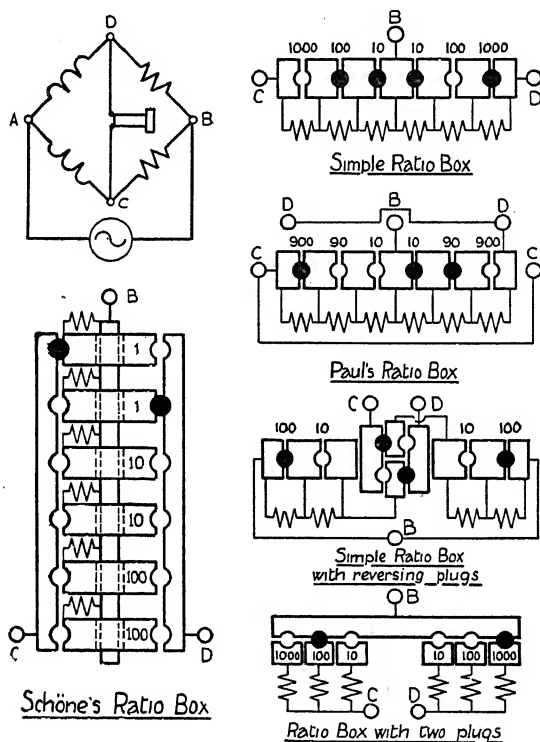


FIG. 27.—RATIO BOXES

Paul, therefore, constructs a ratio box in which the two coil-groups are of 10, 90, 900 each, thus giving the decimal ratios of $10/10$, $100/100$, $1,000/1,000$; $1,000/100$, $1,000/10$, $100/10$ and the reciprocals of these; and, in addition, the required values of $90/10$, $900/100$, $990/10$.

The ratio boxes so far described suffer from the defect that there are as many plugs as there are coils, with their consequent contact errors. It is, however, easy to group the coils so that

only one plug in each ratio branch is necessary. A simple box on this principle is drawn in Fig. 27, arranged to give decimal ratios.

An excellent arrangement for a ratio box is that designed by Schöne,* Fig. 27. Here the bars *C* and *D* form two of the branch points—say the pair to which the detector is attached. The coils are connected between the contact blocks and the common bar *B*. By insertion of one plug between *C* and the blocks, and a second plug between *D* and the blocks, any desired pair of coils may be used to produce any required decimal ratio. Moreover, when using a ratio of unity, reversal of the ratios in the bridge is easily carried out by transference of the inserted plugs to the other end of the contact blocks which are in use, so that equality of the ratio coils may be readily checked (see p. 284).

Effects of Grouping of Coils on Residuals. The coils which are to be put into a resistance box are wound in such a way that their time-constants are as small as possible. Hence care should be taken when grouping them together that "grouping residuals" are kept down to the minimum. Such additional residuals may be introduced by the connections inside the resistance box, by the contact blocks upon the lid, and by the proximity of the coils to one another.

With low resistance coils, less than 100 ohms, the connections from the contact blocks to the coils should have a very low inductance. The normal methods of construction, shown in Fig. 25, in which a rod is fixed to the block or in which the coil spool itself is used as a connector, usually suffice. The small inductance introduced by the rods is practically a constant residual since, whatever value of resistance be plugged in circuit, there are never more than two rods in series.

With high resistance coils, it is necessary to keep all capacity effects in the connecting leads down to a negligible value. With the normal methods of construction, the capacity between the leads and the contact blocks to which the coil is connected act as a permanent capacitive shunt on the coil, whether it be in circuit alone or in combination with others. To reduce this effect the contact blocks should be small, they and the connecting rods being as far apart as is convenient.

Intercapacity between the various coils may also have a pronounced effect upon the time-constant as measured at the resistance box

* O. Schöne, "Ueber eine Stopfelanordnung für Brücken-zweigwiderstände der Firma Siemens and Halske A.G.," *Zeits. f. Inst.*, 18 Jahrgang, pp. 133-135 (1898). The device is used in the high precision ratios of the Leeds and Northrup Company.

terminals, especially at high frequencies. G. A. Campbell* has suggested that each decade or group should be surrounded by a metal screen, as shown by the dotted lines in Fig. 26, so that intercapacities and earth capacities of the coils can be made definite. He suggests that when non-reactive resistances have been assembled—and, for high frequency work, screened as described—the grouping residuals should be allowed for by the addition of capacity in parallel or inductance in series with the resistance box terminals. This auxiliary capacity or inductance should be adjusted until the time-constant of the whole, measured at the resistance box terminals, is as nearly zero as can be secured. A resistance box constructed and adjusted in this manner probably represents the most perfect non-reactive resistance standard which can be constructed for high frequency work, but must be used under the same conditions as when it was adjusted.

An effect which is sometimes important in decade boxes is also due to earth and intercapacity, namely, the “dead-end effect.” When part of a decade is in use, the remaining coils are joined to one terminal of the circuit. The capacity of the free coils to earth and with respect to those in use may affect the time-constant. No known type of decade box entirely avoids this source of trouble, which must, therefore, be reduced to the minimum by good design.†

9. Resistances with Calculable Residuals. Variable Low Resistances. The aim of the various artifices described in the preceding paragraphs is the construction of resistance units which shall be as free from residual inductance and capacitance as possible. The reactance due to the small residuals is then considered negligible in comparison with the resistance, and, over a wide range of frequency, the resistance may be taken as pure or non-reactive. It is, however, possible to approach the matter from a different direction. Instead of endeavouring to prepare resistances which shall be approximately non-reactive, the alternative procedure is to construct resistances in such a way that the residual inductance and capacitance can be calculated from a knowledge of the geometrical form and dimensions of the winding. This method has been frequently employed in cases where reference standards of known time-constant are required, e.g. in testing resistance coils constructed in the ordinary manner; or in setting up a bridge network to measure effective resistance, wherein full allowance must be made for the residuals in all the resistances used to obtain balance. For this purpose resistances made

* G. A. Campbell, “Resistance boxes for use in precise alternating current measurement,” *Elec. World*, Vol. 44, pp. 728–729 (1904).

† See Curtis and Grover, *loc. cit.*, pp. 514–516; and also Wagner, *loc. cit.*, for a full discussion of grouping residuals and their effect on the time constant.

in the form of circles, rectangles, or parallel wires are usually chosen, since accurate formulae can be developed for the inductance and capacitance of such shapes.

Giebe, in 1907, used resistances of the parallel wire type, details of which are given on page 187. Brown, in 1909, also used a parallel wire standard (see p. 63).

Grover and Curtis,* in 1913, used various geometrical standards in their work on the inductance of resistance coils. Two 1-ohm resistances were made of 0.4 mm. diameter manganin wire, in one case bent into a circle 5.8 cm. radius with a gap between the ends of 1.8 cm., and in the other case formed into a rectangle with the long sides 1 cm. apart. Allowance being made in the first case for the gap, and in both cases for the terminals, the inductance of the circle is 419×10^{-9} henry, and of the rectangle 291×10^{-9} henry. Parallel wire resistances up to 5,000 ohms are also described in their paper.

Wagner and Wertheimer† in 1913, and Wagner‡ in 1915, have given a very full description of geometrical resistance standards of manganin wire from which the results tabulated in Fig. 28 are obtained.

For calculable standards of 20 ohms or less, the circle is the most convenient form. The capacity effects in such a standard are negligible, it being necessary merely to calculate the inductance. For a wire of diameter d_1 , bent into a complete circle of mean diameter D cm., Rayleigh and Niven's formula is

$$L = 2\pi D \left[\left(1 + \frac{d_1^2}{8D^2} \right) \log \frac{8D}{d_1} + \frac{d_1^2}{24D^2} - 1.75 \right] \text{cm.}$$

In the resistances of 10 ohm and 1 ohm tabulated in Fig. 28, a gap of a cm. is left in the circle and connecting leads are added perpendicular to the plane of the circle. It is necessary to correct for the gap by multiplying the above formula by $\left(1 - \frac{a}{\pi D} \right)$, and also to add the inductance of the connecting leads. The amended formula stands at the head of the tabulated figures.

For resistances above 20 ohms, the parallel wire form is

* F. W. Grover and H. L. Curtis, "The measurement of the inductances of resistance coils," *Bull. Bur. Stds.*, Vol. 8, pp. 467-470 (1913).

† K. W. Wagner and A. Wertheimer, *Elekt. Zeits.*, 34 Jahrgang, pp. 613-616 (1913).

‡ K. W. Wagner, *Elekt. Zeits.*, 36 Jahrgang, pp. 606-609 (1915).

most suitable. In these the manganin wires are stretched out 1 cm. apart; connecting leads of thicker material—on which the adjustment of the resistance value can easily be made—

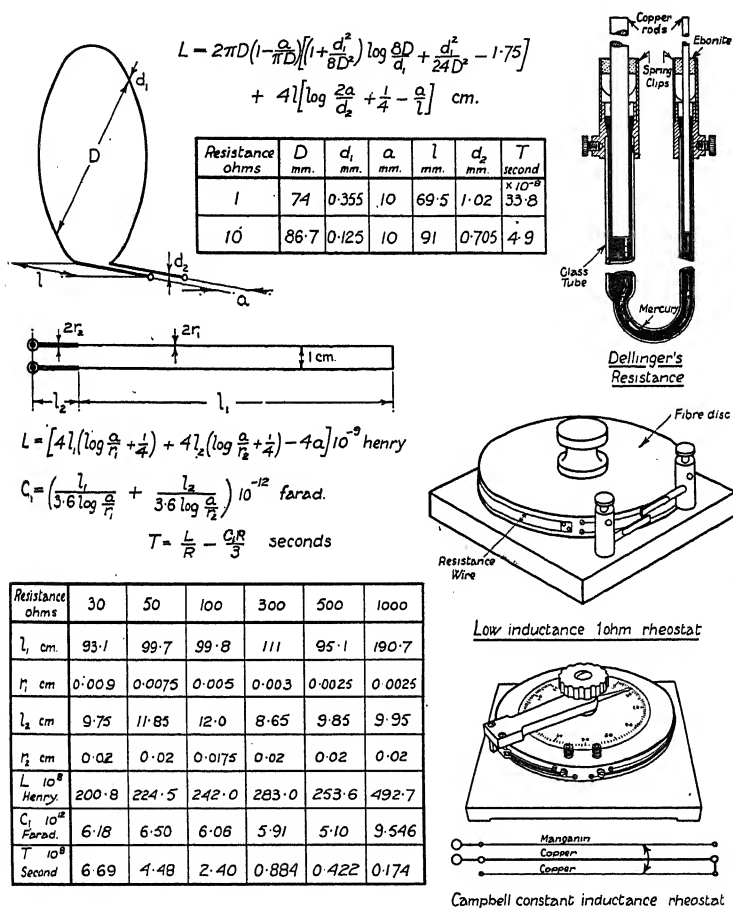


FIG. 28.—RESISTANCES WITH CALCULABLE RESIDUALS.
VARIABLE LOW RESISTANCES

are provided, disc terminals of copper being soldered to them. It is shown on page 37 that such a pair of parallel wires may be treated as a transmission line short-circuited at the distant end, having inductance and capacitance uniformly distributed

along their length. For a pair of wires of unit permeability, having radius r , length l cm., and distance apart a , the inductance is given by

$$L = 4l \left[\log \frac{a}{r} + \frac{1}{4} - \frac{a}{l} \right] 10^{-9} \text{ henry}$$

and the total capacitance between the wires is

$$C_1 = \left[l / \left(3.6 \log \frac{a}{r} \right) \right] 10^{-12} \text{ farad.}$$

For the resistance standard shown in Fig. 28, the expressions for the wires and the connecting leads are added, a/l_1 being neglected.

The impedance operator for such a parallel wire resistance is, approximately,

$$z = R + j\omega \left[L - \frac{C_1 R^2}{3} \right]$$

(see page 38), so that the time-constant is

$$T = \frac{L}{R} - \frac{C_1 R}{3} \text{ seconds,}$$

as tabulated in Fig. 28.

It will be seen from this diagram that the length of such a parallel wire arrangement is fairly great, so that there will be a considerable electrostatic capacity between the wires and earth. This capacity must be taken into account in calculating the time-constant of high resistances. Grover and Curtis* have shown that if such a resistance form one branch of a Wheatstone network, the distributed earth capacity can be allowed for if the potentials of the wires be adjusted so that at every instant the potential of one is as much above that of the earth as the other is below it. They describe a special earthing device by the use of which this condition is fulfilled, the place where the two wires are joined being maintained at earth potential.

When a horizontal parallel wire resistance forms one branch of a Wheatstone network, one terminal is connected to the detector and one to the source of current. Wagner has shown that, in order to avoid capacity troubles in balancing the bridge, especially when using telephones, it is necessary to arrange that the detector branch points are brought to zero potential (see p. 286). In these circumstances it is clearly not possible to apply the Grover and Curtis earthing device, and Wagner† has calculated the effect of earth capacity on the time-constant when one terminal of the resistance is earthed while the other

* Grover and Curtis, *loc. cit.*, pp. 470-479 (1913).

† Wagner, *loc. cit.* (1915).

is attached to the alternator. He finds* that if the horizontal wires have a total capacitance C_e to earth the correction to the time-constant is $RC_e/6$. Taking the figures tabulated in Fig. 28 for pairs in which earth capacity is neglected, the time-constants for wires with one terminal earthed become—

Resistance. Ohms.	C_e Micro-microfarads.	$RC_e/6 \times 10^{-8}$	Uncorrected T . Corrected T Seconds $\times 10^{-8}$	
30	10.2	0.005	6.69	6.69
50	11.07	0.009	4.48	4.49
300	11.25	0.056	0.884	0.94
500	9.71	0.081	0.422	0.50
1,000	18.62	0.310	0.174	0.48

The earth capacities were measured with the wires horizontal. From this table it is clear that earth capacity effects are negligible in parallel wire resistances less than 100 ohms. For higher values the potentials of the terminals of the resistance must be clearly specified before the time-constant can be calculated; unless this is done, and the appropriate working capacity is used, large errors may be introduced.

VARIABLE LOW RESISTANCES. In making a bridge measurement, adjustments of the balancing resistances to the nearest ohm can be made in dial or plug resistance boxes. A fine adjustment rheostat, which can be varied continuously between zero and 1 ohm, is essential for accurate settings with a sensitive bridge method. A simple arrangement is a pair of parallel wires upon which a conducting slider can be moved. The small inductance of such a rheostat can be calculated for any position of the slider and, if necessary, allowed for.

A more convenient arrangement of a low inductance 1 ohm resistance is illustrated† in Fig. 28. In this a loop of No. 22 S.W.G. Eureka wire is fixed upon the edge of an ebonite disc 16.7 cm. diameter. The sides of the loop are 0.51 cm. apart. The disc is capable of rotation about a central axis, contact being made on the wires by means of two spring brushes. The maximum resistance is about 1.2 ohms, the inductance then being about 0.6 microhenry. Such a resistance, used in conjunction with a dial box reading to 1 ohm, proves very satisfactory in obtaining accurate settings. A scale marked on the disc, read against a pointer carried by one of the terminals gives the value of the resistance in any position (these details are omitted from the drawing).

* K. W. Wagner, "Die Theorie des Kettenleiters nebst Anwendungen (Wirkung der verteilten Kapazität in Widerstandssätzen)," *Arch. f. Elekt.*, Bd. 3, pp. 315-332 (1915). Also S. Butterworth, "Notes on earth capacity effects in alternating current bridges," *Proc. Phys. Soc.*, Vol. 34, p. 11 (1921).

† From an example in the laboratory of the City and Guilds (Engineering) College.

In many bridge methods it is very convenient to use a variable low resistance in which the inductance remains constant whatever value the resistance may have. A simple and excellent device which very nearly fulfils this condition is the mercury tube rheostat of Dellinger and Wenner.* In this instrument advantage is taken of the fact that the specific resistance of mercury is about 60 times that of copper. Referring to Fig. 28, a U tube is constructed of ebonite, porcelain, or thin glass, the cross-sectional area of one limb being 10 times that of the other. The limbs are filled with mercury, and at their open ends are fitted with terminal caps, or, if the limbs be of glass, cups are blown upon them in order to contain any overflow of mercury, the terminals dipping therein. Each limb is provided with an amalgamated copper rod which fits the bore of the tube as closely as possible. The resistance is varied by sliding the rods up or down inside the limbs of the U, the copper short-circuiting more or less of the mercury column in virtue of its superior conductivity. The displaced mercury overflows into the terminal caps or the glass cups; spring clips retain the copper rods in any desired position. Settings of resistance can be made with very great precision, and with perfectly uniform variation, since there are no mechanical contacts across which the current must pass. The device can also be constructed in a single tube form with terminal caps at the upper and lower ends. A U tube with limbs 12 cm. long, one being 3 mm. and the other 1 mm. diameter, has an adjustment range for the large limb up to 0.01 ohm, and for the smaller up to about 0.1 ohm. The resistance has the disadvantage of a large temperature coefficient.

Another much-used device is the Campbell constant inductance rheostat made by The Cambridge and Paul Co., illustrated in Fig. 28. In this, a resistance wire of manganin and a parallel copper wire are wound in grooves on the edge of a disc of slate, together with a compensating winding of copper placed between them. A contact carried on a moving arm touches the resistance wire and the copper one in such a way

* J. H. Dellinger, "Note on a variable low resistance," *Phys. Rev.*, Vol. 33, pp. 215-216 (1911).

F. Wenner, "The four-terminal conductor and the Thomson bridge," *Bull. Bur. Stds.*, Vol. 8, pp. 584-585 (1913).

F. W. Grover and H. L. Curtis, *loc. cit.*, pp. 465-466 (1913).

that the resistance is varied by rotation of the arm ; a pointer attached thereto indicates the value of the resistance on a fixed scale. It will be seen from the diagram that no matter what value of resistance the rheostat may be set to, the inductance remains constant and is very small. A range of resistance from 0.10 to 1.1 ohms is obtainable and can be read to 0.01 ohm. Compensated resistance boxes are also made on this principle.

In cases where it is desired to keep the inductance of a circuit strictly constant and to vary its resistance by a definite step, say 0.1 ohm, the following artifice* may be adopted. Let links of two metals having different resistivities, e.g. copper and manganin, be prepared, the links having the same section, length, and shape. The dimensions of the links must be chosen so that the latter differ in resistance by the desired amount. Suitable mercury cups are provided into which one or other of the links may be placed in a perfectly definite position. Substitution of the copper for the manganin link, or *vice versa*, will then produce a pre-determined change of resistance without any change of inductance.

STANDARDS OF SELF AND MUTUAL INDUCTANCE

The standards of self and mutual inductance used in bridge measurements are of two kinds. The first are those which have a single definite value of inductance, and are known as fixed standards. In the second class the inductance may be varied continuously between certain limiting values, without altering the resistance of the inductive coils. Such are known variously as variable standards, inductors, inductometers, etc., and are much used in a.c. bridges.

10. Fixed Standards. Fixed value standards of self and mutual inductance are of two classes, absolute and secondary. In absolute standards the coils are arranged in such a way that the inductance may be calculated from their accurately measured dimensions. Secondary standards, on the other hand, are wound in any convenient manner, and are then calibrated by comparison with absolute standards in a bridge or by other means. They serve as general laboratory standards of reference, whereas the absolute standards are only used in measurements of the highest accuracy, such as would be

* Curtis and Grover, *loc. cit.*, pp. 465 (1913). Also see C. E. Hay, *P.O. Reprint*, No. 53, pp. 21-22 (1912).

undertaken at the National Physical Laboratory and institutions of a similar nature.

SELF-INDUCTANCE. A reference standard of self-inductance, whether absolute or secondary, should be constructed so that it fulfils the following important conditions—

- (i) It should be permanent in form and constant in value.
- (ii) The resistance should be low in comparison with the inductance.
- (iii) The winding should be carried out in such a way that the inductance is the largest possible for a given amount of wire, the coil being of a convenient size.
- (iv) The value of the inductance should be independent of the current strength.
- (v) The effective inductance and resistance of the standard should be little affected by frequency.

In the following paragraphs it will be shown how these various conditions are complied with in the design of absolute and secondary inductances.

In absolute standards of self-inductance, everything is done to construct a coil which is unalterable in value, and of which the dimensions can be measured to the highest attainable degree of accuracy. A brief description of an actual coil will serve to show how this is carried out in practice. The Bureau of Standards* at Washington possesses a standard of self-inductance, having a value of about 0.2 henry, which has been constructed in the following way. A hollow cylinder of white marble, 54 cm. external diameter and 11 cm. in radial thickness, is accurately ground so that it is as perfectly cylindrical and circular in section as modern machine tools can make it. The diameter of the cylinder is accurately measured at a large number of places, so that its true average diameter is known, and also its variations from perfect roundness and cylindrical shape. Upon the surface of the cylinder a single-layer coil is uniformly wound†, the total length of the coil being about 46 cm., there being about 15 turns to the centimetre. The coil

* J. G. Coffin, "Construction and calculation of absolute standards of inductance," *Bull. Bur. Std.*, Vol. 2, pp. 87-143 (1906).

† In most absolute standards the position of every turn is made quite definite by winding accurately drawn, bare copper wire in an accurate screw thread cut in the surface of the marble cylinder.

is wound in sections so that the inductances of portions of the winding may be inter-compared, thereby providing a check on the calculations. From the measured dimensions of the cylinder, and the number of turns in the winding, it is possible to calculate the inductance of the standard to a very high degree of accuracy.

A coil constructed in the manner just described is built on sound mechanical lines, and will, therefore, be permanent in form and unalterable in value. No attempt is made to distribute the wire of the winding in the most economical way, attention being directed more to the precise localization of every turn. By the use of marble, the inductance is made independent of the strength of current in the wire, since marble is practically non-magnetic. In order that the standard may be reasonably free from the effects of frequency, three things are done; (i) the skin-effect in the winding is reduced by the use of small wire; (ii) self capacity of the winding is kept small by using a single-layer coil with the turns reasonably widely spaced; (iii) the insulation resistance of the winding is maintained at as high a value as possible.* These effects are discussed in detail below.

In general laboratory practice calibrated secondary standards are quite sufficient. Wien†, in 1896, introduced the type of coil which, with slight constructional modifications, is in use at the present day. Maxwell‡ has proved that if wire be wound in a channel of square section cut in the rim of a circular bobbin, the inductance of the resulting coil will be a maximum for a given length and thickness of wire when the mean diameter of the coil is 3.7 times the side of the square section of the channel (Fig. 29 (a)). Wien's coils were wound to fulfil this condition, the wire being thus distributed in the most advantageous way,§ i.e. so that a coil is produced which has the largest time constant.

* For a method of winding absolute inductances so that the insulation resistance is high and can be readily tested, see W. E. Ayrton, T. Mather, and F. E. Smith, *Phil. Trans. Roy. Soc., A.*, Vol. 207, pp. 469-470 (1908).

† Max Wien, "Einheitsrollen der Selbstinduction," *Ann. der Phys.* Bd. 58, pp. 553-563 (1896). Coils of 0.001, 0.01, and 0.1 henry are described.

‡ *Treatise*, Vol. 2, Sec. 706, pp. 345-346; see also J. E. Ives, "On the dimensions of large inductance coils," *Phys. Rev.*, Vol. 16, pp. 112-114 (1903).

§ R. E. Shawcross and R. I. Wells, "On the form of coils to give maximum self-inductance for a given length and thickness of wire," *Electr.*, Vol. 75, p. 64 (1915), have shown that Maxwell's figure is derived from an approximate expression for the inductance. Using a more correct formula, they prove (i) that the square channel is best; (ii) that Maxwell's figure for the diameter is too high, the true value being nearly 3; (iii) that it is better to use too large rather than too small a mean diameter, so that Maxwell's figure errs on the right side.

The bobbins on which Wien's coils were wound were made of serpentine,* a material which has been much used for the purpose owing to the ease with which it can be procured and machined. Rosa and Grover† have shown that the inductance of a coil wound on a

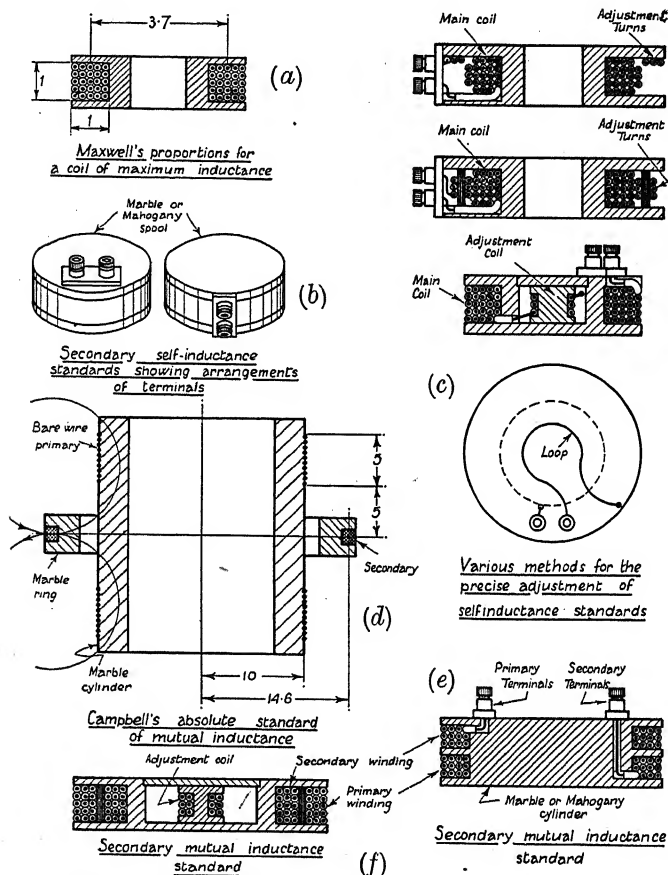


FIG. 29.—FIXED VALUE STANDARDS OF SELF AND MUTUAL INDUCTANCE

* *Serpentine*.—A mineral composed of magnesium silicate with some ferrous inclusions. Chemical analysis shows that about 0.6% of iron may be present, chiefly in the form of magnetite. The mineral may be of a dull red, brown, or greenish colour, veined or mottled. It is widely distributed over the world, and is found in England on the Cornish coast.

† E. B. Rosa and F. W. Grover, "The use of serpentine in standards of inductance," *Bull. Bur. Stds.*, Vol. 1, pp. 337-348 (1905).

serpentine bobbin depends on the strength of the current in the windings, showing that serpentine is quite appreciably magnetic. For example, if a coil of 1 henry be wound on a wooden bobbin—which is certainly non-magnetic—the inductance can be altered by 2 or 3 millihenrys if a lump of serpentine be laid upon the coil. It is desirable, therefore, to use some material for the bobbins which is more nearly non-magnetic.

The best material for the construction of inductance standards is white marble.* This material is practically free from magnetic properties, it possesses a high insulation resistance, and can be easily machined and tooled. Coils wound on it are very permanent in value owing to the rigidity and permanence of form of the marble bobbins.

For ordinary laboratory work, coils may be wound on spools of well-seasoned Cuba mahogany. If the wood is thoroughly dry, and is impregnated with wax to prevent absorption of moisture, coils of great constancy will result. This construction is very cheap and simple, the spool being quite free from magnetic impurities.

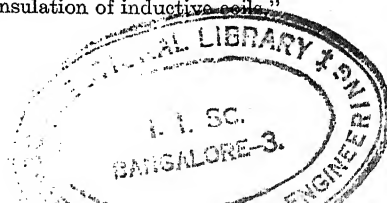
Briefly, a secondary self-inductance standard is a coil of Maxwell's proportions wound upon a rigid, non-magnetic spool of marble or mahogany. After winding, the whole is immersed in hot paraffin wax, which completely fills the interstices between the wires, and when cold sets into a solid mass, which retains its shape permanently and thus holds the wires in position. The greatest care should be taken to see that the wax is not overheated as its insulating properties are thereby greatly impaired.

Frequency Errors. In general, the effective inductance and resistance of a coil will vary with frequency, owing to the following three causes: (i) eddy current effects in the wire; (ii) the self capacity of the winding†; and (iii) imperfect insulation of the coil.‡ As these effects are of considerable importance in accurate measurements, especially at high frequencies, each will be considered in turn, together with the method adopted to make the coil as little dependent on frequency as possible.

* *Marble.*—A mineral formed of a crystalline and compact mass of carbonate of lime. Its magnetic susceptibility is about -0.8×10^{-6} , its permeability 0.999988; its density is about 2.74 and coefficient of linear expansion 0.00001 per °C.

† F. Dolezalek, "Ueber Prazisionsnormale der Selbstinduktion," *Ann. der Phys.*, Bd. 12, pp. 1,142–1,152 (1903). Further information is to be found in E. Orlich, "Über Selbstinduktionsnormale und die Messung von Selbstinduktionen," *Elekt. Zeits.*, 24 Jahrgang, pp. 502–506 (1903); and E. Giebe, "Prazisionsmessungen an Selbstinduktionsnormalen," *Zeits. f. Inst.*, 31 Jahrgang, pp. 6–20, 33–52 (1911).

‡ A. Campbell and J. L. Eckersley, "On the insulation of inductive coils," *Electn.*, Vol. 64, pp. 350–352 (1910).



Eddy current effects may be practically eliminated by winding the coil with stranded conductor, each strand being separately insulated from its neighbours. Dolezalek has shown that the independent strands need not be less than 0.1 mm. diameter, since the effects of eddy currents in smaller wires upon the resistance and inductance of the coil are, at frequencies up to 2,000 cycles per second, less than the effects due to changes of temperature. For higher frequencies, the conductor must be very finely stranded, so that the "skin effect" does not appreciably affect the distribution of current over the section of the wire.*

In large inductances a considerable quantity of wire is wound upon the spool, and such coils will, therefore, possess appreciable self capacity, due to the electrostatic capacity between the individual turns and layers of wire. To a first approximation the self capacity of a coil of inductance L and resistance R may be represented by a condenser of capacitance C joined in parallel with the coil. Referring to Fig. 10, the impedance operator for such an arrangement is

$$z = \frac{R + j\omega[L(1 - \omega^2 CL) - CR^2]}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}.$$

Remembering that C is small, so that its square may be neglected, the effective resistance of the coil is approximately

$$R' = R(1 + 2\omega^2 CL),$$

and the effective inductance is

$$L' = L(1 + \omega^2 CL),$$

L and R being the true inductance and resistance at very low frequency.

When L is small, then C is very minute and the effect of self capacitance is negligible, except when ω is very great. In large inductances, on the other hand, C is important and the effect may be far from negligible even at low frequencies. For example, Dolezalek records the case of a coil for which $L = 0.9365$ henry and $C = 1.6 \times 10^{-10}$ farad. At a frequency of $\omega/2\pi = 10,000/2\pi = 1,590$ the effective resistance of the coil would be nearly 3 per cent higher than at low frequency, while L' would be about 1.5% higher than L .

In practice, the effect of self capacitance is allowed for by measuring L' at two frequencies and therefrom determining L and C . The value of L' can then be calculated at any other frequency. The effect of C on R' is twice as important as its effect on L' , hence correction for self capacitance is essential in measurements of effective resistance in which large self-inductances—or small inductances at high frequency—are used. In order to allow of these corrections being made, some makers mark not only L and R , but C also upon the spool.

The insulation resistance of an inductive coil may be represented by a resistance R_i in parallel with it. If L be the true inductance and

* See, however, S. Butterworth, "Eddy current losses in cylindrical conductors with special applications to the alternating current resistances of short coils," *Phil. Trans. Roy. Soc., A.*, Vol. 222, pp. 57-100 (1921).

R the true resistance of the coil, Fig. 10 states that the impedance operator is

$$z = \frac{R_2\{[R(R + R_2) + \omega^2 L^2] + j\omega LR_2\}}{(R + R_2)^2 + \omega^2 L^2}.$$

The effective resistance of the coil at frequency $\omega/2\pi$ is

$$R' = \frac{R(R + R_2) + \omega^2 L^2}{(R + R_2)^2 + \omega^2 L^2} \cdot R_2,$$

and the effective inductance is

$$L' = \frac{R_2^2}{(R + R_2)^2 + \omega^2 L^2} \cdot L.$$

If the insulation conductance be $G = 1/R_2$ these expressions may be written as

$$R' = R + \frac{R_2(\omega^2 L^2 - R^2) - R(\omega^2 L^2 + R^2)}{(R + R_2)^2 + \omega^2 L^2}$$

and $L' = L/\{(1 + RG)^2 + \omega^2 L^2 G^2\}.$

From the first expression, provided that the second term be positive, R' will always exceed R , the two values being more nearly equal as R_2 becomes greater. Again, L' is always less than L unless G be vanishingly small. Thus, if the insulation resistance be high, the frequency variation of inductance and effective resistance will be small. It should be clearly understood that G is the alternating current conductance of the coil insulation and will itself depend on frequency; it bears no simple relation to the ordinary d.c. insulation conductance.

Summarizing, a secondary standard of self-inductance can be corrected for frequency effects: (i) by ensuring uniform distribution of current in the wire by the use of stranded conductors; (ii) by measuring and allowing for the effects of self capacity in small inductances at high frequencies or in large inductances at all frequencies; (iii) by ensuring high insulation resistance, especially in large inductances, by dipping the coil in wax after winding, and by fixing the terminals on an ebonite block (see Fig. 29 (b)).

Adjustment. Such standards are usually wound to some exact value of inductance, e.g. 0.1 henry, and are precisely adjusted within, say, 0.01 per cent of the marked value. With large inductances, say 1 henry, the desired precision can be attained by winding the coil to the nearest turn. For example, a 1-henry coil of 2,350 turns, with a mean diameter of 15 cm., changes by about 0.04 per cent for single turn adjustment, remembering that the inductance of a coil varies as the square of the number of turns. With smaller inductances, in which

there are fewer turns, the addition or removal of a turn makes a much larger alteration of inductance. For such coils, single turn adjustment is too rough a method, other devices being necessary.* For example, the coil can be wound so that it has an excess of turns; if, then, one or more turns of the last layer are wound at a diameter greater than that of the latter, the self-inductance of the whole coil will be less than if the last layer were uniform in diameter. These auxiliary turns may be put on the coil cheek or on a slip-over tube. Again, if the coil be wound one turn too small, the inductance can be very accurately adjusted by means of a small auxiliary coil sunk in a pit in the main bobbin and connected in series with the main coil. In a variant of this, the auxiliary coil consists of a loop of wire sunk in a groove on the outside of the coil former. In both cases the adjusting turns must be firmly fixed relatively to the main coil (*see* Fig. 29 (c)).

MUTUAL INDUCTANCE. A fixed value reference standard of mutual inductance should satisfy the following conditions—

- (i) It should be permanent in value.
- (ii) The resistances of the two windings should be small and the mutual inductance high.
- (iii) Eddy currents in the windings should be negligible.
- (iv) Capacity effects between the primary and secondary windings should be reduced to a minimum.

Absolute standards of mutual inductance are to be preferred to absolute self-inductances for a variety of reasons: (i) the value of mutual inductance can be calculated with greater certainty from the dimensions of the coils than is the case with self-inductance, since formulae for mutual inductances are of much higher theoretical accuracy than those for self-inductance; (ii) the effective self-inductance of a coil will vary with frequency owing to eddy currents in the wire and to self capacitance of the winding. Both these effects are much smaller in mutual inductances if the primary and secondary windings are reasonably far apart. On account of these considerations, the National Physical Laboratory use an absolute standard of mutual inductance as the ultimate reference standard in bridge measurements.

This standard is constructed on the principles laid down

* G. Moore, "The final adjustment of precision inductances," *Elec. Rev.*, Vol. 88, pp. 504-505 (1921).

by Mr. Campbell.* The primary winding is an accurately measurable single-layer coil of bare copper wire, wound in a screw thread cut on the surface of an accurate marble cylinder. The primary consists of two equal sections in series of 75 turns each, the marble cylinder being 30 cm. diameter. The secondary is a coil of many layers wound in a groove in a marble ring which surrounds the primary cylinder, the ring being adjustable axially and radially with respect to the latter. The secondary has 488 turns in a channel of 1 sq. cm. area and 43.8 cm. mean diameter, and is set midway between the two primary helices. The calculated value of the mutual inductance between the primary and secondary coils is 10.0177₈ millihenrys at 15° C.

When the coils have the relative proportions shown in Fig. 29 (d), Campbell has shown that their mutual inductance is a maximum. The circumference of the secondary coil then lies in zero magnetic field, so that the change in the value of mutual inductance is a minimum for small axial displacements of the secondary and for small changes in its radius. The entire accuracy of construction is thereby thrown on the single-layer primary which can be very precisely measured; it thus becomes possible to use a multilayer secondary in order to get relatively large mutual inductance values. Mechanical details of construction are very similar to those already mentioned in connection with absolute self-inductances.

Secondary mutual inductance standards of fixed value are made for ordinary laboratory work by winding primary and secondary coils in grooves cut in circular bobbins of marble† or of wood‡ (see Fig. 29 (e)). The wire is preferably stranded to reduce eddy current effects. Such standards can be readily

* A. Campbell, "On a standard of mutual inductance," *Proc. Roy. Soc., A.*, Vol. 79, pp. 428-435 (1907). For a description of a similar standard recently made by Mr. Paul for the Japanese Government, *Elec.*, Vol. 88, p. 159 (1922); also D. W. Dye, "Calculation of a primary standard of mutual inductance of the Campbell type and comparison of it with the N.P.L. standard," *Proc. Roy. Soc., A.*, Vol. 101, pp. 315-332 (1922). Further details of the N.P.L. standard are given by A. Campbell, *Proc. Roy. Soc., A.*, Vol. 87, pp. 391-414 (1912).

† A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626-638 (1907). The standard is nominally 0.05 henry.

‡ E. B. Rosa, "Mutual inductances for laboratory use," *Phys. Rev.*, Vol. 24, p. 241 (1907). A mutual inductance of 0.01 henry wound on a mahogany spool was constructed by Prof. T. Mather in 1892 and has been in use for many years in the laboratories of the City and Guilds College.

calibrated, and some correction for the effects of self capacitance will be necessary at high frequencies. The coils can be wound in several sections so that a variety of values of mutual inductance can be obtained.

If such a secondary standard is to be accurately adjusted to some definite value of mutual inductance, Campbell's* second method of construction is convenient (see Fig. 29 (f)). In this the primary consists of a multilayer coil; the secondary consists of two coils concentric with the primary, one being much smaller than the other. The two secondary coils are in series. One turn added to or taken from the small coil produces a very small change in the mutual inductance, which can thus be set with accuracy. For example, consider a coil of radius 10 cm. concentric with which is a coil of a cm. radius; if both coils have 1,000 turns each, the following figures show how M varies with a .

a cm.	1	2	3	4	5	6	7	8	9
M millihenrys	2	8	18	34	55	85	126	181	279

If the primary were of 10 cm. radius and the main secondary of 9 cm. each turn on a central 2 cm. coil would only have one-thirtieth of the effect of one turn on the main secondary.

The particular sources of error in the use of mutual inductance standards will be discussed in the next section.

11. Variable Standards. In adjusting the balance of a bridge, a step-by-step change of self or mutual inductance, such as is provided by a set of fixed value standards, is not sufficiently fine. What is required is an arrangement by which the inductance may be steadily and continuously varied between certain limits without altering the resistance of the part of the apparatus which lies in one of the balancing branches of the network. A piece of apparatus in which this may be performed is called by different writers a variable inductance, an inductor, a variometer, an inductometer, etc.

The desirable features to be possessed by an inductometer are as follows: (i) In self inductometers the time-constant should be large, and the space occupied by the inductometer small. In mutual inductometers the resistances of the windings should be low and the mutual inductance relatively

* A. Campbell, "Inductance measurements," *Electn.*, Vol. 60, pp. 626-627 (1908); also *N.P.L. Researches*, Vol. 4, p. 245 and Fig. 3 (1908).

high. (ii) The range between the maximum and minimum values of inductance should be large. (iii) Inductometers should conveniently possess a linear scale. (iv) Astatic arrangement of the coils is desirable. (v) Inductometers should be free from frequency errors due to capacitance, eddy-current, and insulation effects.

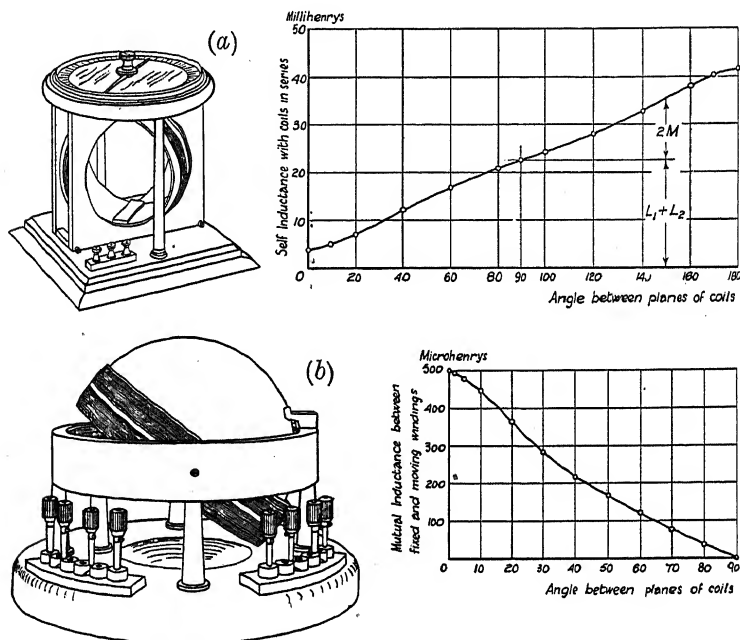


FIG. 30.—AYRTON-PERRY PATTERN INDUCTOMETERS

The simplest type of inductometer is that usually attributed to Ayrton and Perry,* illustrated in Fig. 30 (a). In this instrument two circular coils are arranged one within the other, the inner coil being capable of rotation about a diameter within the outer coil, which is fixed to the base of the apparatus in a

* "The measurement of inductances, capacities, and polarizing resistances," *Electn.*, Vol. 34, pp. 546-547 (1895). The type of apparatus was used by other investigators at an earlier date, notably by Brillouin and by Hughes. See also O. Heaviside, *Electrical Papers*, p. 37 (1892), for use as a variable self-inductance; and Lord Rayleigh, *Phil. Mag.*, 5th series, Vol. 22, pp. 473-477, 498-500 (1886), for use as mutual inductometer.

vertical position. The axis of rotation of the moving coil is vertical, and the angle between the planes of the two coils may be read on a scale of degrees. By using one coil as primary and the other as secondary, the instrument becomes a convenient mutual inductometer; as the inner coil is rotated the mutual inductance varies from a maximum value when the two coils are in the same plane to zero when they are perpendicular, and thence to a negative maximum when they again become coincident. By joining the two coils in series, a variable self-inductance is obtained having a value $L = L_1 + L_2 \pm 2M$ in any given position of the coils when M is the mutual inductance, L_1 and L_2 being the self inductances of the fixed and moving coils respectively. L is shown on a separate scale.

In a typical instrument of this type, the variation of self-inductance is shown by the curve in Fig. 30 (a). The self-inductance with the coils in series varies from 3.64 to 42.22 millihenrys, and the mutual inductance between ± 9.645 millihenrys. The time-constant is 3.95 milliseconds at the maximum self-inductance reading, the resistances of the fixed and moving coils being 5.44 and 5.26 ohms respectively.

In order to avoid eddy current errors at high frequencies, Wien* and others have constructed inductometers in which the amount of metal is reduced to a minimum and the coils are wound with stranded wire. In addition, each coil is wound in sections, so that several overlapping ranges may be obtained from the same instrument. In an inductometer designed by Dr. H. F. Haworth, ebonite is used throughout. The coils are arranged horizontally, one being wound on the inside of a horizontal ebonite ring, and the other upon a pivoted concentric former. Each coil is of stranded wire and has three sections of 5, 10, and 25 turns respectively. The calibration curve and illustration of the instrument are given in Fig. 30 (b). The maximum range of self-inductance

* Max Wien, "Ueber ein Apparat zum variieren der Selbstinduction," *Ann. der Phys.* Bd. 57, pp. 249-257 (1896). In this instrument, constructed of wood, the fixed coil has four windings and the moving coil two windings. The windings may be grouped in series as desired and a range of self-inductance from 0.4 to 120 millihenrys covered almost continuously. Full dimensions and calibration curves are given. H. Hausrath, "Induktions Variometer und Widerstands Kombinationen," *Zeits. f. Inst.*, 27 Jahrgang, pp. 302-312 (1907). Describes an instrument made of Stabilite, the fixed coil having four and the moving coil two windings. Self-inductance varies in eight ranges from 0.385 to 144.2 millihenrys. Dimensions are given.

is up to 2,044 microhenrys, and of mutual inductance 496 microhenrys. The time-constant for the maximum self-inductance is 0.351 milliseconds.

In the Ayrton-Perry and similar inductometers the two coils are made as nearly equal in radius as is possible, the wire being usually wound upon portions of spherical surfaces. The primary and secondary coils being close together, a large mutual inductance for a given bulk is obtained. This arrangement has the disadvantage that the variation of mutual (or self) inductance is far from linear. Lord Rayleigh (*loc. cit.*) has shown that a practically uniformly divided scale can be obtained if the radius of the inner coil is 0.548 of that of the outer coil. The space occupied by the apparatus for a given maximum M is, however, much greater than when the coils are more nearly equal. It is with the object of producing an inductometer with a high time-constant, a linear scale, and a small bulk that the instruments now to be described were designed. In these instruments two discs of ebonite are arranged over one another, the upper disc being pivoted at the centre of the lower. Each disc carries coils, the mutual inductance between the fixed and moving sets of which can be varied by rotation of the upper disc over the lower.*

In order that the apparatus should have the largest time-constant for a given quantity of wire, the fixed and moving coils, when over one another and joined in series, should form a coil of Maxwell's proportions wound on a circular former (Fig. 31 (a)). It is found that an inductometer made in this way has a scale which is far from linear, although from the point of view of space occupied for a given inductance, the apparatus is of the best possible proportions.

To obtain a more uniform scale, G. F. Mansbridge† has designed the inductometer shown in half plan in Fig. 31 (b), the coils being of D or semicircular shape. There are four coils, two embedded in the upper and two in the lower ebonite disc; each coil is wound in sections so that various ranges are obtained. For the higher values, all four coils are in series, the self-inductance varying between 9 and 105 millihenrys. For the lower values, with only a small section of each coil in use, the range is between 0.7 and 12 millihenrys. The

* A. Larsen, "Der komplexe Kompensator, ein Apparat zur messung von Wechselstromen durch Kompensation," *Elekt. Zeits.*, 31 Jahrgang, pp. 1039-1041 (1910). C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, Vol. 39, Part 2, pp. 1525-1529 (1911).

† G. F. Mansbridge, "Improvements in and relating to iron-free variable inductances," *British Patent*, No. 22,206 (1905). The apparatus is made by Messrs. Muirhead & Co., and a similar one by H. Tinsley & Co.

time-constant at the maximum reading is about 1.6 milli-seconds ; the division of the scale is practically uniform over the working range.

The most perfect inductometer of this type is due to Brooks and Weaver.* In this, a much greater time-constant is obtained by the use of link-shaped coils, by the proper proportioning of which a uniform scale can be obtained. Three

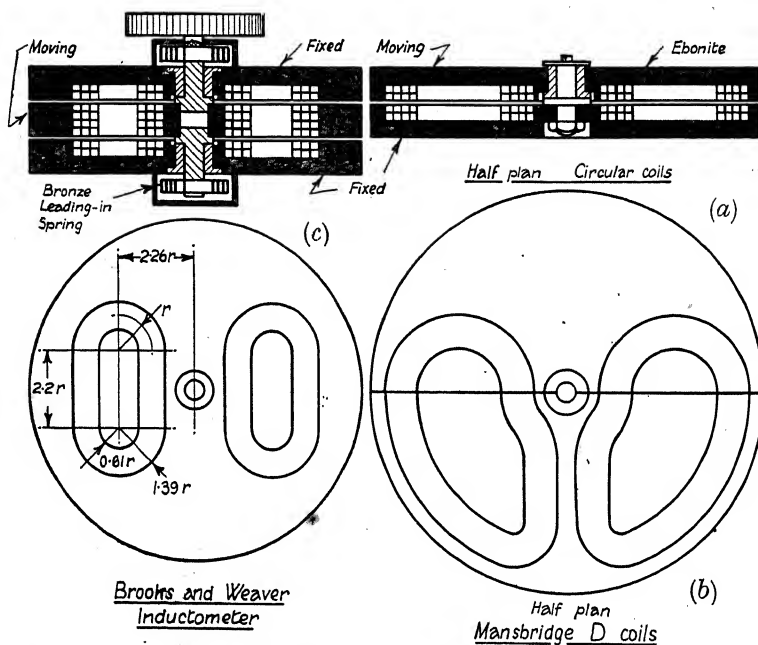


FIG. 31.—DISC PATTERN INDUCTOMETERS

pairs of coils are used ; two being fixed to ebonite or condensite discs, the remaining pair rotating between them. The best results are obtained when the coils have the relative dimensions shown. The coils are connected astatically and are wound with stranded wire. By the use of two sets of fixed coils, the effect of a small axial displacement, due to wear of the bearings, on the calibration is negligible (see Fig. 31(c)).

* H. B. Brooks and F. C. Weaver, "A variable self and mutual inductor," *Bull. Bur. Stds.*, Vol. 13, pp. 569-580 (1917). The instrument is made by the Leeds and Northrup Company in sizes up to 500 millihenrys. Messrs. Tinsley & Co. also make the inductometer.

CAMPBELL'S MUTUAL INDUCTOMETER. Mr. A. Campbell, late of the National Physical Laboratory, has shown that

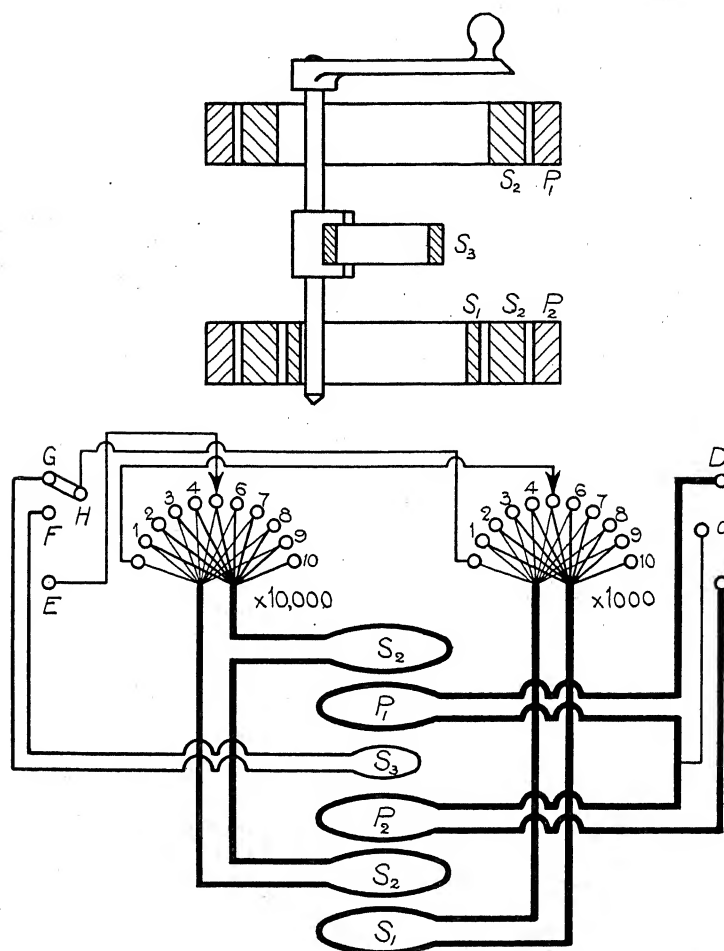


FIG. 32.—CAMPBELL'S MUTUAL INDUCTOMETER

standards of mutual inductance in alternating current bridges have marked advantages over self-inductances, principally for the reason that mutual inductometers can be varied down to zero and then to negative values, while self inductometers have not these properties. This advantage of a mutual

inductometer is of great value in the measurement of small inductances, and in the allowance for the inductance of leads, etc., in other measurements. Mr. Campbell has developed a large number of excellent bridge methods containing mutual inductance, and has devised a special mutual inductometer which is very widely used in practice. The arrangement and connections of the inductometer* are shown in Fig. 32, from which it is seen that the instrument is really a transformer of variable ratio.

The primary winding consists of two equal, coaxial, fixed coils, P_1 , P_2 , wound in the same direction and connected in series. Two terminals make connection with the primary, the point of junction of the two coils being brought to a third† terminal a . In a typical inductometer the total inductance of the primary was 0.10057 henry, and its resistance 40.15 ohms at 20°C.

The secondary winding is composed of three circuits which can be put in series, S_1 , S_2 , S_3 . The coil S_1 is made up of ten sections, each of which has exactly the same mutual inductance with respect to the primary, namely, 1,000 microhenrys in the above-mentioned example. The precise equality of the sections is ensured by the artifice of winding the coil with a stranded‡ rope of wires in the following way. Ten insulated wires of a suitable length are twisted together to form a stranded rope, which is then wound upon a bobbin to form a coil having a predetermined number of turns. It is then found that the mutual inductance between each strand and the primary winding is almost exactly the same.§ By the use of a dial switch, any number of strands may be joined in series and their mutual inductances added.

The coil S_2 is wound in the same way as S_1 , but partly on the lower and partly on the upper bobbin, and has such a number of turns that each of its ten strands has ten times

* A. Campbell, "On the use of variable mutual inductances," *Phil. Mag.*, 6th series, pp. 155-171 (1908); *Proc. Phys. Soc.*, Vol. 21, pp. 69-87 (1910). The instruments are made by the Cambridge and Paul Co.

† For the use of this mid-point in practice, see p. 252.

‡ Stranded inductometers were used by M. Brillouin, *Assoc. Française p. l'avancement d. Sc.*, pp. 333-336 (1881); *Comptes Rendus*, tome 93, pp. 1010-1014 (1881); *Ann. de l'École normale*, tome 11, pp. 339-424 (1882); and by H. Rowland, *Amer. J. Soc.*, 4th series, Vol. 4, pp. 429-448 (1897); *Phil. Mag.*, 5th series, Vol. 45, pp. 66-85 (1898).

§ For the method of final precise adjustment to equality, see G. Moore, *loc. cit.* (1921).

the mutual inductance of a strand in S_1 with respect to the primary, namely, 10,000 microhenrys. The total range of S_2 is thus 100,000 and of S_1 10,000 microhenrys.

The third section of the secondary is the coil S_3 which is mounted on an axis parallel to that of the primary coils. Moving this coil by means of the handle varies its mutual inductance with respect to the primary from a maximum down to zero and then to negative values. With the choice of suitable proportions* for the coil, a semicircular scale may be obtained covering an arc about 30 cm. long, the divisions at the upper and lower values being quite open, so that precise readings are possible. In the inductometer mentioned, the range of S_3 is from -40 to $1,060$ microhenrys. By means of the link shown, connecting F and H puts the coil S_3 in opposition to the coils S_1 and S_2 , thus enabling 1,000 on the scale to be checked against the first section of S_1 .

In normal working, the link connects G and H , the entire secondary winding being in series and connected to the terminals E , F . The range of the inductometer is thus from -0.00004 to 0.111060 henry. A smaller instrument of precisely the same pattern has a range of -3.5×10^{-6} to 0.011104 henry, and by the provision of terminals tapping one-tenth of the primary winding, a particularly low range inductometer, reading up to 0.0011104 henry, is obtained.

The conductors with which the coils are wound are themselves highly stranded, so that eddy currents therein are reduced to a minimum. No unnecessary metal is used in the construction of the inductometer for the same reason.

Errors in Inductometers. Self inductometers are subject to the same sources of error as fixed value self-inductances. They should be constructed, therefore, with a view to permanence of value, high time-constant, freedom from magnetic materials, small self capacity, high insulation resistance, and absence of skin-effect. The methods of construction described above will show how these conditions are met in practice and do not require amplification here. In addition, it should be pointed out that the change of inductance with temperature in inductometers of the Ayrton-Perry type is

* A. Campbell, *loc. cit.* Also see S. Butterworth, "On the coefficients of mutual induction of eccentric coils," *Phil. Mag.*, 6th series, Vol. 31, pp. 443-454 (1916).

small, Taylor* having shown it to be less than 1 part in 40,000 per °C.

Mutual inductometers require more discussion. An ideal mutual inductance is one in which the electromotive force induced in the secondary coil is exactly in quadrature with the primary current. Such an inductance is termed by Silsbee† “pure,” and is very closely represented by a pair of carefully constructed coils arranged not too close to one another and carrying low frequency current. However, at high frequencies these conditions no longer hold, since the effects of the self and intercapacities of the coils, of imperfect insulation, and of eddy current losses become appreciable. Arising from these effects a component of secondary electromotive force appears in phase with the primary current, so that the secondary electromotive force is no longer $j\omega Mi$ but $(\sigma + j\omega M)i$. The mutual inductance is then said to be “impure,” σ being called the “impurity.”

The effects of impurity are important at the higher acoustic frequencies, above about 2,000 cycles per second, particularly in stranded inductometers of the Campbell type when the primary and secondary have a common point, as in many bridge methods. Butterworth‡ has shown how the effects of impurity may be determined and allowed for; the reader is referred to his exhaustive paper for further details, some of these being discussed in Chapter IV.

The principal source of impurity in Campbell inductometers lies in the relatively great self capacitance of the coils, brought about by the proximity of the strands. Mr. Butterworth has designed an inductometer to work successfully at high frequencies in which this defect is removed. In this, the primary consists of a fixed coil; the secondary consists of three coils having mutual inductances of 1, 3, and 6 units with respect to the primary. By means of a suitable dial switch the mutual inductances of the coils can be added or subtracted to cover by unit steps a range from 1 to 10 units. A movable coil, as in the Campbell instrument, carries down the range from 1 to zero.§

STANDARDS OF CAPACITANCE

The standards of capacitance used in bridge measurements are of two kinds, absolute and secondary. Condensers of

* A. H. Taylor, “On the possible variation of inductance standards with temperature,” *Phys. Rev.*, Vol. 20, p. 394 (1905).

† F. B. Silsbee, *Bull. Bur. Stds.*, Vol. 13, p. 414 (1916).

‡ S. Butterworth, “Capacity and eddy current effects in inductometers,” *Proc. Phys. Soc.*, Vol. 33, pp. 312-354 (1921).

§ See N.P.L. Report for 1922, pp. 82-83.

the former type are constructed so that their capacitance can be calculated from their accurately measured dimensions. Secondary standards, on the other hand, have their capacitance determined by comparison with an absolute standard or otherwise.

Absolute condensers are only very rarely used in alternating current bridges; their principal application is to determine the ratio of the electromagnetic unit of electricity to the electrostatic unit. For this purpose condensers of quite small capacitance are suitable, and are made of some simple geometrical shape for which the capacitance can be easily calculated. For this reason they have the form of concentric spheres, flat plates or coaxial cylinders, and the dielectric is air.*

All condensers used in alternating current bridges are secondary standards of a capacitance considerably greater than that of the absolute condensers just mentioned. Secondary standards are calibrated and should have the following properties—

(i) They should be true condensers, i.e. the current taken by them when supplied with a sinusoidal p.d. should lead on the p.d. by $\pi/2$ and should be free from harmonics.

(ii) They should be free from losses and absorption effects in the dielectric.

(iii) The capacitance should be definite and permanent, and the standard should be compact for a given value of capacitance so that inaccuracy due to earth capacities may be very small.

(iv) They should be independent of frequency, wave-form, and temperature.

(v) The insulation resistance should be great and the condensers should be capable of withstanding high voltages.

The condensers used in practice fulfil these conditions in varying degree, according to the way in which they are constructed and to the material used as the dielectric. The dielectrics most frequently employed are air or other gas, oil, mica, treated paper, and glass. In accurate standards the only satisfactory materials are air and mica; paper

* See, for example, E. B. Rosa and N. E. Dorsey, *Bull. Bur. Stds.*, Vol. 3, pp. 433–604, 605–622 (1907), for accurate details of all these types and for references. For a cylindrical condenser see J. J. Thomson, *Phil. Trans. Roy. Soc.*, Vol. 174, pp. 707–721 (1884); and J. J. Thomson and G. F. C. Searle, *ibid.*, Vol. 181, p. 603 (1891).

condensers are sufficiently good for less accurate work or in cases where dielectric imperfections are of little importance. The other materials are only used where capacitance is required without the advantages of permanence, accuracy, and perfection, e.g. tuning condensers, or in filter circuits. Secondary condensers are, therefore, conveniently classified according to the dielectric employed in them, each group having special properties and uses which will now be described.

12. Air Condensers. Gases, such as air, when used as the dielectric in a condenser subjected to an alternating potential difference are almost entirely free from dielectric losses. A properly constructed air condenser is, therefore, the closest approach to a perfect condenser which can be attained in practice, i.e. one in which the current leads on the p.d. by a quarter period.

Absolute air condensers have usually only a single pair of electrodes, so that their capacitance is very small. To increase the value in secondary air condensers, it is necessary to use several sets of electrodes or plates in parallel, the two sets of plates forming the condenser being held in a suitable framework and separated from one another by supports of solid insulating material. Obviously the amount of solid insulation used to support the plates of an air condenser should be reduced to the minimum, and should stand in a weak electrostatic field. It should have the highest possible insulation resistance; for this purpose ebonite is much used, but has the disadvantage of yielding in course of time. Amberite* and fused quartz are preferable in air condensers of the highest class, as they are more permanent, though these materials are somewhat difficult to work.

Since the dielectric constant of air is low, it follows that a large number of electrodes in parallel must be used, each of large area, to produce a reasonable value of capacitance. An air condenser is, therefore, a very bulky standard, and the bulk is added to on account of two other important facts which impose a limit on the closeness of successive plates. Firstly, the dielectric strength of air is not very high, and, secondly, particles of dust may bridge across between the plates when the condenser is subjected to high voltage. This

* *Amberite*.—A synthetic material composed of amber chips moistened with ether and compressed into a solid mass.

produces internal leaks and greatly impairs the insulation of the condenser. To reduce this effect it is usual to make the distance between successive plates not less than 2 mm.; if care be taken to dry the air thoroughly, condensers can be made with 1 mm. air space or less.

Air condensers are of two classes: (i) in which the capacitance is fixed; (ii) in which the capacitance can be continuously varied. The former serve as fixed value reference standards; the latter as fine adjustments for use in balancing bridges, being frequently used in conjunction with standard mica condensers to carry the capacitance down continuously to the lowest values.

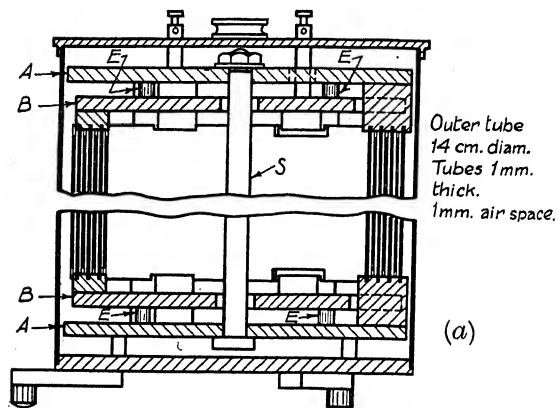
FIXED STANDARDS. Fixed value air condensers are made on the same plan as absolute condensers, but with several sets of plates in parallel to give adequate capacitance. The principal forms are made with cylindrical or with flat electrodes, both types being developed in several different designs.

An early form of cylindrical condenser is that designed for the Standards Committee of the British Association by Glazebrook and Muirhead.* It consists of 24 concentric brass tubes, 12 forming one electrode and 12 the other. These tubes are mounted upon two stepped brass cones, the lower one standing upon three pillars of ebonite. The upper cone is supported by the case of the condenser. The capacitance is about 0.024 microfarad.

A modern example of a cylindrical condenser is due to Giebe† and illustrated in Fig. 33 (a). This is composed of seven concentric brass tubes, four forming one electrode and three the other. One set of cylinders is fixed between the plates *AA* upon brass blocks, three projecting from each plate. The other set of cylinders is mounted in a similar way between the plates *BB*. The plates *BB* and the cylinders are slotted out to clear the projections from *AA*, upon which the first set of cylinders is mounted. The plates *AA*, *BB* are insulated from one another by corrugated ebonite cylinders *E*, the whole being clamped solid by the spindle and nut *S*. The lower plate *A* stands on the metal base of the condenser, which, in turn, is supported on ebonite feet. A metal case surrounds

* R. T. Glazebrook, "On the air condensers of the British Association," *Elecn.*, Vol. 25, pp. 616-619, 637-640 (1890).

† E. Giebe, "Normal Luftkondensatoren und ihre absolute Messung," *Zeits. f. Inst.*, 29 Jahrgang, pp. 269-279, 301-315 (1909).



Giebe's cylindrical air condenser

(b)

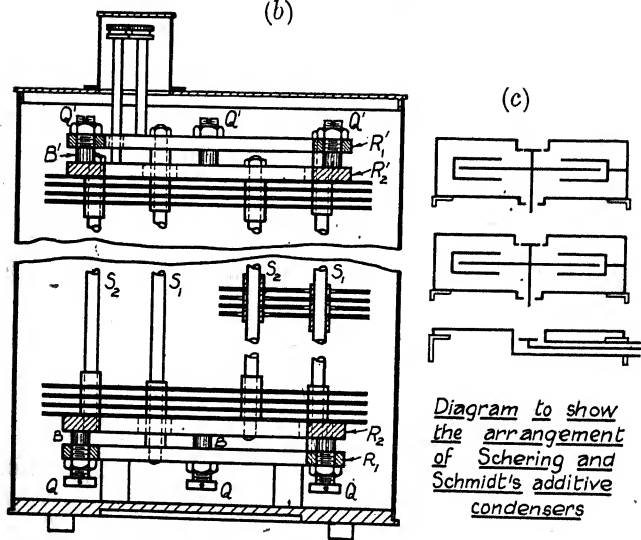


Diagram to show the arrangement of Schering and Schmidt's additive condensers

Giebe's plate air condenser

FIG. 33.—PRECISION AIR CONDENSERS

the cylinders and is connected to A , thus shielding the condenser from external electrostatic influences. With the dimensions given in the diagram, the capacitance is about 0.0109 microfarad.

Flat plate condensers of various forms are widely used. In a simple type the electrodes consist of square sheets of plate glass, each plate being coated on both sides with tin-foil. The glass sheets are arranged in a pile with small spacing pieces of glass at the corners to insulate successive plates from one another. The glass plates serve merely as convenient supports for the tin-foil coatings, the dielectric being the air between successive plates. The tin-foils on alternate plates are connected together to form the two sets of electrodes.*

In another type, due to Lord Kelvin,† 45 brass plates, each 10.13 cm. square, are prepared; 22 form one electrode and 23 the other. The plates of each set are suitably supported on metal rods, the second set of plates being at 45° to the first. With an air space of 3 mm. between successive plates the capacitance is about 0.001 microfarad.

Giebe‡ has constructed a precision flat plate condenser in which the plates are circular instead of square. Referring to Fig. 33 (b), the construction can be readily made out. The plates are of magnalium,§ 71 in number, each 1 mm. thick and 20 cm. diameter; 35 form one electrode and 36 the other, with a space of 2 mm. between successive plates. A bronze ring R_1 is fixed to the base of the condenser and carries four adjusting screws which support a second bronze ring R_2 upon short cylinders of amber, B . Four equally spaced brass rods, S_1 , are screwed into R_1 and pass through clearance holes in R_2 . Four other rods, S_2 , midway between S_1 are fixed to R_2 . The condenser plates are provided with eight holes, four being 5 mm. diameter and four 12 mm. diameter, so spaced that the plates can be dropped over the rods. The condenser is assembled in the following way: a plate is put on the rods to rest on R_2 , the rods S_2 passing through the 5 mm. holes. Distance tubes of a length to give 2 mm. air space are put on S_1 , and a plate of the second electrode put on the rods with the

* For a variety of simple condensers of this type, with and without guard rings, see W. E. Ayrton and T. Mather, *Practical Electricity*.

† Lord Kelvin, *Proc. Roy. Soc.*, Vol. 52, pp. 6-10 (1892-3); see also *Dictionary of Applied Physics*, Vol. 2, pp. 655-656.

‡ E. Giebe, *loc. cit.*

§ *Magnalium*.—A light, strong alloy of aluminium and magnesium.

5 mm. holes on S_1 . Distance tubes 5 mm. long are then slipped on S_2 , and a second plate of the first set put on. This procedure is carried out until the whole condenser is assembled. A bronze ring, R_2' is then put on the holes in it fitting S_2 and clearing S_1 , nuts on the top of S_2 clamping the set of plates between R_2 and R_2' . Suitable distance tubes are then put on S_1 and a ring R_1' added, the second set of plates being clamped between this and R_1 by nuts on S_1 . The air gaps between the two sets of plates are adjusted by moving the ring R_2 with its plates by means of the screws Q ; the two sets of plates are clamped together by the screws Q' insulated by amber rods B' . Terminals are attached to R_1' R_2' , the whole condenser being surrounded by a case joined to the ring R_1 and its associated plates. The condenser is about 30 cm. high, weighs 19 kilogrammes, and has a capacitance of about 0.01 mfd. It will break down at a voltage of 3,000.

In a second type, Giebe uses 107 plates with 1 mm. air gaps. The breakdown voltage in this case is 900 and the capacitance 0.03 mfd.

The insulation resistance of condensers constructed on this plan is very high, being of the order of 10^9 to 10^{10} ohms. If care be taken to remove dust in the condenser with 1 mm. gaps, and to dry the air by means of metallic sodium, the insulation resistance can be raised to 10^{15} ohms. In illustration, such condensers, when charged at 120 volts, lose only 5 per cent of their charge in 8 days.

Giebe's condensers are very permanent in value, and his later researches (*see* p. 113) have shown them to be practically perfect capacitances. They have a small temperature coefficient, of the order of 2 or 3 parts in 100,000 per degree.

In much standardizing work, units of larger capacitance are frequently required, and can be obtained by connecting several air condensers in parallel. As the total capacitance is not very large, the unknown capacitance of the leads used to connect the component condensers in parallel may cause an appreciable error in their combined value. In order that air condensers may be paralleled and have a definite capacitance equal to the sum of their separate values, Schering and Schmidt* have devised the principle illustrated in Fig. 33 (c).

* H. Schering and R. Schmidt, "Ein Satz Normal-Luftkondensatoren mit definierter Schaltungskapazität," *Zeits. f. Inst.*, 32 Jahrgang, pp. 253-258 (1912).

Their condensers are built on the plan of Giebe's flat plate instruments, but differ in the arrangement of the terminals. A base plate, forming one terminal of the assembled condenser, carries at its centre an insulated metal socket forming the second terminal. The condenser units contain two sets of plates, one connected to the case and the other insulated therefrom; the insulated set bears a central socket above and a central spring spigot below. Any condenser can be piled on top of any other or on the base, the spigot pressing firmly in the socket; to ensure central registration of each unit, the upper portion of each case is machined to fit interchangeably a recess in the base of the one above it.

Their condensers had 20 cm. diameter magnalium plates with capacitances of $0.005 \mu\text{F}$. (37 plates), 0.002 (15 plates), and two of 0.001 (9 plates). The air spaces were 5 mm. in the first two condensers and 5.8 mm. in the second pair. Tests were made by the Maxwell commutator method to measure the capacitances of the separate condensers and their combined values when assembled in various ways, as follows—

Condenser	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Measured value	0.0010001	0.0010001	0.0019997	0.0050002
Combination	<i>A + C</i>	<i>B + C</i>	<i>A + B + C</i>	<i>A + B + C + D</i>
Measured value	0.0029994	0.0029994	0.0039990	0.0090010
Calculated value	0.0029998	0.0029998	0.0039999	0.0090001

Plate condensers with a definite parallel connected capacity have been made by Mr. Paul on a principle similar to that of Schering and Schmidt.

VARIABLE STANDARDS. A continuous variation of capacitance is frequently required in bridge work, and is usually provided by a variable air condenser. In its commonest form, it consists of a set of fixed plates between which a set of moving plates can be passed, so that the active area of the condenser is continuously variable. The two sets of plates are generally approximately semicircular, as in Fig. 34, the moving set being rotated about a central axis so as to be more or less interposed within the fixed set. By proper proportioning of the plates, the changes of capacitance can be made over a wide range nearly proportional to the angle turned through by the moving plates, as the calibration curve shows.

In the commonest construction, semicircular plates of sheet aluminium are used, set on rods with distance pieces between the plates so that they are properly spaced out. The rods supporting the fixed plates are fastened to the ebonite cover of the condenser; the moving plates are assembled on the

central spindle which passes through a bearing in the cover and bears an ebonite knob by means of which the plates can be rotated. The entire condenser is enclosed in a metal case or in a glass vessel lined with tin-foil, serving to screen the condenser from external electrostatic influences. In practice, it is best to join the moving plates to the case. (See page 112.)

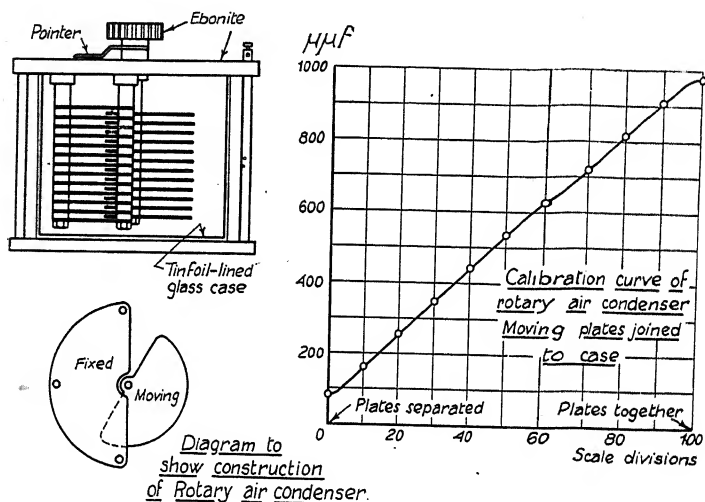


FIG. 34.—VARIABLE AIR CONDENSER AND CALIBRATION CURVE

This construction has many disadvantages. In course of time the upper bearing may wear and the moving plates descend somewhat, thus altering the value of the condenser. In the best types, therefore, the shaft is supported in bearings at both ends, or, alternatively, in a properly fitted conical bearing at the upper end. The shaft is continued a sufficient distance through the cover, so that the presence of the hand on the operating knob does not appreciably affect the capacitance. Connection is made with the moving plates by a flat spiral spring, one end being soldered to the shaft and the other to a terminal fixed on the ebonite cover.

In condensers of the best class the plates are not made separately from sheet metal and then assembled, but are machined out of the solid from magnalium castings in the

manner introduced by Seibt.* A stout construction is thus secured with air spaces of definite dimensions. The capacitance is indicated by a pointer attached to the central shaft moving over a scale marked on the ebonite cover. In the most accurate instruments the pointer carries a vernier, and a slow motion adjustment is provided by gearing the operating knob to the central shaft instead of attaching it directly thereto.† The scale is usually one of degrees and is calibrated. Such rotary condensers are made in various sizes, up to 3,000 μF .

Variable condensers are usually of the rotary pattern, though other types have been suggested. Lord Kelvin, for example, has described a form in which concentric cylinders are arranged to slide one within the other. Briggs‡ has made a condenser in which the air spaces are altered in thickness, but neither of these types is so convenient as the rotary pattern.

13. High Voltage and Compressed Air Condensers. Bridge methods are sometimes used to measure the dielectric losses in power cables at high voltages. The standard condensers against which the cable samples are compared are air condensers suitably built to withstand the high voltage.

Owing to the presence of dust streaks reducing the insulation resistance in air condensers charged at high tension, it is necessary to provide large air spaces; hence the capacitance is usually small. Moreover, there will be brush discharges and corona losses necessitating special design to reduce the imperfections due to these causes. Monasch§ and, more recently, Semm|| have designed suitable condensers. In these, care is taken to round off all sharp edges and to provide guard plates and screens, so that brush effects are reduced to a minimum; both experimenters use the cylindrical pattern.

The dielectric strength of air increases proportional to the pressure, and, at the same time, brush discharge and consequent losses at high voltage are much reduced. Condensers for very high voltages are sometimes made,** therefore, with a steel case so that the air within

* G. Seibt, *Jahrb. d. D. Tel.*, Vol. 5, p. 407 (1911); *Dictionary of Applied Physics*, Vol. 2, pp. 656-657.

† See Schering and Schmidt, *loc. cit.*, for a variable condenser of excellent design with a range from 0.0001 to 0.002 μF .

‡ L. J. Briggs, "A new form of electrical condenser having a capacity capable of continuous adjustment," *Phys. Rev.*, Vol. 11, pp. 14-21 (1900).

§ B. Monasch, *Ann. der Phys.*, Bd. 22, pp. 905-942 (1907).

|| A. Semm, *Arch. f. Elekt.*, Bd. 9, pp. 30-34 (1921); also W. Petersen, *Hochspannungstechnik*, p. 92, 104 (1911). For a flat plate condenser used up to 10,000 volts, see C. A. Butman, *Elec. World*, Vol. 71, pp. 502-506 (1918). Also see G. B. Shanklin, *Gen. Elec. Rev.* vol. 19, pp. 842-853, 1916.

** See F. Laws, *Electrical Measurements*, pp. 353-354 (1917); and *Dictionary of Applied Physics*, Vol. 2, pp. 123 (1922).

can be raised to a high pressure and these advantages secured. Though used in radiotelegraphy, condensers on this plan do not seem to have had much application in bridge work.

14. Earth Capacity Effects and other Sources of Error in Air Condensers. The capacitance measured across the terminals of an air condenser is not simply the intercapacity between its electrodes, since, for a given capacitance, the bulk of the condenser is so large that earth capacities of the electrodes cannot be neglected. The effects are extremely important when the capacitance is less than $100 \mu\mu\text{F}$. It is shown on page 13 that the working capacity of a condenser is $C = c_{12} + (c_1 c_2 / (c_1 + c_2))$, where c_{12} is the intercapacity between the electrodes, c_1 and c_2 being the capacities of the electrodes to earth. In practically all air condensers a metallic screen or case completely surrounds the electrodes, and can, at will, be joined to one or other set of plates or simply left free. It is usual to connect one electrode to the case, in rotary variable condensers the moving system being so joined and the case arranged to be as near earth potential as possible. By this means the working capacity of the condenser will be but slightly influenced by the presence of the hand upon the operating knob. In this instance the terminal capacity is $C = c_{12} + c_1$ simply.

In any particular arrangement of electrodes and screen the effects of earth capacity can be readily allowed for by Orlich's method of making three measurements from which c_1 , c_2 and c_{12} can be found. Let the condenser have three terminals, 1, 2, 3, connected respectively to the two electrodes and the case. Measure the capacitance between 1 and 2, (a) with 1 connected to the case, and (b) with 2 connected to the case. In (a) the measured capacitance is $C_a = c_{12} + c_2$, and in (b) is $C_b = c_{12} + c_1$. Now join 1 and 2 together and measure their combined capacitance to the case, i.e. $C_c = c_1 + c_2$. Then from these three measurements, c_1 , c_2 , and c_{12} are easily calculated, and thence the working capacity for any method of connection.

In a test recorded by Orlich* the following values were obtained : $C_a = 25.6 \mu\mu\text{F}$, $C_b = 24.1 \mu\mu\text{F}$, and $C_c = 9.0 \mu\mu\text{F}$, from which $c_1 = 3.75 \mu\mu\text{F}$, $c_2 = 5.25 \mu\mu\text{F}$, $c_{12} = 20.35 \mu\mu\text{F}$. With both electrodes insulated from the screen, $C = 22.53 \mu\mu\text{F}$, which exceeds c_{12} by over 10 per cent due to the earth capacities.

* E. Orlich, *Kapazität und Induktivität*, p. 179.

This example shows the extreme importance of specifying exactly the arrangement of the connections to the terminals and case of an air condenser, owing to the large effect of earth capacities.

In precise work, air condensers are found to possess certain small imperfections due to three distinct causes. If it were possible to arrange that the dielectric between the plates were entirely air, the condenser would be quite free from dielectric loss (at ordinary voltages) and would behave as a perfect capacitance. It is necessary to support the plates and to insulate them from one another by pieces of solid dielectric in which, under alternating voltage, some absorption loss is bound to occur. Hence the amount of solid dielectric in an air condenser should be kept down to the absolute minimum, and should be made of some material having low absorption; it should be so placed that it lies in a weak electric field. Ebonite, or, better still, amber or quartz, are suitable materials. The imperfection occurs not only in the dielectric of c_{12} but also in the dielectric interposed in c_1 and c_2 , the earth capacities. Hence in variable condensers the ebonite top should be made of the best possible material.

A second cause of imperfection is faulty insulation resistance. Hence the solid materials employed should be very good insulators. The materials mentioned are excellent for the purpose, but ebonite is rather liable to deteriorate under the action of sunlight and dust, especially when used for the tops of variable condensers; amber or quartz are free from this objection and, in addition, keep their shape more permanently than ebonite, which in time yields to pressure, especially under the action of heat. To avoid faulty insulation in the air between the plates, some drying agent should be kept within the condenser case and the air should be freed from dust.

The third cause of imperfection lies in the resistance of connections within the condenser. Even if the condenser itself be perfect, internal resistance of connections will occasion losses and produce imperfection. This defect is most frequently encountered in variable condensers in the connection between the terminal and the moving plates. A properly designed flat spring device mentioned above overcomes the difficulty.

The researches of Giebe and Zickner* on the causes of

* E. Giebe and G. Zickner, "Verlustmessungen an Kondensatoren," *Arch. f. Elekt.*, Bd. 11, pp. 109-129 (1922).

imperfection in condensers have shown that in precise work no air condenser can be considered *absolutely* perfect, but that by proper attention to construction, to insulation and to connection in use, a very close approximation to an ideal condenser can be secured, in which the imperfections are too slight to be taken into account in any but the most refined experiments.

15. Liquid Condensers. The capacitance of air condensers is usually small, but can be increased by making the case leak-proof and filling it with a suitable oil. This is frequently done in the case of rotary variable condensers when larger capacitances are required. A suitable oil is paraffin* of high boiling point, thoroughly dried before being put into the condenser. The capacitance is approximately doubled, but there is a small dielectric loss. Castor oil is not so satisfactory, for, although it multiplies the capacitance about four times, the power-factor of the condenser greatly increases at high frequencies. Paraffin is practically free from frequency effects.

16. Condensers with Solid Dielectrics. General. The capacitance obtained in fixed-value air condensers is small, being of the order of a few hundredths of a microfarad or less, and the much larger values frequently required in bridge work cannot be obtained without an enormous increase in the number of electrodes and consequent bulk of the condenser. In order to overcome these defects, high value condensers are constructed with some solid dielectric, having a dielectric constant greater than that of air and of high breakdown strength, so that very thin layers of dielectric can be interposed between the plates, with a consequent increase in capacitance. In practice, many solid materials are used in the manufacture of condensers, e.g. glass, ebonite, waxed paper, mica, but the two materials last named are the only ones used in condensers which are to serve as standards.† Both paper and mica are easily obtainable in the form of very thin sheets and have a high breakdown strength, especially in the case of the latter material.

Condensers made with solid dielectrics show certain imperfections when used with alternating current, due to energy losses produced by dielectric hysteresis, imperfect insulation, and internal resistance. If a condenser be charged with a

* *Dictionary of Applied Physics*, Vol. 2, p. 123. Also S. H. Anderson, "Effect of frequency on the capacity of a condenser with kerosene for the dielectric," *Phys. Rev.*, Vol. 34, pp. 34-39 (1912).

† For a consideration of these and other materials see *Dictionary of Applied Physics*, Vol. 2, pp. 116-120.

certain quantity of electricity and is then discharged, the discharge is found to consist of two parts: (i) the so-called free charge, which is given up during the first instants of the discharge period; and (ii) the bound charge, which is only given up very slowly as the short-circuit between the condenser plates is continued. This bound charge may be a considerable fraction of the original charge in the condenser and may take days to be completely removed. The phenomenon has its seat in the physical nature of the dielectric and is referred to as *absorption*; many theories have been advanced in explanation of absorption, but it cannot be said that the laws governing it are very well understood.

If, now, an absorptive condenser be taken through a cyclic charge and discharge, as in an a.c. circuit, the absorption occasions a dissipation of energy, since the charge admitted into the condenser in one interval of time is not entirely returned in the next discharge interval. If the instantaneous value of the charge throughout a cycle be plotted as a function of the instantaneous charging voltage, a closed curve will result, the area within the curve representing energy lost per cycle due to absorption effects. By analogy with a similar effect in ferromagnetism the phenomenon is referred to as *dielectric hysteresis*. The importance of the effect will increase at high frequencies.

Energy is also dissipated in condensers due to leakage conductance through the dielectric or over the insulation between the terminals. Again, ohmic resistance in the internal connections to the plates, or, at high frequencies with thin plates,* in the electrodes themselves, will also produce losses.

In a general way, all these losses are taken together and are referred to as the condenser energy loss. In consequence, the current through such an *imperfect condenser* will not lead on the applied voltage by $\pi/2$, but will differ therefrom by an angle θ , called the *phase-difference* or *loss angle* of the condenser. The angle θ is a measure of the imperfection of the condenser, since it depends upon the losses therein; the power-factor is clearly $\sin \theta$ and will depend on the frequency. A *perfect condenser* is one in which there is no energy loss, so that

* J. C. Coffin, "The effect of frequency upon the capacity of absolute condensers," *Phys. Rev.*, Vol. 25, pp. 123-135 (1907).

$\theta = 0$; this ideal is very nearly attained in a condenser with air or other gas for the dielectric.

So far as its effect in an a.c. circuit is concerned, an imperfect condenser can be represented by a perfect capacitance in combination with a resistance. The equivalent capacitance

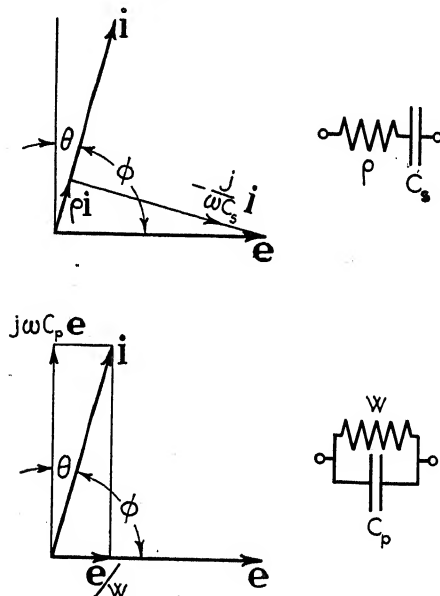


FIG. 35.—ILLUSTRATING THE CIRCUITS EQUIVALENT TO AN IMPERFECT CONDENSER

and resistance can be arranged either in series or in parallel. The resistance is chosen to dissipate the same power as in the imperfect condenser, and in combination with the capacitance to give the same phase-difference. The equivalent series arrangement is usually more convenient in practice, but the parallel arrangement is sometimes useful; both are illustrated in Fig. 35.

In the case of the series circuit, Fig. 10 gives $z = \rho - (j/\omega C_s)$; from which, numerically,

$$\tan \phi = 1/\omega \rho C_s$$

$$\text{and} \quad \tan \theta = \omega \rho C_s$$

For small values of θ , as in good condensers,

$$\cos \phi = \sin \theta \approx \tan \theta$$

In the parallel circuit, Fig. 10 shows that

$$z = W(1 - j\omega C_p W)/(1 + \omega^2 C_p^2 W^2);$$

so that, numerically,

$$\begin{aligned}\tan \phi &= \omega W C_p \\ \tan \theta &= 1/\omega W C_p.\end{aligned}$$

The two systems are very simply related; clearly the effective resistances and reactances of the two circuits must be equal, i.e.

$\rho = W/(1 + \omega^2 C_p^2 W^2)$ and $1/\omega^2 C_s = C_p W^2/(1 + \omega^2 C_p^2 W^2)$.
so that

$$W = \rho \left(1 + \frac{1}{\omega^2 C_s^2 \rho^2} \right) \text{ and } C_p = C_s / (1 + \omega^2 C_s^2 \rho^2).$$

With good condensers, $\tan \theta$ is small, so that

$$C_p = C_s \text{ and } W = \rho / \omega^2 C_s^2 \rho^2;$$

hence the equivalent capacitance is then approximately the same whichever circuit be assumed, and W is a very large resistance.

The importance of the phase-difference, θ , as an index of the perfection of a condenser cannot be over-emphasized, since it is the best single test which can be made of the suitability of a condenser to serve as a standard. In a good condenser, e.g. a mica standard, θ is a very small angle; in a poor condenser θ may be large, even several degrees. A large phase-difference is usually accompanied by instability of the condenser and by large variations of capacitance with frequency and temperature. The measurement of θ involves certain difficulties and necessitates special methods, which are dealt with in Chapter IV (*see also* p. 280).

17. Paraffined-paper Condensers. Condensers of large capacitance, say up to 20 microfarads, are conveniently made with paper for the dielectric. A thin, tough paper, free from loading materials, is used, and after being thoroughly dried is impregnated with paraffin wax. Other materials, such as beeswax, oil, and shellac are sometimes employed, but are usually less satisfactory. The plates of the condenser are sheets of tin-foil, alternate sheets being joined together to form the two electrodes. Between each pair of tin-foils, two

or three sheets of paraffined paper are put, the completed condenser being compressed to form a solid mass, and frequently confined between wood or metal clamping plates.

While paraffined paper condensers are very useful in bridge work, they suffer from certain serious defects which render them unsuitable as reference standards of capacitance, condensers with mica dielectric being vastly superior, both from the point of view of permanence and of freedom from dielectric imperfections. If frequently calibrated, they serve quite well in ordinary testing work, but are particularly useful in cases where accuracy and perfection are not requisite, e.g. in wave-filters, oscillator circuits, etc., especially when large capacitance is required, since large mica condensers are extremely costly.

On account of the fact that paraffined paper condensers are less constant than mica condensers, and have much larger phase-differences and frequency errors, Grover* concludes that paper condensers form poor standards of capacitance. Moreover, it does not seem that any definite rules can be formulated to say, in general, how paper condensers behave with changes of frequency and temperature. The following facts are, however, fairly well established for condensers used in bridge work, though commercial condensers may depart very considerably from these rules.

(i) The changes of capacitance with frequency and temperature are greater than for mica condensers.

(ii) The capacitance usually diminishes with increase of frequency, the variation with frequency being greater at higher temperatures.

(iii) At constant frequency the capacitance generally falls with rise of temperature. In a condenser of small phase difference the temperature coefficient of capacitance is usually negative and is nearly constant, not exceeding about -5 parts in 10,000 per °C. In condensers of large power-factor the temperature coefficient may be positive, or may change from negative to positive, its value increasing considerably at high temperatures and low frequencies.

(iv) The phase difference, and hence the power factor, depends very much on the materials used in the condenser, and particularly on the dryness of the paper dielectric. Grover found values of the phase difference ranging between 6' and 22° in good and in poor condensers respectively, corresponding to power-factors between 0.0017 and 0.37. The values in poor condensers rise rapidly at low frequencies and high temperatures.

* F. W. Grover, "The capacity and phase-difference of paraffined paper condensers as functions of temperature and frequency," *Bull. Bur. Stds.*, Vol. 7, pp. 495-578 (1911).

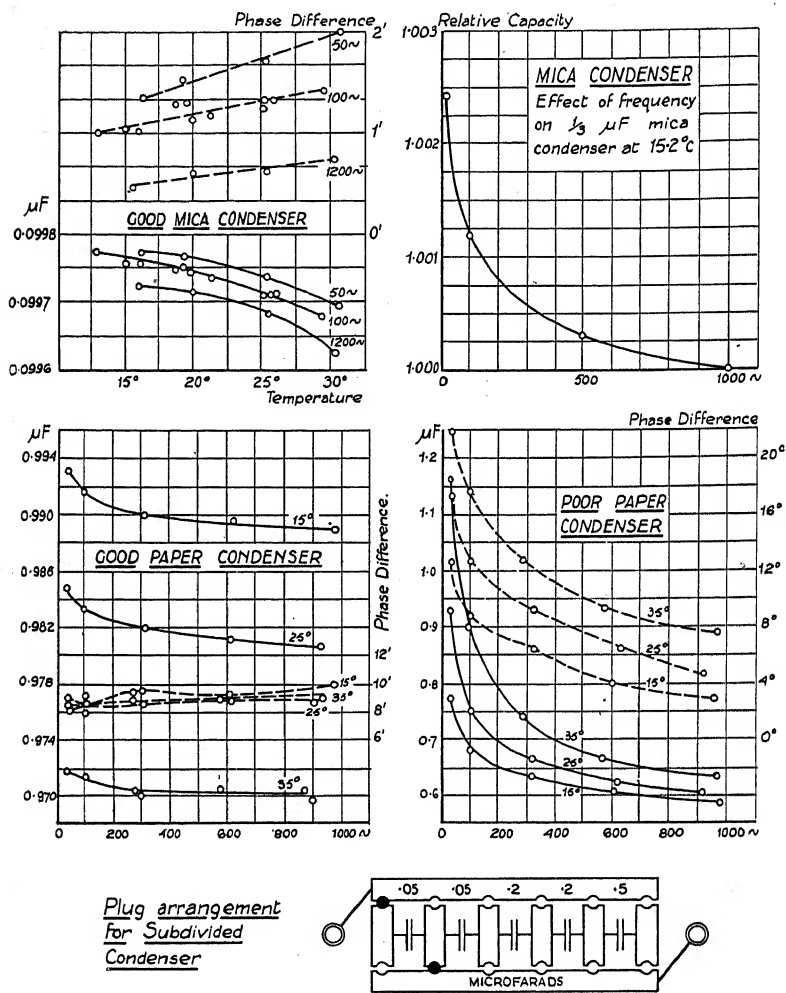


FIG. 36.—ILLUSTRATING THE PROPERTIES OF PARAFFINED-PAPER AND MICA CONDENSERS. ARRANGEMENT OF SUBDIVIDED CONDENSER

(Capacitance shown by full lines ; phase difference by dotted lines)

In Fig. 36 curves are drawn to show the comparative behaviour of good and poor paraffined paper condensers over a range of temperature and frequency, the values being taken from Grover's paper just cited.

18. Mica Condensers. Although fixed-value air condensers form the ultimate reference standards in bridge measurements, mica condensers are the usual working standards, since they can be made of adequate capacitance with small bulk. They can be constructed in such a way as to have very small imperfections, and their variations with temperature and frequency are regular and definite.

Mica* has the advantage that it can be split into very thin plates of very great crushing strength, and has very large electrical constants. Specimens 0.1 mm. thick have a dielectric strength of about 1.2×10^6 volts/cm.; the dielectric constant lies between 4 and 8, and the insulation resistivity is exceedingly high. The mica used for the construction of standard condensers is of the best clear ruby variety and is split into very thin sheets. As it is not possible to obtain very large sheets, a considerable number of thin laminae are required to make even a small capacitance; for example, a 0.1μ F. condenser made in the City and Guilds College laboratories had 20 sheets of mica, each 8.0 cm. \times 6.5 cm. and 0.05 mm. thick. Moreover, the material is expensive, so that mica condensers are not usually made of greater capacitance than 1μ F.

The sheets of mica are arranged in a pile with alternate sheets of tin-foil, the operation being carried out under melted paraffin wax. All trace of moisture must be carefully removed, and air bubbles must be excluded. The assembled pile is then taken out of the wax and is squeezed so that much of the wax is removed, the smallness of the temperature coefficient and other properties of the condenser depending very considerably on the amount of paraffin left between the plates. The completed condenser is then adjusted to its approximately correct value and is firmly clamped between stout brass or stabilite plates, since unclamped condensers do not remain permanent in value. The two sets of electrodes are soldered to suitable connecting leads, and the condenser is sealed up in

* Mica is a mineral form of silicate of alumina with potash (Muscovite) or magnesia (Biotite). It is found in the form of monoclinic (pseudo-hexagonal) crystals which cleave readily into very thin laminae.

an air-tight case to prevent it from absorbing moisture from the atmosphere.

Another process of construction is sometimes adopted in which the tin-foil plates are done away with, the electrodes being formed of a coating of silver deposited chemically upon the mica laminae. Such condensers, however, show somewhat erratic changes of capacitance, especially when tested at different voltages. These instabilities have been traced to deposits of silver upon flakes of mica not properly attached to the main deposit.

A very thorough investigation of the properties of mica condensers has been made by Curtis,* in order to determine the conditions under which they can be used as reliable working standards of capacitance. He shows that, although several condensers may have different properties from one another, any individual condenser behaves in quite a regular and definite way under specified physical conditions. His conclusions are summarized below—

(i) The capacitance diminishes with increase of frequency, the rate of decrease becoming small at high frequencies. Curtis calls the limiting value to which the capacitance tends as the frequency approaches infinity, the *geometric capacity*, since it depends only on the dimensions of the condenser and the dielectric constant. In a good condenser, with a small phase difference, the change with frequency† from 50 up to about 2,000 cycles per second may be about 1 part in 1,000; but in poor condensers it may be three or four times as much.

(ii) The temperature coefficient of capacity‡ at constant frequency is generally negative, the capacitance falling from 1 to 3 parts in 10,000 per °C. rise. If much paraffin be left between the plates, abnormal values as high as 10 in 10,000 may occur; Curtis has shown that by squeezing out much of the paraffin, the temperature coefficient can be reduced, and may be made zero or even caused to become positive. Since the dielectric of mica and wax is a poor conductor of heat, a condenser will only slowly attain the temperature of its surroundings; for this reason the N.P.L. keep mica condensers in a

* H. L. Curtis, "Mica condensers as standards of capacity," *Bull. Bur. Stds.*, Vol. 6, pp. 431-488 (1910); also F. W. Grover, "Simultaneous measurement of the capacity and power factor of condensers," *Bull. Bur. Stds.*, Vol. 3, pp. 371-431 (1907).

† See also B. V. Hill, *Phys. Rev.*, Vol. 26, pp. 400-405 (1908); and S. Butterworth, "On the use of Anderson's bridge for the measurement of the variations of the capacity and effective resistance of a condenser with frequency," *Proc. Phys. Soc.*, Vol. 34, pp. 1-7 (1922).

‡ See E. M. Terry, *Phys. Rev.*, Vol. 21, pp. 193-197 (1905); also D. Owen *Proc. Phys. Soc.*, Vol. 27, p. 52 (1915).

constant temperature room for 24 hours before making measurements. With silvered mica condensers the temperature coefficient is small and positive.

(iii) The phase difference of a good condenser at normal temperatures and frequencies may lie between $1'$ and $5'$, corresponding to a power factor between 0.0003 and 0.0015. Values as low as 0.0001 are sometimes met with in the very best condensers. The phase difference increases with rise of temperature and with fall of frequency. Abnormal values of θ are not infrequently found in small condensers and are a sure indication of poorness of quality.

(iv) Finally, a mica condenser forms a very permanent standard; its capacitance will remain constant within 1 or 2 parts in 10,000 for years. The temperature and frequency variations are definite and regular, and can be made quite small. Mica condensers should always be firmly clamped; the tests made by Curtis show that unclamped condensers are unstable and are affected by changes of atmospheric pressure. Clamped standards are but little affected by pressure changes.

In Fig. 36, characteristic curves for a good condenser are shown, from which the reader may see how nearly the above rules are followed in an actual example.

19. Grouping of Condensers in Working Standards. Standard condensers are arranged in a variety of ways according to the uses to which they are to be put. Single value mica standards are frequently used, a mica condenser being sealed up in a box provided with suitably insulated terminals. In most bridge work a subdivided condenser is very convenient; several mica condensers, totalling 1 microfarad, are assembled in a box. A system of blocks and plugs enables the sections to be combined in any desired fashion, one such arrangement being shown in Fig. 36. In this the condensers may be grouped in series or parallel, or any section can be discharged. Simpler plug arrangements are sometimes provided by which the parallel grouping only is possible, the same end being also served by a suitable system of dial switches and contact blocks.

The plug arrangement has a disadvantage when applied to small condensers, that the capacitance of the blocks is included in the measurements and the error is different for various settings of the condenser. Such small condensers are frequently mounted together in a box, each having its own pair of terminals, so that connection capacitances are reduced to the least possible.

SOURCES OF ALTERNATING CURRENT

20. General Remarks. On page 6 it has been shown that the modern alternating current bridge is developed from the older ballistic bridge to which some rotating commutator, such as a "secohmmeter," has been applied in order to increase the sensitivity of adjustment. Such a commutator supplies the bridge with a current which is periodically interrupted or reversed, and, together with its battery, may be looked upon as a crude form of periodic or alternating current source.

Another simple device, which is much used in rapid laboratory tests, is the "buzzer," shown in Fig. 37 (*a*). This piece of apparatus is the same in principle as an ordinary trembler bell, a rapidly vibrating armature making and breaking the current supplied to the bridge.* This current may also act as the maintaining current of the buzzer, as the diagram shows, or two independent currents may be used. The frequency of interruption is not usually very steady, and the armature is easily put out of adjustment. Hence, a buzzer is unsuitable in any case where the bridge balance conditions or the sensitivity of the detector depend upon constancy of frequency.

In all precise measurements with modern bridge methods it is usual practice to use a vibration galvanometer tuned to a definite frequency. Moreover, in many bridges the conditions of balance depend upon the frequency of the applied current. Hence, an ideal source of alternating current should have a constant and known frequency. The wave-form of the current should preferably be sinusoidal, so that methods in which balance depends upon frequency can be used with telephones, and also in order that tests may be carried out under simple, known wave-form conditions. Absolute purity of wave-form is not so very essential if the source is to be used in conjunction with a tuned detector, owing to the enormous sensitivity of such an instrument to the frequency for which it is tuned and its insensitiveness to all others.

21. Interrupters. A very common source of current with a constant, known frequency is a small induction coil or transformer in combination with an interrupter. The bridge current is taken from the secondary winding; the primary

* See K. Perlewitz, "Elektrische Signalhuppen," *Elekt. Zeits.*, 29 Jahrgang, pp. 445-450 (1908).

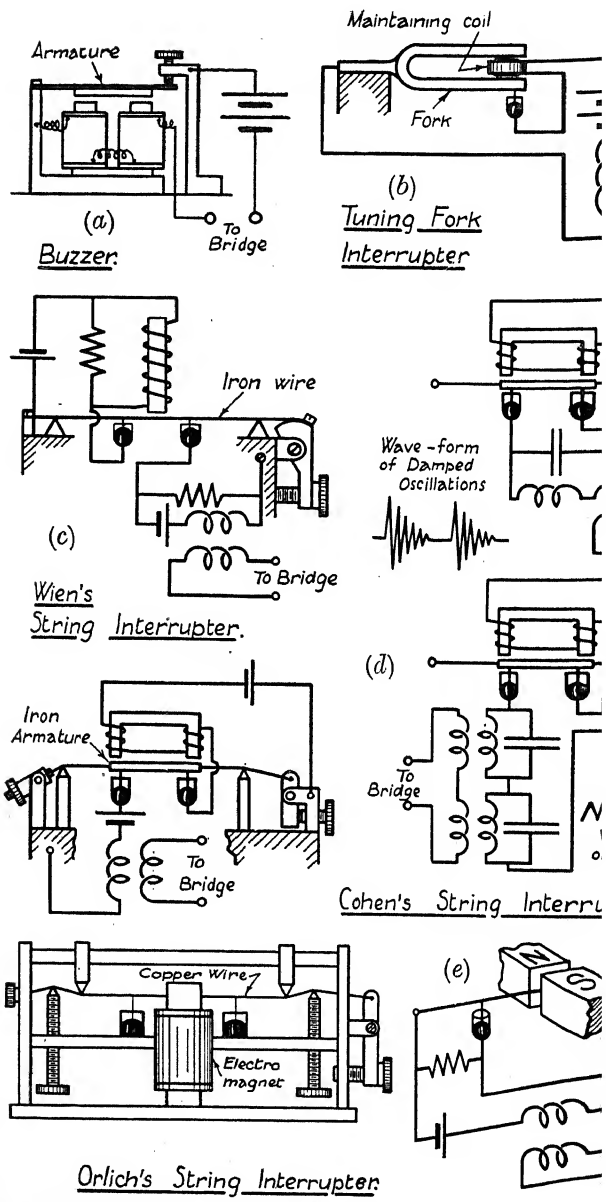


FIG. 37.—INTERRUPTERS

circuit contains a battery and an interrupter working at a constant frequency.*

A simple interrupter is an electrically maintained tuning fork† of suitable frequency, operating a contact by means of which the primary circuit is regularly opened and closed (see Fig. 37 (b)). The frequency is fixed by the pitch of the fork and is, therefore, very constant; the wave-form at the secondary terminals is not sinusoidal. Frequencies lying between 5 and about 1,000 cycles per second are obtainable.

Instead of using a tuning fork as the source of constant frequency oscillations, a stretched wire is often employed. The arrangement is then called a "string" interrupter, and was first extensively used by Max Wien.‡ In its simplest form, a string interrupter consists of a thin iron or steel wire stretched over two bridges which determine its vibrating length. The natural frequency of the wire is adjusted by varying the distance between the bridges and by altering the tension of the wire by means of a screw (see Fig. 37 (c)). Near the middle of the wire two platinum contacts are attached so that, when the wire is at rest, they just touch the surface of mercury contained in two adjustable cups. Over the centre of the wire an electromagnet is fixed, its exciting winding being connected in series with a battery through the left-hand half of the wire and the corresponding mercury cup. The right-hand mercury cup is used to act as an interrupter in the primary circuit of the induction coil.

The action of the apparatus is simple. Current flowing in

* D. Dvorák, "Ueber verschiedene Arten selbthätiger Stromunterbrecher und deren Verwendung," *Zeits. f. Inst.*, 11 Jahrgang, pp. 423-439 (1891).

† For various methods of electrically maintaining a tuning fork, see S. P. Thompson, "Note on a mode of maintaining tuning forks by electricity," *Proc. Phys. Soc.*, Vol. 8, pp. 72-76 (1887); W. G. Gregory, "On a method of driving tuning forks electrically," *Proc. Phys. Soc.*, Vol. 10, pp. 288-290 (1890); W. H. Eccles, "The use of the triode valve in maintaining the vibration of a tuning fork," *Proc. Phys. Soc.*, Vol. 31, p. 269 (1919); S. Butterworth, "The maintenance of a vibrating system by means of a triode valve," *Proc. Phys. Soc.*, Vol. 32, pp. 345-360 (1920). Also, H. M. Dadourian, "On the characteristics of electrically operated tuning forks," *Phys. Rev.*, 2nd series, Vol. 13, pp. 337-359 (1919).

‡ F. Niemöller, "Ueber einen neuen Stromunterbrecher," *Ann. der Phys.*, Bd. 6, pp. 302-304 (1879). Max Wien, "Das Telephon als optischer Apparat zur Strommessung," *Ann. der Phys.*, Bd. 42, pp. 593-621 (1891), Bd. 44, pp. 681-688 (1891); "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 689-712 (1891). For the theory of maintained oscillation of a wire, see S. Butterworth, "On electrically maintained vibrations," *Proc. Phys. Soc.*, Vol. 27, pp. 410-424 (1915).

the winding of the electromagnet magnetizes the latter and attracts the iron wire pulling it upward. This breaks contact at the left-hand cup, whereupon the wire flies back and re-makes the maintaining circuit. The monocord is thus maintained in continuous vibration, in the course of which the contact at the right-hand cup is made and broken, so that the primary of the induction coil carries an interrupted current. Sparking at the surface of the mercury is minimized by the use of the shunt resistances shown in the diagram, or by covering the mercury with a layer of paraffin oil. A range of frequency from 50 to 500 cycles per second can be obtained.

Cohen* has described a slight modification of the string interrupter which enables it to be used for the production of alternating as distinct from interrupted current. The apparatus is similar to that just described. A steel wire, the tension and length of which can be varied, carries at its centre a soft iron armature and the two platinum contacts. The maintaining magnet is of horse-shoe form and can be adjusted relatively to the wire until steady vibrations are produced.

In the first diagram (Fig. 37 (d)) the device is used as an interrupter, the frequency being adjustable between 20 and 500 cycles per second by altering the position of the bridges and the tension of the wire.

The second diagram shows how the interrupter may be used to produce damped high frequency oscillations. The wire is maintained in steady vibration, so that it makes and breaks the circuit of a battery connected to an oscillatory circuit composed of an inductance and condenser in parallel. Each time the interrupter breaks contact a train of damped waves is produced in the oscillatory circuit, the frequency of the waves depending only on the value of inductance and capacity used. There is no oscillation on making the contact, since the maintaining battery acts as a short-circuit to the oscillating portion of the circuit. The wave form of the current in the secondary consists of a series of damped oscillations of a known frequency, the trains of waves following one another at the frequency of the interrupter. The particular instrument described can be used to produce oscillations varying between 560 and 2,500 cycles per second by simple adjustment of the inductance and capacity of the oscillatory circuit, the interrupter working at a constant rate.

By arranging two oscillatory circuits tuned to the same frequency, and suitably proportioning the inductances, capacitances, and resistances, the secondary oscillations can be made to be nearly continuous and approximately sinusoidal, as the third diagram shows.

* B. S. Cohen, "The production of small variable frequency alternating currents suitable for telephonic and other measurements," *Proc. Phys. Soc.*, Vol. 21, pp. 283-297 (1910). For another variant see *Dictionary of Applied Physics*, Vol. 2, pp. 395-396.

The reader is referred to Cohen's paper for a detailed description of the uses of the apparatus in telephonic research, and for the excellent series of wave-form oscillograms taken with the interrupter working under various conditions.

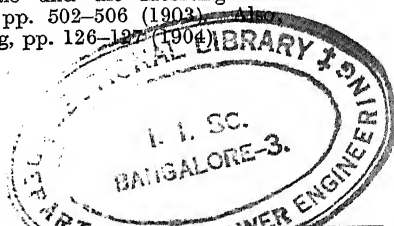
The disadvantage of the string interrupters hitherto described is that the maintaining circuit which is made and broken by the vibrating wire is highly inductive, since it contains the windings of a powerful electromagnet. Hence there will be troublesome sparking at the surface of the mercury in the contact cups. In order to avoid this trouble, Arons* has introduced a type of interrupter in which the wire is made of copper and lies in the field of a strong permanent magnet. The mercury contacts are only called upon to break the slightly inductive circuit of the wire itself. Orlich† has improved this instrument, in the pattern used at the Physikalische Reichsanstalt, by the use of an electromagnet and various devices by means of which the frequency may be accurately adjusted (Fig. 37 (e)). The direction of the maintaining current in the wire should be such that the force acting on the wire tends to move it upward out of the magnetic field; the circuit is then broken and the wire flies back, and so on, so that the oscillations are maintained. Any small spark at the contacts can be suppressed by the connection of resistances (or condensers) across the cups, as shown. A frequency of about 500 cycles per second can be obtained.

The output of string interrupters is not very large, but is usually sufficient for a bridge measurement. The frequency is easily controlled and is reasonably constant. In the Orlich pattern, care should be taken not to pass an excessive current along the wire, since the consequent heating and slackening of the string will cause the frequency to alter considerably. This will be particularly troublesome when sharply-tuned vibration galvanometers are in use, causing an enormous fall in sensitiveness.

22. Microphone Hummers. The microphone, which is so much used in telephony for the production of speech currents, forms a useful source of alternating current for bridge work.

* L. Arons, "Ein neuer electromagnetischer Saitenunterbrecher," *Ann. der Phys.*, Bd. 66, pp. 1177-1178 (1898).

† E. Orlich, "Über Selbstinduktiononormale und die Messung von Selbstinduktionen," *Elekt. Zeits.*, 24 Jahrgang, pp. 502-506 (1903).
"Saitenunterbrecher," *Zeits. f. Inst.*, 24 Jahrgang, pp. 126-127 (1904).



In principle the microphone consists of a hollow chamber or capsule containing granules of carbon in loose contact with one another. Mechanical vibrations, even of very small amplitude, acting upon the microphone produce very large alterations in the electrical resistance of the contacts between the granules. Hence, by subjecting a microphone to suitable mechanical vibrations and connecting it in series with a battery, the current through it will be made to vary. This varying current can be arranged to act upon the source of vibrations in such a way that the oscillations are maintained. Such a device is then called a "microphone hummer," and forms a convenient source of alternating current.

In its simplest form, a hummer can be made by placing a microphone transmitter opposite the diaphragm of an ordinary magnetic telephone receiver (see Fig. 38 (a)). The microphone is connected in series with a battery and the primary of a transformer; the secondary of the transformer is connected to the receiver. The action of the apparatus is then as follows: An external disturbance—a stray sound, or some change in the battery circuit—acting on the microphone will change the current through it. This change of current will produce a transient current in the secondary circuit, and will cause a movement of the telephone diaphragm. Sound waves will be emitted and will fall upon the microphone, producing further transient changes in its resistance and in the current through it, these reacting on the telephone *via* the transformer; and so on. The system, once disturbed, will continue in action and will emit a loud note, the frequency of which will depend upon the length of the air column between the telephone and microphone, and upon a variety of other circumstances.* An additional winding on the transformer will provide current for the bridge.

Unfortunately, the frequency of a simple hummer is not very constant, and is much affected by external effects. Larsen† has shown (Fig. 38 (b)) that the action can be made much more definite by joining the microphone to the telephone by means of a tube. The air column is now confined, so that external effects are small; the action can be adjusted until the best

* F. Gill, "Note on a humming telephone," *Journal, I.E.E.*, Vol. 31, pp. 388-395 (1902).

† A. Larsen, "A new high frequency generator," *Elecn.*, Vol. 67, p. 827 (1911).

conditions are found by means of a tuning tube sliding upon the connecting pipe. Still further improvement can be secured

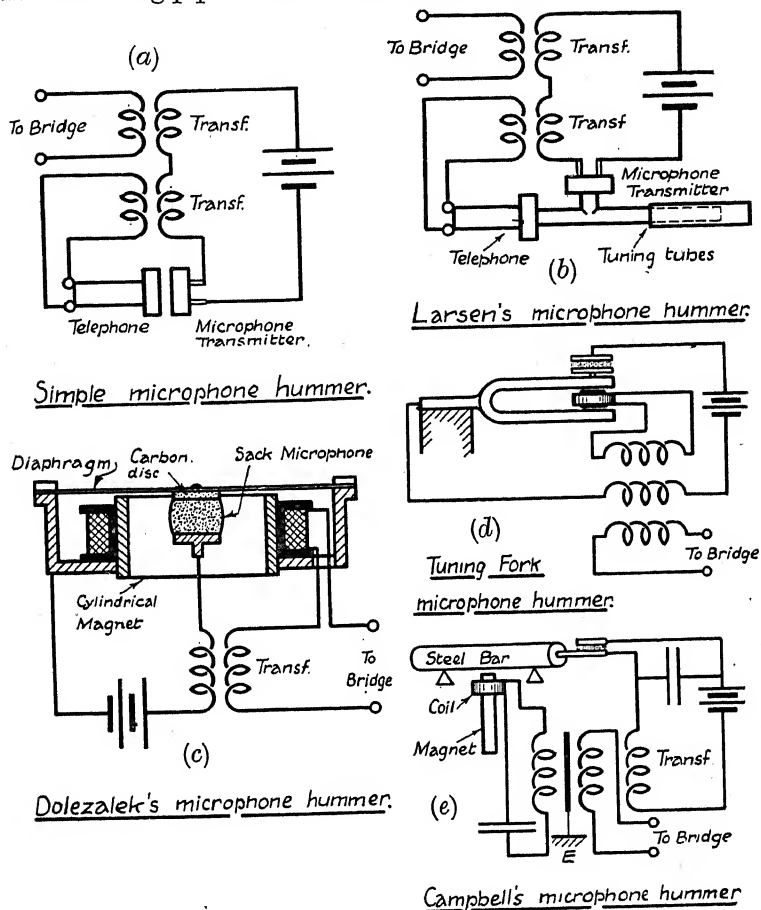


FIG. 38.—MICROPHONE HUMMERS

by tuning, in addition, the column of air behind the diaphragm of the telephone receiver. Frequencies lying between 600–1,100 cycles per second are obtainable.

Dolezalek* has greatly improved the microphone hummer

* F. Dolezalek, "Messeinrichtung zur Bestimmung der Induktionskonstanten und des Energieverlustes von Wechselstromapparaten," *Zeits. f. Inst.*, 23 Jahrgang, pp. 240–248 (1903).

by combining in one piece of apparatus the telephone and microphone elements (*see* Fig. 38 (c)). In this instrument a telephone diaphragm carries at its centre a small microphone consisting of a silk bag filled with carbon granules. The diaphragm is made of thin sheet iron and stands above a cylindrical, hollow, permanent magnet surrounded by a coil. When the diaphragm is set in motion, the resistance of the microphone is changed and the current from the battery varied. By the action of the transformer, currents will flow in the coil and will affect the magnet in such a way that the motion of the diaphragm is maintained. The frequency is very constant and can be adjusted by substitution of diaphragms of different thicknesses; a range of from 300 to 1,000 cycles per second is covered. The secondary circuit is frequently tuned by means of a condenser so that the maximum effect is obtained. The resulting oscillations are very nearly sinusoidal.

In the hummers just described, the microphone currents are used to maintain in vibration the diaphragm of a telephone receiver; they can, however, be used to act in a similar way upon quite other dynamical systems. For example, a common arrangement is a tuning fork maintained in oscillation by the microphone current. The fork* acts on the microphone, in series with which is the primary of a transformer. The secondary winding is joined in series with the maintaining magnet, and an additional winding provides the bridge current. The fork† usually has a frequency of 800 or 1,000 cycles/second and forms a very constant source of current. The output is a fraction of a watt (*see* Fig. 38 (d)).

A further development of the tuning fork hummer is the vibrating bar hummer of A. Campbell.‡ In this (Fig. 38 (e)) the vibrating element is a steel bar 2.5 cm. diameter and 23.7 cm. long, supported upon knife-edges at two nodal points. At one end the bar carries a microphone button, the microphone being connected in series with a battery and the primary of a small transformer. Beneath the centre of the bar a magnet

* For other arrangements, *see* R. Appleyard, "A new method of electrically driving tuning forks," *Tel. J. and Elec. Rev.*, Vol. 26, p. 57 (1890). J. E. Taylor, *Journal I.E.E.*, Vol. 31, pp. 395-398 (1902).

† For precautions necessary to attain success with low frequency (50 to 100 cycles/second) hummers, *see* A. Campbell, "A note on low frequency microphone hummers," *Proc. Phys. Soc.*, Vol. 31, p. 84 (1919).

‡ A. Campbell, "On the electric inductive capacities of dry paper and of solid cellulose," *Proc. Roy. Soc. A.*, Vol. 78, pp. 196-211 (1907).

is fixed, a coil wound upon it carrying current from the secondary of the transformer and serving to maintain the vibrations of the bar when once started. Tuning condensers are connected in these circuits as shown. Current for the bridge is taken from a third winding on the transformer, an earthed screen being put between the two secondaries so that errors due to earth capacity are avoided when the hummer is used on a bridge.

With a bar of the dimensions given, a frequency of about 2,000 cycles/second is obtained ; other bars can be substituted and a range of frequency from 500 to 3,000 cycles/second, or higher, covered. The frequency depends only on the physical properties of the bar and is constant within 1 or 2 parts in 1,000 ; the wave-form is practically pure, though the output is small. By resonating the circuit which supplies the bridge it is possible to obtain 50 to 100 volts on the supply terminals, which is useful when high impedance networks are used.

Another simple use of the microphone is found in the reed hummer, used in the British Post Office. This instrument is identical with an ordinary hummer, the telephone diaphragm being replaced by a vibrating steel reed carrying the microphone button. The action is just the same as that of the telephonic device.

23. Alternators. As will have been gathered from the above remarks, the output required from a source of current for bridge work is not very great, the interrupters and hummers described above usually supplying sufficient output for most purposes. For much laboratory work, especially when outputs somewhat larger than those given by these devices are required, a small motor-driven alternator forms a convenient source of current. Many types of alternator have been devised to work at telephonic, or even higher, frequencies and to produce a wave form reasonably free from harmonics ; the principle and action of some of these machines will now be considered.

Inductor Alternators. Some of the earliest alternators designed for bridge work were arranged to act on the inductor principle. A magnetizing coil is arranged to produce a flux in a magnetic circuit in which the rotating element of the machine is included. The latter is shaped in such a way that, as it rotates, the reluctance of the magnetic circuit is periodically varied. The flux is thus caused to change

harmonically and links a secondary or armature winding in which an electromotive force is induced.

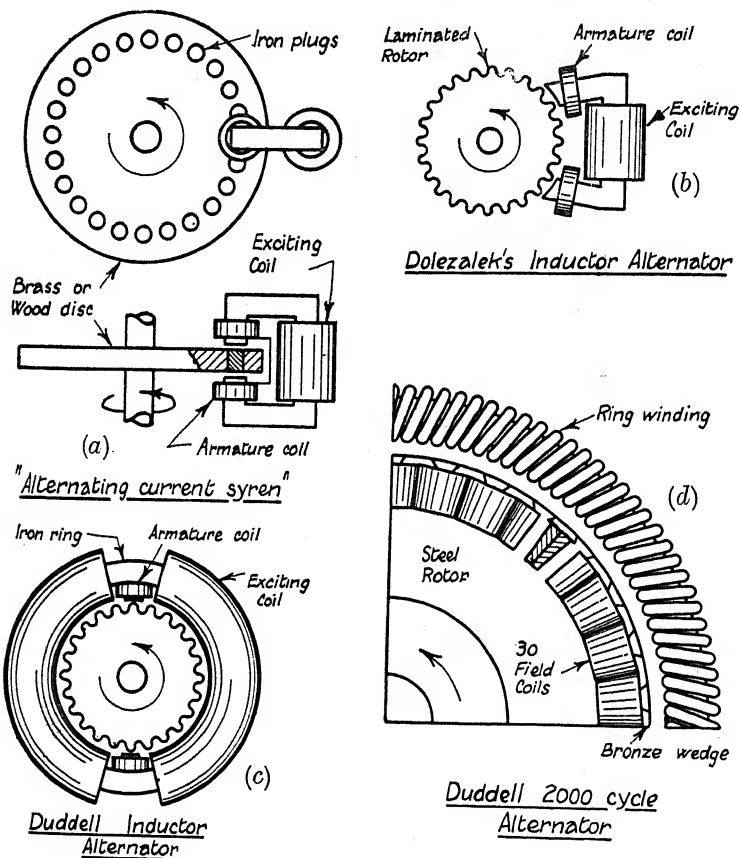


FIG. 39.—TYPES OF ALTERNATORS

The simplest type of inductor alternator* is the so-called "alternating current syren" shown in Fig. 39(a). In this a

* Max Wien, "Ueber die Magnetisirung durch Wechselstrom," *Ann. der Phys.*, Bd. 66, pp. 859-953 (1898).

E. Orlich, "Ueber Selbstinduktionsnormale und die Messung von Selbstinduktionen," *Elekt. Zeits.*, 24 Jahrgang, pp. 502-506 (1903).

B. V. Hill, "The variation of apparent capacity of a condenser with the time of discharge and the variation of capacity with frequency in alternating current measurements," *Phys. Rev.*, Vol. 26, pp. 400-405 (1908).

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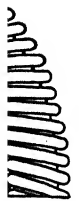
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disc of wood, ebonite, or brass is caused to rotate at a high speed between the poles of an electromagnet. Holes are drilled at equidistant intervals round a circle near the rim of the disc, soft iron plugs or studs being driven into the holes; two or more rows of studs are sometimes provided, together with means to move the electromagnet to face any desired row. The pulsating flux links small coils wound upon the pole tips, these coils acting as an armature winding from which the bridge current is taken. Frequencies up to about 3,000 cycles/second are easily obtained, the disc being direct-coupled to the shaft of a small shunt motor.

In a modification* the studs are replaced by slots cut in the rim of the disc, filled with packets of transformer iron. Wien† has constructed such a machine to give 8,500 cycles/second and an output of 20 volts, 0.2 amps. The wave form is not pure, and by resonating the second harmonic Wien was able to obtain a frequency of 17,000.

A common type of inductor alternator uses as the rotating element a packet of thin soft iron stampings clamped tightly together upon a shaft. The periphery of the iron disc is cut into the form of teeth, an electromagnet being placed opposite the rim of the disc so that the teeth, when moving past the poles, will produce fluctuations in the magnetic field. Small armature coils are wound on the poles of the magnet, and have a high frequency e.m.f. induced in them by the varying flux. Frequencies up to 6,600, with an output of 7 watts, have been attained by the device as designed by Dolezalek‡ and shown in Fig. 39 (b). The disc is direct driven by a small shunt motor run at a constant speed.

Duddell§ has adapted the toothed disc for the production of extra-high frequency currents, up to 120,000 cycles/second, his arrangement being illustrated in Fig. 39 (c). In his machine

* J. Puluj, "Bestimmung des Koeffizienten der Selbstinduktion mit Hilfe des Elektrodynamometers und eines Induktors," *Elekt. Zeits.*, 12 Jahrgang, pp. 346-350 (1891).

† Max Wien, "Ueber die Erzeugung und Messung von Sinusströmen," *Ann. der Phys.*, Bd. 4, pp. 425-449, 1901.

‡ F. Dolezalek, "Messeinrichtung zur Bestimmung der Induktionskonstanten und des Energieverlustes von Wechselstromapparaten," *Zeits. f. Inst.*, 23 Jahrgang, pp. 240-248 (1903). See also *Elecn.*, Vol. 64, p. 449, (1910), for an illustration of a similar machine.

§ W. Duddell, "A high frequency alternator," *Proc. Phys. Soc.*, Vol. 19, pp. 431-442 (1905). A list of references to other papers on high frequency alternators is given.

the electromagnet takes the form of a ring concentric with the disc, the exciting coils being connected in such a way that the flux passes along a diameter. The disc was specially driven by means of a belt, speeds as high as 35,000 r.p.m. being attained. The armature coils were wound upon the inwardly-projecting pole pieces from which the flux issued.*

Heteropolar Alternators. Heteropolar alternators are machines of the ordinary type in which a system of alternate north and south poles is moved relatively to an armature winding. For low frequencies an ordinary alternator is sufficiently good up to about 200 cycles/second. Much attention has been given to the design of machines which will give a sinusoidal wave form, even when heavily loaded, at frequencies lying between 200 and 10,000 for measurements within the telephonic range.

A typical machine of this class was designed by the late Mr. W. Duddell† to give an output of about $\frac{1}{2}$ kilowatt at a frequency of 2,000 cycles per second (see Fig. 39 (d)). The rotor formed the field magnet system and ran at a speed of 8,000 r.p.m. The rotor disc is of steel, 20 cm. diameter, with 30 polar projections milled out of the solid; the field coils are slipped over the poles and secured in place by bronze wedges. The stator is a smooth ring provided with a Gramme winding. Slots are avoided, so that the wave form is free from tooth ripples; also the necessarily long air-gap makes the effect of armature reaction very small, so that the wave form remains pure when the machine is loaded. The rotor runs in long bronze bearings and is driven by a motor and link belt.

The laboratories of the City and Guilds (Engineering) College contain two small high-frequency alternators which are specially designed‡ for bridge work. The first works up to 250 cycles per second and has 6 poles; the second gives 1,000 cycles per second and has 24 poles. The field systems of both machines are stationary, the field coils being wound on every other pole.

* For another form of high frequency inductor alternator, see A. Franke, "Die elektrischen Vorgänge in Fernsprechleitungen und Apparaten," *Elekt. Zeits.*, 12 Jahrgang, pp. 447-452, 458-463 (1891).

† W. Duddell, "A 2,000 frequency alternator," *Proc. Phys. Soc.*, Vol. 24, pp. 172-180 (1912).

‡ H. F. Haworth, "The measurement of electrolytic resistance using alternating currents," *Trans. Far. Soc.*, Vol. 16, pp. 365-391 (1921).

For another high frequency machine, see F. Addey, "The experimenting room," *Journal, P.O.E.E.*, Vol. 3, pp. 1-16 (1910-11).

The armatures of both machines are wound on the smooth rotor. In the low frequency machine a simple Gramme ring winding is used. The high frequency machine has a smooth rotor with former-wound coils held on its surface by steel binding wire. The alternators are mounted with their axes vertical, and are direct coupled to $\frac{1}{4}$ h.p. shunt wound motors supplied from a 200 volt battery. The wave forms are very nearly sinusoidal.

24. Methods of Maintaining Alternators at Constant Speed.

In modern bridge measurements the balance detector usually employed is the vibration galvanometer. This instrument, as will be shown on page 152, is tuned to respond to currents of one definite frequency and is insensitive to currents whose frequencies are even slightly removed from the critical value. Moreover, in some of the methods of measurement which are now in regular use, the frequency must be known as one of the conditions for balance. For both these reasons it becomes necessary to maintain strictly constant the frequency of the current supplied to the bridge. It is proposed now to show how this can be done when an alternator is the source of current.

In order to avoid any effects due to backlash or slip, the alternator is best driven by a direct current shunt motor, the shafts being rigidly connected. The motor should be run from a well-charged battery of ample output, so that variations of voltage of the battery, and consequent changes of speed of the motor, are small. In order that the small changes of load on the alternator, produced by adjusting the bridge to obtain balance and by changes of bearing friction, shall not affect the speed, it is frequently arranged that the motor shall at the same time drive a small shunt generator. This acts as a permanent constant load which can be arranged to swamp the effect of the alternator (Fig. 40 (a)).

While these precautions are in many cases sufficient, it is necessary in very exact work with sharply tuned detectors, to have some means of checking even the smallest casual fluctuations of speed. One simple way is to apply a pair of telephones to the alternator, and to compare the note emitted by them with that of a standard tuning fork, regulating the motor speed by means of its field rheostat, or by light pressure of the hand upon the rotating shaft, until constant beats (or unison, as the case may be) are maintained. The frequency is thus known and is controlled.

A device of much greater sensitiveness is the Maxwell condenser bridge shown in Fig. 40 (b). In the use of this, a commutator, by means of which a condenser can be charged and discharged in a branch of a Wheatstone bridge, is mounted on the motor shaft. If the condenser be charged and discharged n times per second, then very nearly

$n = R/SPC$. Hence with given resistances the auxiliary bridge is balanced for a certain value of n , i.e. for a certain speed of the motor shaft.* The observer merely has to watch the galvanometer G and keep it at zero by small adjustments of the motor field rheostat or otherwise. This method is much used at the Bureau of Standards Laboratory;† the speed, and hence the frequency, can be kept constant on the average to about 1 part in 10,000, and is known from the balance resistances of the Maxwell bridge.

Both the above methods have the disadvantage that two observers are necessary, one to keep the speed constant by the use of the auxiliary telephone or bridge, while the other makes the tests on the alternating current bridge. To bring the work within the scope of a single observer, automatic speed regulators have been devised.

One of the simplest devices was developed from the centrifugal governor by Giebe,‡ following a suggestion of von Helmholtz. In Fig. 40 (c) the connections are shown; R is a small resistance in the field circuit of the motor whose speed is to be kept constant. The governor is mounted on the motor shaft and is shown diagrammatically in the figure. M is a mass which is supported upon a guide wire W stretched tightly in the frame F ; spiral springs S , of which the tension can be varied, draw the mass inward. When the shaft is running, centrifugal force tends to cause the mass to move outward along W against the tension of the springs. At a sufficiently high speed the mass moves out so far that C_1 and C_2 , a pair of light platinum contacts joined to the slip-rings ss , make contact and cut out of circuit the resistance R . The speed of the motor driving the shaft then falls, since its field is strengthened, the contacts open again. The speed thereupon rises and the whole cycle of operations is repeated. The driven shaft thus fluctuates about its average speed, and with proper adjustment great constancy is attained. Tests show that the variations from the mean speed do not exceed 1 part in 10^6 .

The theory of the regulator is very simple. With the shaft at rest, let r_1 be the distance of the centre of gravity of M from the axis of rotation and T_1 the initial pull of the springs. When the device is

* The device was introduced at the National Physical Laboratory about 1902, and is widely used for the maintenance of constant speed. *Dictionary of Applied Physics*, Vol. 2, p. 127.

† E. B. Rosa and F. W. Grover, "The absolute measurement of inductance," *Bull. Bur. Stds.*, Vol. 1, pp. 128-129 (1905). E. B. Rosa and F. W. Grover, "The absolute measurement of capacity," *idem*, pp. 175-181 (1905). E. B. Rosa and N. E. Dorsey, "A new determination of the ratio of the electromagnetic to the electrostatic unit of electricity," *idem*, Vol. 3, pp. 557-561 (1907). F. W. Grover, "The capacity and phase difference of paraffined paper condensers as functions of temperature and frequency," *idem*, Vol. 7, pp. 504-505; also pp. 505-508 (1911).

‡ E. Giebe, "Ein empfindlicher Tourenregler für Elektromotoren," *Zeits. f. Inst.*, 29 Jahrgang, pp. 205-216 (1909).

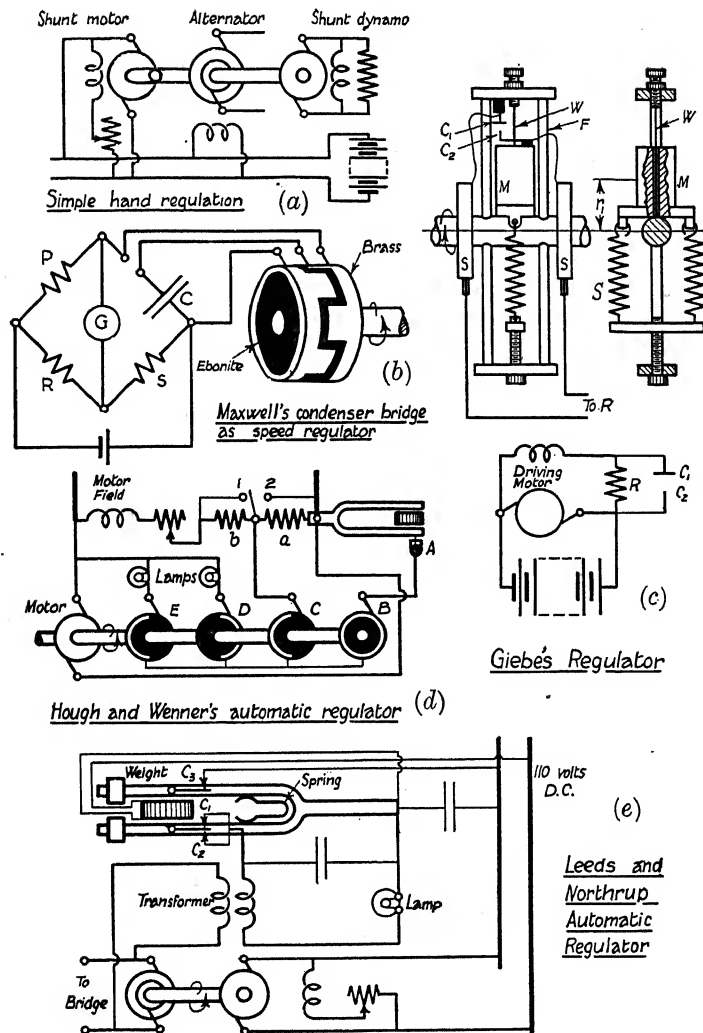


FIG. 40.—METHODS OF MAINTAINING ALTERNATORS AT CONSTANT SPEED

running at angular velocity ω , the mass will fly out to a radius r and the equation of equilibrium will be $c(r - r_1) + T_1 = Mr\omega^2$, where c is the force per unit extension of the springs. Writing $e_1 = T_1/c$ for the original extension of the springs, it is easy to prove that equilibrium is stable if $r_1 \geq e_1$; and the change of r for a small change in ω tends to infinity as $r_1 = e_1$. Hence if the springs are given such a tension as would bring the centre of gravity of the mass to the centre of the shaft—were its motion in that direction unimpeded—the apparatus is extremely sensitive to small changes of angular velocity. Putting this condition in the equation makes the critical velocity at which the most sensitive adjustment occurs $\omega = \sqrt{c/M}$. At speeds above this the apparatus is unstable, the mass flying out to the end of its travel and remaining there until the speed falls to the critical value. To change the speed at which the device will satisfactorily regulate it is necessary to change either the mass or the springs.

Other devices are developed from an arrangement described by Lebedew* in which the motor driven shaft is made to run in synchronism with an electrically maintained tuning fork of suitable frequency. As modified by Hough and Wenner,† the apparatus is arranged in the manner illustrated in Fig. 40 (d). The switch is, in the running position, put on contact 1. The maintained tuning fork and the contact rings B , C mounted on the motor shaft are then arranged so that the resistance a is short-circuited during a greater or lesser fraction of a revolution, according as the speed of the shaft is above or below the normal value. To do this, the fork is provided with a contact A which is closed during half a vibration of the prongs. Contact B is a slip-ring in electrical connection with the half-ring C . Since C and A are in series, the resistance a is shorted only when both the contacts are simultaneously closed. For normal speed the contact C is arranged to close a quarter period later than contact A , so that a is cut out for a quarter of a revolution and is in circuit for $\frac{3}{4}$. If now the shaft runs too quickly the lag of C is reduced, a is shorted for a greater time and the motor speed will fall. On the other hand, when the speed falls too low, a is cut out for a shorter time, the motor field is weaker, and the speed rises. Two other half-rings, D , E , in electrical connection with B , serve to show when the arrangement is synchronized. To do this, the switch is thrown over to contact 2, putting b in place of a ; b is about $\frac{1}{4}$ of a . The motor is then started and run up to such a speed that the lamps glow equally brightly; the switch is then put on to 1 and the apparatus continues to run in synchronism, regulating the speed as described.

The Leeds and Northrup Co. have an ingenious modification of this device, shown in Fig. 40 (e), in which the fork is synchronized with the alternator instead of with a half-ring commutator. The tuning fork is of a special design arranged to work over a range of 50 to 75 vibrations

* P. Lebedew, "Ueber die ponderomotorische Wirkung der Wellen auf ruhende Resonatoren," *Ann. der Phys.*, Bd. 59, pp. 118-119 (1896).

† R. H. Hough and F. Wenner, "The operation of a direct current shunt wound motor in synchronism with a tuning fork," *Phys. Rev.*, Vol. 24, p. 535 (1907).

per second. The variation is secured by sliding weights on the prongs and by springs pressing on them. The contact C_3 operates the maintaining circuit of the fork. A small transformer is connected to the alternator, the secondary winding being joined to a lamp through the contacts C_1 and C_2 . If the fork and the alternator be running synchronously, two contacts per vibration will be made at the fork, two impulses of alternating voltage being applied to the lamp. Clearly, if the speed of the alternator be constant, these impulses will always take place at the same points in each voltage cycle.

Suppose now that the speed of the machine were to change, tending to put the alternating current and the fork out of synchronism. The voltages impressed on the lamp in successive cycles would thus be different, since contacts are not now made at the same points in each cycle. If the speed had increased, the contacts would be made at higher values of the voltage curve, tending to increase the current through the lamp; the increased load then slows down the motor until it runs in step again. Should the speed decrease, the voltage impulses are lower, the current in the lamp is less, and the machine tends to increase its speed.

To avoid sparking, condensers are shunted across the contacts; the transformer serves to isolate the lamp-load current from the maintaining current. The speed can be kept constant to about 2 parts in 10,000. Variations of alternator load or of the voltage on the motor up to 10 per cent can be tolerated without synchronism being destroyed.

25. Oscillators. All the sources of current hitherto described depend for their action, in part at least, upon certain mechanical movements, e.g. the vibrations of a fork or bar, the rotation of a shaft or the movement of a microphone. Only by special arrangements* can the mechanical difficulties be overcome and the frequency be maintained sufficiently steady. Moreover, it is difficult to ensure that the wave form is purely sinusoidal.

In order to overcome these objections completely, many devices have been designed to produce alternating current of suitable frequency by purely electrical methods. Such arrangements are known as oscillators and all work on the following principle.

Consider a circuit having inductance L and capacitance C . If the resistance of the circuit be sufficiently small, the circuit will possess a natural frequency given approximately by $f = 1/2\pi\sqrt{LC}$. That is, if the circuit be disturbed and then left to itself, a sinusoidal current of this frequency will flow in it, this natural oscillation being gradually damped out by

* A complete solution of the problem is obtained by the use of a valve-maintained tuning fork as an oscillator, perfect steadiness of frequency and freedom from trouble being secured. Such a device is made by the Cambridge & Paul Co.

the resistance of the circuit. The frequency of these lightly damped oscillations is fixed entirely by the properties of the circuit, and is therefore constant. If some means can be found to maintain these natural oscillations, i.e. to make up for the dissipation of energy in the circuit, a sinusoidal current of fixed frequency and amplitude will be produced. This condition is realized in an oscillator by the devices described below.

The action of an oscillator has a mechanical analogy in that of an ordinary pendulum clock. Here the pendulum is a mechanical system having a definite natural period, and is acted upon when in motion by certain frictional damping forces; it is the mechanical parallel of the oscillatory circuit. The energy required to run the driving train of the clock corresponds to the energy which the oscillatory circuit is to supply to the bridge. The source of energy is the clock spring. The action is then as follows: the pendulum being disturbed from its position of rest is maintained in steady motion by receiving impulses at the right instant from the escapement wheel, the energy of these impulses being drawn from the mainspring. The same escapement regulates the passage of energy from the spring to the driving train at each beat of the pendulum. Hence the oscillations of the pendulum are maintained and regulate the flow of energy to the gears which operate the hands, turning the latter at a regular rate fixed only by the vibration constants of the pendulum. The student will find that these processes are followed, by analogy, in an electric oscillator, the current in an oscillatory circuit being maintained by a kind of escapement action, so that energy absorbed in resistance damping and in the doing of external work is automatically supplied.

Vreeland's Oscillator. In Vreeland's oscillator* the oscillations in a circuit are maintained by means of a special type of mercury arc lamp shown diagrammatically in Fig. 41. A large, exhausted, glass bulb has at its lower part a tube containing a mercury kathode *K*, and two horizontal side tubes each containing a carbon anode, *AA*. A battery supplies current for the mercury vapour arc which, if everything is symmetrical, spreads equally from the kathode upon the two anodes. Across *AA* an oscillatory circuit is connected, consisting of an adjustable condenser in series with two coils, one placed behind and the other in front of the bulb.

* F. Vreeland, "A sine wave electrical oscillator of the organ pipe type," *Phys. Rev.*, Vol. 27, pp. 286-293 (1908). B. Liebowitz, "Electrical oscillations from mercury vapour tubes," *Phys. Rev.*, 2nd series, Vol. 6, pp. 450-477 (1915). A. Tobler and K. Schild, "L'oscillateur Vreeland et son emploi dans les mesures à courant alternatif," *Journal Tél.*, Vol. 40, pp. 121-124, 145-148, 169-172, 193-196 (1916).

If the arcs be slightly disturbed so that one is stronger than the other, the potential of the anode emitting the stronger arc will be lower relatively to that which emits the weaker arc. There will thus be a difference of potential across the oscillating circuit and a current will start to flow in it. If the coils be wound in such a direction that their magnetic field causes the arc to be still further deflated toward the stronger side, the potential difference across the oscillatory circuit will increase until the condenser is fully charged. The condenser then commences to discharge, the arc is deflected to the other side, and the oscillations are maintained by the impulses given by the arcs to the two anodes, these impulses being controlled by the oscillations themselves.

The working circuit is taken from a third coil placed close to one of the deflecting coils; this is omitted from the figure. In order to keep the high frequency oscillations out of the battery, choking coils, L, L , are inserted; a resistance R serves to adjust the arc. The arcs are started by means of the auxiliary mercury anode A_0 ; by closing the key and tilting the bulb, the mercury in A_0 and that in K are brought into contact. A current flows between these electrodes and on restoring the tube to the vertical this current starts an arc which spreads to the anodes AA .

With the coils in a fixed position, the frequency can be adjusted over a range of from 160 up to 4,000 cycles per second merely by adjustment of the condenser. For any given capacity, the frequency is steady and the wave form of the working current is very closely sinusoidal. An output of about 3 watts is obtainable.

Triode Valve Oscillator. The Vreeland oscillator has been practically ousted from its position as the only means for producing an alternating current of steady frequency by the recent development of the thermionic valve to fulfil the same purpose. The valve forms about the best obtainable source of current for bridge work; the frequency is quite constant, the wave form is very nearly sinusoidal and the output is ample. The apparatus is easily procured and assembled, and is quite cheap.

It is not proposed here to enter into the theory of the action of the valve, this being adequately treated in books on wireless telegraphy. Attention will be directed merely to the description of the valve oscillator and its mode of working.

The three-electrode valve or triode consists of a highly evacuated glass bulb containing within it three elements : (i) a tungsten filament heated by the passage of electric current through it ; (ii) a plate of tungsten or molybdenum, known variously as the "anode" or "plate" ; (iii) interposed between the filament and the anode is a third electrode, called the "grid," consisting of a piece of metal gauze, perforated foil, or merely of a wire spiral. The negative terminal of the filament, and the terminals of the anode and grid constitute the three electrodes.

A simple method of connection is shown diagrammatically in Fig. 41, this figure having reference to a small oscillator in use in the City and Guilds College. The filament is kept at white heat by current from a 6 volt accumulator. Between the anode and the filament an oscillatory circuit, consisting of an inductance of about 300 millihenrys and a condenser in parallel, is connected through a 200 volt battery, so that the anode is strongly positive with respect to the filament. Inductively coupled to this circuit is a coil joining the grid to the filament.

The action is then, roughly, as follows. A stream of negative electrons steadily emitted from the hot filament crosses the bulb and impinges on the anode, a steady current thus flowing through the oscillatory circuit from the high voltage battery. If the current in the oscillatory circuit be disturbed, e.g. by switching on, oscillations will be set up in this circuit, having a frequency fixed by the inductance of the anode coil and the capacitance of the condenser in parallel with it. These oscillations act inductively on the grid and modify its potential. Now the function of the grid is to act as a control electrode upon the electronic emission from filament to anode, the unique property of the valve being that small variations of grid potential will produce changes in anode current. If, therefore, the relative directions of winding of the grid and anode coils be correct, this amplifying property of the valve may result in such modifications of the anode potential that the oscillations are maintained, energy being drawn from the battery at the right instant to make up for the losses in the oscillatory circuit and for the energy taken by the working circuit. The working circuit is coupled inductively to the other coils by a third winding, as shown in the diagram. It is sometimes an advantage to tune the grid coil also ; indeed,

the number of possible connections for the maintenance of oscillations by the valve is very large indeed, the reader being referred to books dealing with the subject for further information.

In the apparatus mentioned the valve is of the "French" or "R" type, taking a filament current of 0.75 ampère. The anode coil has an inductance of about 300 millihenrys and is shunted by a good paper condenser which can be varied in steps of 0.01

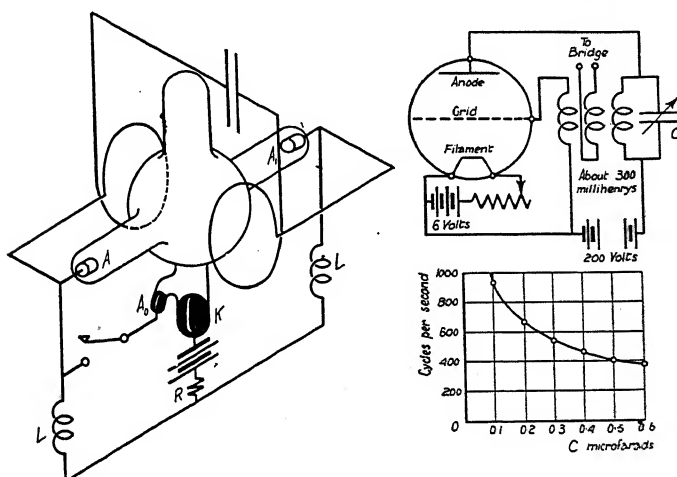


FIG. 41.—VREELAND'S MERCURY TUBE OSCILLATOR.
THE TRIODE VALVE OSCILLATOR

microfarad. The high voltage battery consists of small storage cells. With the cells well charged, it is easy to obtain an output of 30 milliamperes through 60 ohms at 400 cycles per second. By adjustment of the condenser C the frequency can be varied from about 350 cycles per second upwards, being almost exactly proportional to $1/\sqrt{C}$.

By suitable design, a triode oscillator can be made to give a pure wave form and to cover an enormous range of frequency. From 300 cycles per second up to the values used in radio telegraphy—millions per second—the apparatus is compact and easy to construct. For the lower frequencies, however, very large coils and condensers are necessary, though oscillators have been constructed to work at values as low as 5 cycles per second. In modern bridge work, with the use of

highly-sensitive, sharply-tuned, vibration galvanometers, the valve oscillator becomes almost a necessity and is by far the best source to use for such work.

26. Wave-Filters and Interbridge Transformers. *Wave-Filters.* In precise work it is necessary to use some "filtering" device in all cases where absolute purity of wave form is essential to sharp balance. Two types of filter are in use: (i) in which all harmonic components of the impure wave, except one, are *partially* suppressed; (ii) in which one component is *entirely* suppressed.

Fleming and Dyke* have used the first process. Their alternator, whose wave form contained very pronounced harmonics, supplied a variable inductance and a condenser in series, the inductance being adjusted until a chosen harmonic was in resonance. To enhance the harmonic still further, this first filter circuit was coupled by means of a transformer to a second filter circuit adjusted to resonance. A transformer in the second circuit supplied the bridge. The chosen harmonic was thus exaggerated to such an extent that the other components were of negligible importance. All the coils were free from iron.

In many alternators or hummers there is usually one particular harmonic which causes trouble and which it is desired to suppress. Campbell† has described a number of ways in which this suppression may be carried out. One of these is shown in Fig. 71(a). Suppose the telephone in the figure is replaced by the primary of a small transformer from whose secondary the bridge is to be supplied. Then if M and C be such that $2\pi f = 1/\sqrt{MC}$, no current of frequency f —which is that of the undesirable harmonic—can flow in the transformer. This method is suitable only for higher frequencies owing to the inconveniently large value of MC at low frequency. For frequencies of the order of 100 or less, Campbell describes several other types of sifter.

Interbridge Transformers. It is always best to connect the source, whether it be alternator, interrupter, hummer, or oscillator, to the bridge via a small transformer whose windings are well insulated from one another. Between the two windings an earthed metallic screen is put, by means of which capacity effects between the source and the bridge are reduced to a minimum. Such capacity effects are readily detected if, after the bridge has been balanced, the connections to the transformer secondary are reversed. In general, small adjustments of the branches will be required to restore balance; the mean of the two values represents the true balance, provided that the capacity effects are small.

An interbridge transformer should have a very small leakage field so as not to interfere inductively with the bridge. Various windings

* J. A. Fleming and G. B. Dyke, "On the power factor and conductivity of dielectrics when tested with alternating currents of telephonic frequency at various temperatures," *Journal, I.E.E.*, Vol. 49, pp. 323-431 (1912).

† A. Campbell, "On wave-form sifters for alternating currents," *Proc. Phys. Soc.*, Vol. 24, pp. 107-111, 158-159 (1912). For a more perfect type of filter see G. A. Campbell, *Phil. Mag.*, Vol. 5, p. 313 (1903).

should be provided so that the working voltage may be varied as desired. A closed iron ring wound with uniformly distributed toroidal windings with an earth screen to separate the primary coils from the secondary is very suitable.*

The choice of a suitable ratio of transformation has also an important effect on the sensitivity of the bridge. Suppose Z to be the impedance of the source, t_1 the number of turns in the primary winding to which it is joined, the secondary coil of t_2 turns being connected to the bridge. Then, if $t_2/t_1 = T$ be the ratio of transformation, the effective impedance of the source expressed in terms of the secondary winding of the transformer is T^2Z . This artifice is of great service when measuring small condensers by the methods shown on pages 194 to 211. For sensitivity it is pointed out that, since the branches of the network have a high impedance, a high impedance source and detector are requisite. This can be artificially produced by connecting a source of normal impedance to the bridge through a step-up ($T > 1$) transformer of appropriate ratio. A precisely similar method can be used to adapt detectors to high or low impedance bridges, *see* pp. 151 and 173.

DETECTORS

To determine when an alternating current bridge is balanced, some detecting instrument capable of giving an indication with a small alternating current or voltage is required. In most cases the detectors usually employed are either telephones or vibration galvanometers, but there are many other devices which are frequently of service. In this section detectors will be classified and described, their special fields of usefulness being indicated.

27. The Synchronous Commutator. On page 6 and in Fig. 1(d) it has been pointed out that one of the earliest applications of alternating current to bridge work is found in the secohmmeter. In this method a crude type of a.c. is supplied to the bridge by means of a reversing commutator and battery, a second commutator successively reversing the connections to a ballistic galvanometer so that the transient impulses in the instrument act in the same direction and are additive, producing a steady deflection. Owing to the sensitiveness of the modern moving-coil ballistic galvanometer, several devices have been introduced to enable this instrument to be used as the detector in bridges to which a sinusoidal current is supplied. Obviously, it is necessary to have some commutating device in the galvanometer circuit working synchronously with the alternating

* For the design of a suitable transformer *see* *Dictionary of Applied Physics*, Vol. 2, p. 393.

supply and reversing the galvanometer connections at each half-wave of the supplied current. For use with a two pole alternator, a simple two-part commutator mounted on the alternator shaft is all that is required.

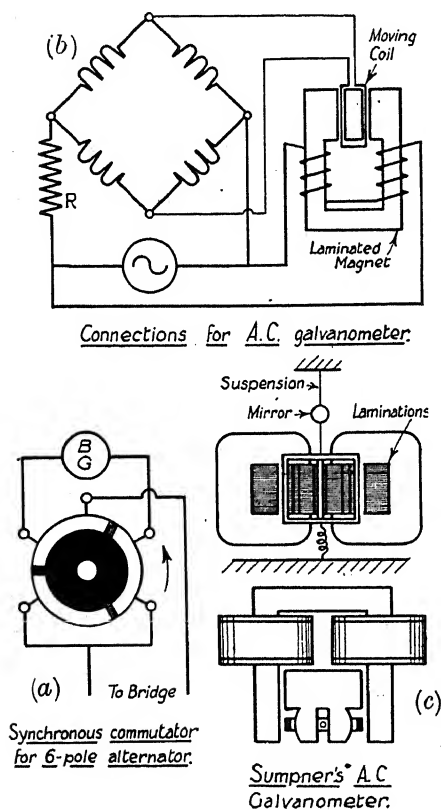


FIG. 42.—THE SYNCHRONOUS COMMUTATOR. A.C. GALVANOMETERS

natively, some form of thermal ammeter† can be used, especially at frequencies in the radio range.

By adjustment of the position of the brushes, or the cam,

Fig. 42 (a) illustrates a rather more elaborate arrangement designed by Dr. H. F. Haworth for application to a 6-pole alternator. The commutator is driven by the alternator shaft; contact is made with it by five small carbon brushes, these being mounted on a fibre ring surrounding the commutator so that they may be turned into the most advantageous position. When used at low frequencies, with a good moving coil ballistic galvanometer, very precise balance settings can be secured.

Contacts between stationary brushes and a rotating commutator are frequently troublesome. Sharp and Crawford* have designed, therefore, a special reversing key in which the contacts are operated by a cam driven in synchronism with the alternator. For higher frequencies a galvanometer can be used in combination with a crystal‡ or valve rectifier. Alter-

* C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, Vol. 39, part 2, pp. 1518-1521 (1911).

† See, for example, S. H. Anderson, *Phys. Rev.*, Vol. 34, pp. 34-39 (1912).

relatively to the alternator, the reversals of the galvanometer can be made to take place at any desired part of the current wave; the galvanometer can be set, therefore, to be sensitive to one component of the current and to be insensitive to the component in quadrature therewith. It is easy with a synchronous commutator, therefore, to make sensitive independent adjustments of the bridge for reactance and then for resistance, an advantage shared with other instruments now to be mentioned.

28. Electrodynamometers. Any sensitive electrodynamometer can be used as a bridge detector. In use, the instrument can be connected in various ways, e.g. the fixed coils may be joined to the source and the moving coil across the detector branch points. Then, if i_1 , i_2 be the instantaneous values of the currents in the fixed and moving elements, the average torque and the deflection of the moving coil will be proportional to $\int i_1 i_2 dt$ taken over a period, i.e. to $I_1 I_2 \cos \phi$, where I_1 and I_2 are the r.m.s. values of the currents and ϕ is the phase-difference between them. Assuming I_1 not to be zero, the moving coil will not deflect if either (i) $I_2 = 0$, as in the ordinary null adjustment of the bridge; or (ii) $\cos \phi = 0$, $\phi = \pi/2$, the fixed and moving coils carrying currents in quadrature. Rowland* has used a dynamometer in both these ways, giving various methods of connecting the fixed and moving coils to the bridge, though other experimenters have employed the instrument before him.†

The dynamometer must be very sensitive‡, having a light moving coil hanging from a long suspension to give a very weak control. Electrostatic forces between the fixed and moving coil are minimized by means of a tin-foil screen surrounding the latter and connected to one of its terminals. The inductance of the moving coil is preferably compensated by connecting in series with it a shunted condenser; the condenser and its shunt are adjusted until the entire combination has zero effective inductance.

* H. Rowland, *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429-448 (1897).

† A. Oberbeck, *Ann. der Phys.*, Bd. 17, pp. 816-841, 1040-1042 (1882). J. Puluj, *Elekt. Zeits.*, 12 Jahrgang, pp. 346-350 (1891); O. Troje, *Ann. der Phys.*, Bd. 47, pp. 501-512 (1892); H. Martienssen, *Ann. der Phys.*, Bd. 67, pp. 95-104 (1899); A. Blondel, *Écl. Elect.*, t. 21, pp. 138-141 (1899).

‡ See A. Palm, "Spiegel Elektrodynamometer mit besonders hoher Empfindlichkeit," *Zeits. f. Inst.*, 33 Jahrgang, pp. 368-373 (1913). Also *Dictionary of Applied Physics*, Vol. 2, p. 402.

29. Alternating Current Galvanometers. The sensitiveness of an electro-dynamometer can be greatly increased by providing the fixed coils with an iron core built up of laminations, since the strength of the magnetic field in which the moving coil lies is made much greater. The instrument then resembles an ordinary moving-coil galvanometer except that the field is produced by an alternating current electromagnet with laminated core; a laminated core may also be inserted within the moving coil. The arrangement is then called an alternating current galvanometer and was first introduced by Stroud and Oates,* who showed that the instrument is capable of very great sensitiveness. In use, the electromagnet is excited by the alternator supplying the bridge, and therefore takes a current approximately $\pi/2$ out of phase with the alternator voltage. A resistance R , Fig. 42 (b), connected between the bridge and the source ensures that the bridge current is nearly in phase with the supply voltage. With the moving coil connected across the detector points, the instrument will be very sensitive to components of current in quadrature with the alternator voltage, i.e. it is in the best condition to measure reactances. If the instrument is to be sensitive to resistance adjustments in the network, the magnet current and the bridge current must be brought more nearly into phase.

A more sensitive form of instrument was later designed by Abraham† in which the inductance of the moving coil was compensated by the addition of a shunted condenser in series with it, thereby improving its performance.

In practical working, these instruments possess a troublesome feature due to the strong electromagnetic control exerted on the moving coil when on closed circuit, by transformer action from the main field. Stroud and Oates noticed the trouble but did not explain it; they showed, however, that it could be overcome in practice by use of a false zero. The theory of the control was briefly given by Taylor,‡ but a complete theory of the instrument was not supplied until much later by Weibel. Instruments of the Stroud and Oates

* W. Stroud and J. H. Oates, "On the application of alternating currents to the calibration of capacity boxes, and to the comparison of capacities and inductances," *Phil. Mag.*, 6th series, Vol. 6, pp. 707-720 (1903).

† H. Abraham, "Galvanomètre à cadre mobile pour courants alternatifs," *Comptes Rendus*, t. 142, pp. 993-994 (1906).

‡ A. H. Taylor, "Theory of control of the alternating current galvanometer," *Phys. Rev.*, Vol. 25, pp. 61-63 (1907).

type have been used, very largely in America, by other observers.*

Sumpner† has made a thorough investigation of the design and construction of iron-cored a.c. instruments, and has described a galvanometer‡ suitable for bridge work which is diagrammatically shown in Fig. 42 (c). In this, a strong alternating current electromagnet of special construction is used, so arranged that the flux is proportional to and in quadrature with the applied voltage. The moving coil is suspended in the specially shaped air-gap and has a low inductance. The control-effect of the alternating field on the moving coil, which in this instrument is small, can be allowed for by adjustment of the bridge balance until reversal of the moving coil produces no deflection; alternatively, Abraham's compensation or Stroud and Oates' false zero method are applicable. Owing to the phase-selectivity of the instrument, bridges can be balanced for inductance or capacitance with an accuracy of 1 part in 10,000 of the measured values when the p.d. over the coils or condensers concerned is only 1 or 2 volts. At the same time, it is not necessary to adjust simultaneously the resistance balance of the bridge. Sumpner and Phillips described numerous bridge tests showing the high sensitiveness of the galvanometer.

More recently, Weibel§ has given an exhaustive study of high sensitivity alternating current galvanometers. He has obtained their equation of motion, and shows that their performance is similar to that of d.c. moving coil galvanometers since there may be any degree of damping, chiefly due to the circuit with which the instrument is connected; the period and damping depend on the circuit, the former being shortened when there is inductance present and lengthened by capacitance. The deflection is proportional to the applied p.d. on

* E. M. Terry, *Phys. Rev.*, Vol. 21, pp. 193-197 (1905), describes a means of overcoming the control difficulty; A. Trowbridge and A. H. Taylor, *Phys. Rev.*, Vol. 23, pp. 475-488 (1906), use Terry's instrument as a differential galvanometer.

† W. E. Sumpner, "The use of iron in alternate current instruments," *Journal I.E.E.*, Vol. 34, pp. 144-170 (1905); "New iron-cored instruments for alternate current working," *Journal, I.E.E.*, Vol. 36, pp. 421-467 (1906).

‡ W. E. Sumpner and W. C. S. Phillips, "A galvanometer for alternate current circuits," *Proc. Phys. Soc.*, Vol. 22, pp. 395-409 (1910).

§ E. Weibel, "A study of electromagnet moving coil galvanometers for use in alternating current measurements," *Bull. Bur. Stds.*, Vol. 14, pp. 23-58 (1919).

the moving coil, and depends on the component of the p.d. in phase with the exciting current; hence the phase-selectiveness of the galvanometers. He has described the design procedure and the methods of measuring the intrinsic constants of the instruments; galvanometers having a sensitiveness at low frequencies exceeding that of the telephone and the vibration galvanometer, comparable with the best d.c. moving coil instruments are described, one being highly sensitive at 2,100 cycles per second. He points out the necessity for shielding the moving coil from electrostatic effects, as mentioned on page 147.

Other forms of a.c. galvanometer working on the moving needle principle* have been proposed, but have not found much application.

30. The Telephone. The detector most used in a.c. bridge work is the ordinary telephone receiver and at the higher audio frequencies, say above 700 cycles per second, is also the most sensitive. In principle the modern telephone is little different from the well-known type introduced by Bell in 1875. In its simplest form it consists of a thin iron diaphragm clamped by its rim so that it is close to the pole of a permanent bar-magnet. A coil wound near the pole of the magnet carries the current to be detected; the modification of the permanent field by the alternating field of the coil produces periodic attractions of the diaphragm, thus setting it into forced vibration and causing it to emit an audible note. In the more modern form, the permanent magnet is of horse-shoe form so that both poles act on the diaphragm, the whole device being contained in a flat watch-like case. In detail, the watch pattern telephone has been somewhat modified with a view to obtaining greater sensitiveness.†

The sensitivity of the telephone is very high, but is far from being constant for currents of differing frequencies, due to the fact that the diaphragm possesses definite normal modes of vibration. When the frequency of the current approaches one of these natural frequencies, the amplitude of vibration of the diaphragm increases enormously‡ owing to resonance, and the

* W. S. Franklin and L. A. Freudenberger, "A new type of alternating current galvanometer of high sensibility," *Phys. Rev.*, Vol. 24, pp. 37-41 (1907); also Max. Wien, *Ann. der Phys.*, Bd. 4, p. 445 (1901).

† K. W. Wagner, "Ueber die Verbesserung des Telephons," *Elekt. Zeits.*, 32 Jahrgang, pp. 80-83, 110-112 (1911).

‡ See A. E. Kennelly and H. A. Affel, *Proc. Amer. Acad.*, Vol. 51, p. 421 (1915).

sensitivity is correspondingly high; hence the sensitivity/frequency curve is sharply peaked. Wien* records results for a Bell telephone having resonance at 1,100, 2,800, and 6,500 frequency, and for a Siemens instrument, 750, 2,350, 5,400. In general, most telephones used in connection with speech transmission show a maximum of current sensitivity somewhere between 700 and 1,200 cycles per second.

The sensitiveness of a telephone is not entirely determined by the amplitude of oscillation of the diaphragm, but depends also on the acuteness of hearing of the experimenter, and is, therefore, different for different persons. The combined sensitiveness of telephone receiver and observer is best expressed by the least current which will cause an audible sound. Lord Rayleigh† and Max Wien‡ have given results for a number of receivers, the results of the latter observer for the Siemens telephone cited above being as follows—

Frequency	64	128	256	512	720	1,024	1,500	2,030	2,400	4,000	16,000
Current in Micro-amps.	12	1.5	0.135	0.027	0.008	0.0135	0.024	0.03	0.01	0.3	17

Since a telephone is more sensitive at certain frequencies than at others, care must be taken to select a telephone suitable for the work in hand. The resistance of the telephone should also be chosen to correspond with the resistance of the network; or its resistance should be adapted to the bridge by the interposition of an interbridge transformer§ of suitable ratio (see p. 145). For ordinary bridges of medium impedance a telephone of 150 ohms resistance is suitable. For use in bridges with high impedance branches such a telephone can be combined with a step-up transformer, the winding with few turns being joined to the telephone, while that with many turns is connected to the bridge branch-points. Conversely, the use of a step-down transformer will adapt the instrument to a low impedance bridge. Capacity effects between the observer and the telephone are sometimes disturbing and can best be allowed for by some device such as the Wagner earth point (see p. 286). Stray magnetic fields from the bridge may also

* Max Wien, "Die Akustischen und elektrischen Constanten des Telephons," *Ann. der Phys.*, Bd. 4, pp. 450-458 (1901).

† Lord Rayleigh, *Phil. Mag.*, Vol. 38, p. 294 (1894).

‡ Max Wien, *loc. cit.*, p. 456; see L. W. Austin, *Bull. Bur. Stds.*, Vol. 5, pp. 153-157 (1909). E. W. Washburn, *Phys. Rev.*, 2nd series, Vol. 9, pp. 437-439 (1917).

§ G. Chaperon, *Comptes Rendus*, t. 108, p. 799 (1889).

cause trouble; in this case the instrument should be set up out of inductive range and be connected to the observer by flexible ear-tubes. When choosing a telephone, it should be remembered that it is the *effective* resistance and reactance of the instrument at the frequency of the test which must be considered.

31. Tuned Detectors. It has been pointed out in the last section that a telephone shows preferential sensitivity for currents of certain frequencies, namely, those which will set the diaphragm into resonant vibration in one of its normal modes of oscillation. It would appear advantageous to have a telephone or other piece of apparatus which would be capable of being *tuned* to resonance with the current to be measured, so that the greatly magnified oscillation and increase of sensitivity can be secured. Such instruments are called *tuned detectors* and are very widely used.

A. Campbell* has shown that an ordinary telephone can be tuned by pressing on the diaphragm at an eccentric point by means of an adjusting screw. A more accurate and sensitive device is the mono-telephone of Abraham,† in which the diaphragm is replaced by two stretched strips carrying an armature which is arranged to face the magnet/coil system. Tuning is effected by varying the tension of the strips. Mercadier‡ has introduced a form of telephone tuned to a fixed frequency; ordinary sensitive telephones made to respond to fixed frequencies are made by the Cambridge & Paul Co. A highly sensitive telephone is that of S. G. Brown, in which the usual diaphragm is replaced by a vibrating iron reed carrying a light conical aluminium diaphragm; this instrument is not tunable in the sense considered here.

Wien§ has shown that the ordinary telephone can be rendered enormously more sensitive by acoustically tuning it. This is done by causing the sound emitted by the telephone to pass directly into a suitable Rayleigh acoustic resonator.

The most important tuned detectors are the so-called *vibration or resonance galvanometers*. These are really galvanometers working on the usual moving coil or moving magnet principle, in which the natural frequency of the moving system can be tuned to be in resonance with the alternating current to be measured. The forced vibration of the moving system will

* *Dictionary of Applied Physics*, Vol. 2, p. 404.

† H. Abraham, "Rendement acoustique du téléphone," *Comptes Rendus*, t. 144, pp. 906-908 (1907).

‡ E. Mercadier, *Comptes Rendus*, t. 130, pp. 1382 (1900).

§ Max Wien, "Ueber die Anwendung von Luftresonatoren bei Telephontönen," *Phys. Zeits.*, 13 Jahrgang, pp. 1034-1037 (1912).

be much greater when currents of the resonating frequency pass through the instrument than when currents of any other frequency are used. By the use of the principle of resonance, a great increase in sensitiveness is produced and the instrument shows marked frequency selectiveness. Hence when the instrument is tuned to the fundamental of a given source of current, harmonics in the wave form produce little effect, so that when a vibration galvanometer or other tuned detector is used in an a.c. bridge, the balance obtained is the same as if the harmonics were absent. The source, therefore, need not have a pure wave form; it must, however, have constant frequency, since the high sensitivity is confined to a narrow range of frequency near the resonance value.

Vibration galvanometers are constructed to cover a range of frequency from about 1,000 cycles per second downward. At the lower values, up to about 300 cycles per second, they are much more sensitive than the telephone, and approach in sensitivity the best d.c. galvanometers.

Vibration galvanometers are of two kinds, according as the tuned system is a moving magnet or a moving coil; the two types correspond to the Kelvin suspended needle and the d'Arsonval d.c. galvanometers. A brief discussion of each will now be given.

32. Moving Magnet Vibration Galvanometers. Moving magnet vibration galvanometers generally resemble in principle of construction the Kelvin galvanometers used with d.c. The moving part consists of a suspended magnetized system, consisting of small permanent magnets or of soft iron polarized by an auxiliary magnetic field. Tuning of the natural period of the suspended part is effected by altering the length and tension of the suspension, or by varying the strength of the auxiliary magnetic field, if any. The moving magnets are deflected by the current to be measured passing round suitable deflecting coils; the amplitude of the resulting oscillation is determined by observing the breadth of the band of light reflected on to a scale from a mirror mounted on the vibrating part. The principal sources of damping are air friction and elastic hysteresis in the suspension; the controlling forces are supplied either by torsion of the suspending fibre, by magnetic action of an auxiliary field, or by combinations of both these causes.

The earliest vibration galvanometer is the "optical

telephone" of Max Wien.* In principle this instrument is the same as an ordinary acoustic telephone, but differs in two important respects: (i) it works in resonance with the applied current and is, therefore, highly sensitive; (ii) the resulting motion of the diaphragm is caused to operate a magnifying lever which carries a mirror, from which a beam of light is reflected to a distant scale, where it is drawn out by the vibration into a luminous band. Fig. 43 (a) illustrates two types of optical telephone in diagrammatic outline. The early pattern is practically a telephone with optical magnification. In the later pattern a more effective magnet system is used, in which four deflecting coils, wound so that their relative polarities are as shown by the small letters. With a d.c. resistance of 400 ohms the sensitivity is quoted as about 3 mm. per microamp. at 1 metre. Wide range of tuning is effected by changing the diaphragm; fine adjustment by slightly moving the polarizing magnet.

Rubens† has improved Wien's arrangement by replacing the diaphragm by a galvanometer system consisting of 20 small iron needles soldered to a brass wire (see Fig. 43 (b)). By altering the length and tension of the wire, the instrument can be tuned over a range of 50 to 200 cycles per second. The needles lie in the magnetic field produced by a pair of permanent magnets, upon the poles of which coils are wound in such a direction as to exert a couple on the suspended system. Fine tuning is obtained by altering the positions of the magnets relative to the needles. The sensitivity‡ at 100 cycles is 1.5 mm. per microamp. at 1 metre.

Schering and Schmidt§ have described a modification of Rubens' type in which the polarizing field is produced by an electromagnet excited by d.c., adjustment of the strength of which tunes the moving system. The latter consists of a small sheet-iron needle suspended by a short phosphor bronze wire. A range from 8 up to 140 cycles per second is

* Max Wien, "Das Telephon als optischer Apparat zur Strommessung," *Ann. der Phys.*, Bd. 42, pp. 593-621, Bd. 44, pp. 681-688 (1891).

† H. Rubens, "Vibrationsgalvanometer," *Ann. der Phys.*, Bd. 56, pp. 27-41. (1895).

‡ F. Wenner, *Bull. Bur. Stds.*, Vol. 6, p. 365 (1910); see also E. B. Rosa and F. Grover, *idem*, Vol. 1, p. 291 (1905), for a resonance curve.

§ H. Schering and R. Schmidt, "Ein Vibrationsgalvanometer mit elektromagnetischer Abstimmung für niedrige Frequenzen," *Zeits. f. Inst.*, 38 Jahrgang, pp. 1-11 (1918).

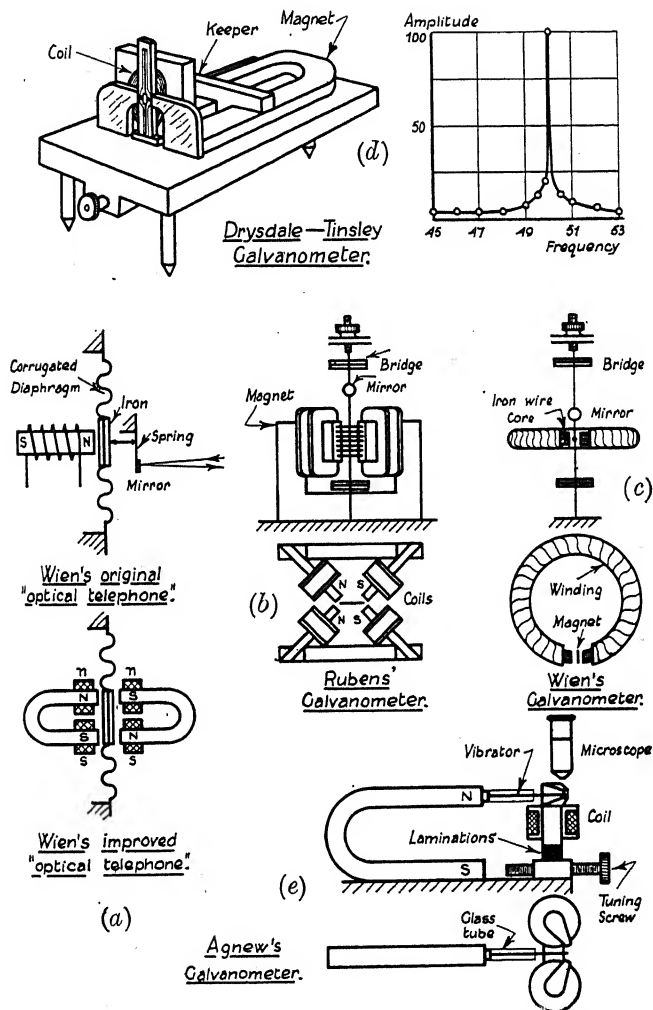


FIG. 43.—MOVING MAGNET VIBRATION GALVANOMETERS

covered, and the sensitivity is about 10 mm. per microamp. at 1 metre.

Wien* has described a highly sensitive galvanometer shown in Fig. 43(c), in which the current to be detected passes through a coil wound on a ring made of iron wires. The moving system consists of a set of very small magnets fastened to a brass wire so that they are held parallel to the faces of a gap in the ring. Coarse tuning is effected by varying the length and tension of the suspension wire; fine tuning by altering the width of the gap in the electro-magnet. The sensitiveness is inversely proportional to the square of the frequency, and at 100 cycles per second is quoted as 70 mm. per microamp. at 1 metre; Wells† has shown that the deflection is proportional to the voltage and shows how the deflection varies with the suspension length when a current of constant frequency flows in the coil.

Drysdale‡ has designed a galvanometer for use at low frequencies (see Fig. 43(d)). The moving system consists of a light mirror carrying a soft iron needle, the whole being suspended by a taut silk thread in the field of a powerful permanent horse-shoe magnet. Behind the moving needle the deflecting coil is situated so that the deflecting field is perpendicular to the control field and, hence, sets the needle into vibration. Since the natural frequency of the suspended needle is proportional to the square root of the control field, tuning is effected by altering the field due to the permanent magnet by the expedient of sliding an iron keeper along the magnet limbs. A range of frequencies between about 40 and 160 is thus covered. The deflecting coils are interchangeable and can be chosen to have an impedance suitable for the work in hand. The damping is very slight, resulting in sharp resonance. At 50 cycles, the current sensitivity varies between 1 mm. and 70 mm. per microamp. at 1 metre when coils having impedances between 1.26 and 6,000 ohms are used. The corresponding voltage sensitivities are 0.8 to 0.012 mm. per microvolt.

A galvanometer identical in principle with that of Drysdale

* M. Wien, "Ueber die Erzeugung und Messung von Sinusströmen," *Ann. der Phys.*, Bd. 4, pp. 425-449 (1901).

† R. T. Wells, "Note on the vibration galvanometer," *Phys. Rev.*, Vol. 23, pp. 504-506 (1906).

‡ H. Tinsley, "A magnetic shunt vibration galvanometer," *Electr.*, Vol. 69, pp. 939-941 (1912).

has been described by Blondel and Carbenay;* in it the controlling field is produced by a pair of coils carrying a direct current instead of by a permanent magnet. The instrument is tuned by adjustment of this current, and has a range up to 1,500 cycles per second.

Agnew† has designed a galvanometer on a new principle, shown in Fig. 43 (e). The moving system consists of a fine steel wire attached to one pole of a permanent magnet by one of its ends; the other end vibrates in the field of an electro-magnet whose winding carries the alternating current. The vibration of the wire is observed by a microscope mounted above it. With 270 ohm coils and a magnification of 50/100, 0.05 microamp. can be detected. The frequency to which the instrument responds is varied by changing the vibrator; fine tuning is secured by screwing an iron rod nearer to or farther from the lower magnet pole. The instrument is sturdy, and very quickly responsive.

Disadvantages of Moving Magnet Galvanometers. Moving magnet instruments are much affected by stray magnetic fields of the resonant frequency and must be set up at some distance from the bridge. They are also very susceptible to disturbance by mechanical vibrations of the appropriate frequency, and are best arranged on some form of anti-vibration support.

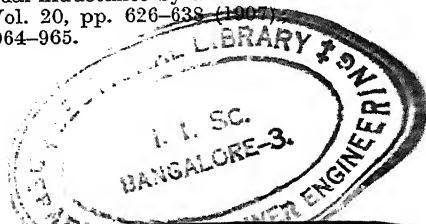
33. Moving Coil Vibration Galvanometer. The moving coil vibration galvanometer is really a d'Arsonval of short periodic time, arranged with a small light coil of few turns mounted on a suspension which can be tuned by alteration of its length and tension. The coil stands in the field of a permanent magnet and vibrates when the alternating current to be measured is passed through it. The amplitude of the oscillation is observed in the usual way by reflecting a beam of light to a distant scale from a mirror carried by the coil.

The development of the m.c. instrument is due to Campbell,‡ who has designed various types of galvanometer, some being

* A. Blondel and F. Carbenay, "Analyse harmonique des différences de potential alternatives par la résonance mécanique d'un barreau de fer aimanté," *Annales de Phys.*, t. 8, pp. 97-158 (1917).

† P. G. Agnew, "A new form of vibration galvanometer," *Bur. Stds. Scientific Papers*, No. 370, pp. 37-44 (1920).

‡ A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626-632 (1907); also *Dictionary of Applied Physics*, Vol. 2, pp. 964-965.



shown in Fig. 44. In the original type the coil is suspended by a silk thread above and bifilar suspensions below, and is tunable over a range from 50 to 800 cycles per second. The modern long-range pattern has metal bifilars both above and below, and is tunable over a range from 40 to 1,000 cycles per second; the short range type, on the other hand, is provided with unifilar strip* suspensions, originally introduced by Hausrath,† and covers a range from 10 to 400 cycles per second.

The instrument is capable of very high sensitivity, especially at low frequencies; Campbell‡ has produced a galvanometer giving 400 mm. at 1 metre per microamp. at 10 cycles per second. The curves given in Fig. 44 for a typical Campbell bifilar galvanometer show the sensitivity of the instrument when tuned to respond to different frequencies. The sensitiveness falls inversely as the frequency for lower values, but much more rapidly at higher frequencies owing to the enormous increase in the damping with short suspensions. The effective resistance varies between about 500 ohms and 35 ohms over the range of frequency, 50 to 1,000 cycles per second.

The damping is chiefly due to elastic hysteresis in the suspension strips, which are usually of phosphor bronze. The small value of damping is indicated by the sharp resonance curves shown in Fig. 46, p. 169. Owing to the narrow range of high sensitiveness, moving coil galvanometers must be used with a constant frequency source; moreover, in common with other tuned detectors, they show great frequency selectiveness.

An ordinary bifilar oscillograph of the Duddell pattern can be used as a vibration galvanometer by removing the oil damping; it is applicable at frequencies above about 2,000.§

* A. Campbell, "On vibration galvanometers with unifilar torsional control," *Proc. Phys. Soc.*, Vol. 25, pp. 203-205 (1913). A. Campbell, "On vibration galvanometers of low effective resistance," *Proc. Phys. Soc.*, Vol. 26, pp. 120-126 (1914). Both types are made by the Cambridge & Paul Co.

† H. Hausrath, "Die methoden zur Eisenuntersuchung bei Wechselstrom und ein Apparat zur Darstellung dynamischer Hysteresiskurven," *Phys. Zeits.*, 10 Jahrgang, pp. 756-762 (1909).

‡ A. Campbell, *Proc. Phys. Soc.*, Vol. 31, pp. 85-86 (1919). For other types of m.c. galv., see P. G. Agnew and F. B. Silsbee, *Trans. Amer. I.E.E.*, Vol. 31, pp. 1635-1644 (1912); H. Zöllich, "Ueber ein hochempfindliches Vibrationsgalvanometer für sehr niedrige Frequenzen," *Arch. f. Elek.*, Bd. 3, pp. 369-383 (1915); A. Blondel, *Annales de Phys.*, t. 10, pp. 195-354 (1918).

§ Mühlenthorst, *Dissertation*, Münster i. W. (1905). E. Giebe and H. Diesselhorst, *Zeits. f. Instk.*, 26 Jahrgang, p. 151 (1906).

Duddell,* by re-designing the instrument, greatly increased the sensitiveness and extended the range downwards. In his galvanometer the current passes through a loop of fine bronze

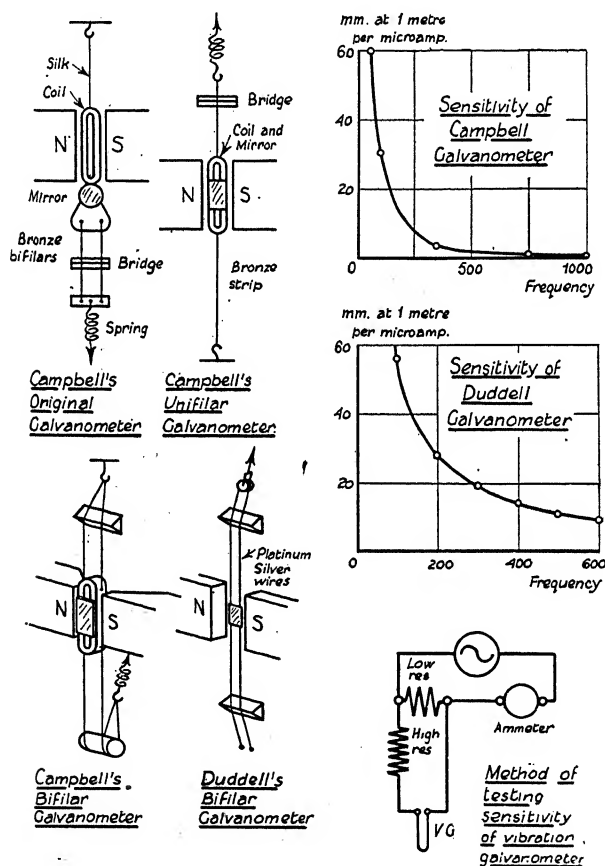


FIG. 44.—MOVING COIL VIBRATION GALVANOMETERS

or platinum-silver wire stretched by means of a spring over two ivory bridges which determine its vibrating length. The wires lie in the strong field of a permanent magnet, and carry a small mirror at the middle of their length. Rough tuning

* W. Duddell, "On a bifilar vibration galvanometer," *Proc. Phys. Soc.*, Vol. 21, pp. 774-787 (1910).

is secured by changing the distance apart of the bridges, which can be moved simultaneously by a right and left-hand screw, so that the mirror is always midway between them. Fine tuning is done by altering the tension of the loop. Instruments of this pattern cover a range from 100 to 1,800 cycles per second and have an effective resistance of about 250 ohms.

The sensitivity is high, being about 50 mm. at 1 metre per microamp. at 100 cycles per second, and falls off inversely as the frequency, provided the loop is not too short. Haworth* has shown how the sensitiveness, especially to applied voltage, may be further increased by the use of an electromagnet and by using the instrument *in vacuo*. In the latter case an increase of 30 per cent is obtainable, since a large part of the damping is due to air friction.

Schering and Schmidt† have constructed a galvanometer of Duddell's pattern having wires as long as 100 cm. and extending the frequency down to 25 cycles. At 50 cycles, 10 mm. per microamp. at 1 metre was obtained.

34. Vibration Electrometers. Small condensers possess very large impedance when used in a.c. circuits at low frequencies. When measured in a bridge they form branches of high impedance, and to secure sensitiveness the impedance of the detector must also be high. A vibration electrometer is, therefore, useful in such cases and was first employed by Greinacher.‡ His instrument was developed from the ordinary Wulf electrometer.

Curtis§ has designed an instrument by means of which condensers of $10^{-3} \mu F$ can be measured at 50 cycles with ten times the accuracy attainable with a vibration galvanometer. It consists of an aluminium vane mounted vertically upon a tunable bifilar suspension, the vane corresponding to the "needle" of a quadrant electrometer. Four rectangular plates are placed two in front and two behind the vane, and

* H. F. Haworth, "The maximum sensibility of a Duddell vibration galvanometer," *Proc. Phys. Soc.*, Vol. 24, pp. 230-237 (1912); "Vibration galvanometer design," *idem*, Vol. 25, pp. 264-272 (1913).

† A. Schering and R. Schmidt, "Ein empfindliches Vibrationsgalvanometer für niedrige Frequenzen," *Arch. f. Elek.*, Bd. 1, pp. 254-258 (1913).

‡ H. Greinacher, "Ueber die Verwendung des Vibrations-Elektrometers in der Wheatstoneschen Brücke," *Phys. Zeits.*, 13 Jahrgang, pp. 388-389, 433-434 (1912); "Ueber das Vibrationsselektrometer und dessen Verwendung bei Wechselstrommessungen," *Arch. f. Elek.*, Bd. 1, pp. 471-476 (1913).

§ H. L. Curtis, "A vibration electrometer," *Bull. Bur. Stds.*, Vol. 11, pp. 535-552 (1915).

are connected diagonally. The vane is charged at a high, steady potential; the two pairs of plates are connected to the bridge. The suspended vane is tuned to resonance with the applied p.d. and, to reduce damping, the whole instrument is enclosed in a glass bell-jar, from which the air is exhausted to a certain degree. The sensitivity is of the order of 10,000 mm. at 1 metre per microamp., the deflection being read by reflected light from a mirror carried by the vane. The instrument is not suitable for frequencies over 100.

35. Method of Using Vibration Galvanometer. Before endeavouring to balance the bridge in which a vibration galvanometer is to be used, the instrument must be tuned. To do this a small current is passed through it, frequently by connecting the galvanometer into the unbalanced bridge and heavily shunting it. The length and tension of the suspension, or other tuning adjustment, is then varied until the band of light on the scale attains its maximum breadth; the moving system is then in resonance with the applied current. The bridge may now be balanced, the galvanometer shunt being gradually removed to give increased sensitiveness as the deflection is reduced to zero.

Great care should be taken to see that the spot of light reflected from the mirror is quite sharp when no current is applied to the instrument. The mirror should be a good one, and should give a good image of the lamp filament or the cross-wire used to show the deflection. A "Point'o'ite" lamp forms an excellent source of illumination, especially for instruments of the Duddell pattern in which a very small mirror is used. The instrument should be set up so that it is free from mechanical vibration and is unaffected by stray alternating magnetic fields of the resonance frequency.

For an additional adjustment, possible in some instruments, for greatest voltage sensitivity, *see* page 173.

36. Theory of the Vibration Galvanometer. Owing to the importance of the vibration galvanometer as a sensitive detector in bridge work and in other classes of measurements, e.g. testing of current transformers, the analysis of alternating wave forms, etc., it is of interest to examine the theory of the instrument to find the best conditions under which to use it and also to determine data for design. The theory of apparatus working in mechanical resonance with a periodic

current has been known for a long time,* but the application to the theory of vibration galvanometers has only recently been made.

For the purposes of theory vibration galvanometers may be divided into two classes, according to the possible number of degrees of freedom possessed by the moving system. The first class includes instruments of the moving coil or the suspended magnet type where a bifilar or a flat strip suspension is used; the system has only one degree of freedom, namely, rotation about the axis of suspension, provided that the moving system is symmetrically mounted on the axis and that the latter possesses lateral rigidity. In the second class, represented by the Duddell string type of galvanometer, there is an infinite number of degrees of freedom.

The theory of galvanometers with one degree of freedom has been given by Wenner.† His terms and theory will be adopted here, but his analytical methods will be superseded by the symbolic vector method as worked out by Butterworth‡ and extended by him to cover the case of instruments with an infinite number of degrees of freedom.

37. Vibration Galvanometers with One Degree of Freedom.

General. In order to fix ideas, let the theory of the widely-used Campbell galvanometer be considered. In this instrument a moving coil vibrates in a magnetic field and is controlled by a bifilar suspension, the length and tension of which can be varied. With obvious modification of detail, the theory will also apply to moving magnet instruments and also to the acoustic telephone.

When at rest the coil of the galvanometer lies with its plane in the direction of the magnetic field. Suppose the field to have strength H , and that the plane of the coil has area A . If the coil has N turns and carries a current whose instantaneous value is i , the couple deflecting the coil from its initial position will be $HANi \cos \theta$, where θ is the angle between the field and the plane of the coil. When θ is small, the deflecting couple is, very nearly, $HANi = Gi$; G is called the *constant of displacement*.

* H. L. F. von Helmholtz, *On the Sensations of Tone*, 2nd edn., Appendix 8, pp. 398-400 (1885).

† F. Wenner, "Theoretical and experimental study of the vibration galvanometer," *Bull. Bur. Stds.*, Vol. 6, pp. 347-378 (1910).

‡ S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75-94 (1912).

The deflecting couple is opposed by three retarding couples—

(i) A restoring couple, due to the elasticity of the suspension, proportional to the displacement. If c is the *constant of restoration*, the couple is $c\theta$.

(ii) A damping couple due to air-friction, imperfect elasticity of the suspension, etc. It is usual to take this couple as proportional to the angular velocity of the coil. If b be the *constant of damping*, the couple is $b \cdot d\theta/dt$. Blondel and Carbenay* have described a moving magnet galvanometer in which it was necessary to assume the damping proportional to the square of the velocity; other workers have also made this correction to the theory of oscillating bodies, especially in cases where air-damping at high velocities is involved.

(iii) A kinetic reaction couple proportional to the angular acceleration of the coil. If a be the *constant of inertia* of the coil, the couple is $a \cdot d^2\theta/dt^2$.

The equation of motion is obtained by equating the total resisting couple to the deflecting couple, i.e.

$$a \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} + c\theta = Gi.$$

The quantities a , b , c , G are expressed in absolute units, and are called by Wenner the *intrinsic constants* of the instrument.

Two distinct problems now arise: (i) the conditions of motion of a tuned galvanometer carrying a current $i = i_1 \cos \omega t$; (ii) the behaviour of a galvanometer when connected in a circuit, such as a bridge network, which contains an alternating e.m.f. The first problem is concerned with the current sensitivity of the galvanometer, and is similar to the theory of the oscillograph worked out so thoroughly by Blondel.† The second problem involves the consideration of the effect of the voltage induced in the galvanometer coil as it swings in the magnetic field, since this back e.m.f. will have an important influence on the voltage sensitivity of the instrument.

38. Vibration Galvanometers with One Degree of Freedom.
Current Sensitivity. Let the galvanometer carry a current

* A. Blondel and F. Carbenay, "Analyse harmonique des différences de potentiel alternatives par la résonance mécanique d'un barreau de fer aimanté," *Annales de Phys.*, t. 8, pp. 97-158 (1917); see also B. O. Pierce, *Proc. Amer. Acad.*, Vol. 44, pp. 63-88 (1909); and R. Grammel, *Phys. Zeits.*, 14 Jahrgang, pp. 20-21 (1913).

† A. Blondel, "Théorie des oscillographes," *Écl. Élec.*, t. 33, pp. 115-125 (1902), and t. 36, pp. 326-346 (1903).

$i = i_1 \cos \omega t$ represented by a vector i . Then the complete solution of the equation of motion consists of the sum of two parts: (i) a transient portion, representing the natural free motion of the coil, involving the initial conditions of the moving system at the time of switching on the current; (ii) a periodic forced oscillation corresponding to the steady vibratory motion of the coil after the initial disturbances have subsided. It is with (ii) that we are chiefly concerned, but a few words are first necessary regarding (i).

The *transient part of the solution*, θ_r , is obtained by solving the equation

$$a \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} + c\theta = 0.$$

By the ordinary theory of differential equations if the roots of the auxiliary equation

$$ax^2 + bx + c = 0$$

are $m_1, m_2 = \{-b \pm \sqrt{b^2 - 4ac}\} / 2a = -\alpha \pm \beta$, the solution is

$$\theta_r = A\epsilon^{m_1 t} + B\epsilon^{m_2 t} = \epsilon^{-\alpha t} \{A\epsilon^{\beta t} + B\epsilon^{-\beta t}\}$$

provided $b^2 < 4ac$;

or $\theta_r = \epsilon^{-\alpha t} \{A + Bt\}$

when $b^2 = 4ac$; A and B are found from the initial conditions.

Hence owing to the factor $\epsilon^{-\alpha t}$ the transient part of the solution is gradually damped out. In vibration galvanometers b is usually small, so that $4ac$ is greater than b^2 ; in this case

$$\theta_r = \epsilon^{-\alpha t} \left\{ C \cos \left(\frac{\sqrt{4ac - b^2}}{2a} t + \phi \right) \right\};$$

C and ϕ are again determined by the initial conditions. The free galvanometer system thus performs *damped natural oscillations* of frequency $(\sqrt{4ac - b^2}) / 4\pi a$. If the system had no

damping the *undamped natural frequency* would be $f_0 = \frac{1}{2\pi} \sqrt{\frac{c}{a}}$; or $\omega_0 = \sqrt{c/a}$.

When the damping is such that the system just ceases to be capable of free oscillations $\beta = 0$ and the galvanometer is *critically damped*. The appropriate value of b is $b_c = 2\sqrt{ac}$. This degree of damping is approximately that aimed at in oscillographs, but is not nearly approached in vibration

galvanometers; it is useful, however, to express the actual damping in terms of the critical value, i.e. b/b_c is the *degree of damping*.

The effect of the transient oscillations of the coil is to produce initially when the galvanometer is switched into circuit an unsteady motion, beats being observed between the free and forced oscillations. These beats gradually die out until the instrument is in a steady state of forced oscillation corresponding to the current i , as the records of the motion of a moving coil galvanometer obtained by Zöllich* clearly show.

Hence for the present purpose the transient part of the motion may be neglected, attention being confined to the forced periodic motion of the coil.

The *periodic forced oscillation* is obtained by solving

$$a \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} + c\theta = Gi_1 \cos \omega t = \sqrt{2}GI \cos \omega t.$$

Clearly θ must be a periodic function of ωt ; let it be represented by the harmonic vector θ . Then by the methods shown on pages 25-32

$$\{(c - a\omega^2) + j\omega b\}\theta = Gi,$$

$$\theta = \frac{Gi}{(c - a\omega^2) + j\omega b} = \frac{G}{\sqrt{(c - a\omega^2)^2 + \omega^2 b^2}} \cdot \angle -\tan^{-1} \frac{\omega b}{c - a\omega^2} \cdot i$$

from page 36.

Hence the amplitude of the motion of the coil is

$$\theta_1 = \frac{Gi_1}{\sqrt{(c - a\omega^2)^2 + \omega^2 b^2}} = \frac{\sqrt{2}GI}{\sqrt{a^2(\omega_0^2 - \omega^2)^2 + \omega^2 b^2}}$$

the motion lagging in phase by an angle

$$\psi = \tan^{-1} \frac{\omega b}{c - a\omega^2};$$

i.e. $\theta = \theta_1 \cos(\omega t - \psi).$

For a given value of the current I it is required to make the amplitude of the motion as large as possible. Clearly, θ_1 is proportional to $G = HAN$; since the area and turns of the coil are fixed, the amplitude increases as the field in which the coil lies is made stronger. Hence *the magnet should be powerful* and can be an electromagnet (see, however, p. 173).

* H. Zöllich, "Über ein hochempfindliches Vibrationsgalvanometer für sehr niedrige Frequenzen," *Arch. f. Elek.*, Bd. 3, pp. 369-383 (1915).

Further increase of amplitude is secured by "tuning" the instrument. As Max Wien has pointed out, there are two ways of tuning a galvanometer to resonance with a source of supply: (i) by adjusting the frequency of the source; (ii) by adjusting the constants of the galvanometer until resonance is attained.

FREQUENCY TUNING. Let the galvanometer be such that a , b , c , and G are fixed, and let ω be continuously varied; then θ_1 will be a maximum when ω reaches the value given by

$$\omega_f^2 = \frac{c}{a} - \frac{1}{2} \left(\frac{b}{a} \right)^2 = \omega_o^2 - \frac{1}{2} \left(\frac{b}{a} \right)^2$$

the amplitude then being

$$\frac{2 \sqrt{2} G a I}{b \sqrt{(4ac - b^2)}} = \frac{\sqrt{2} G I}{b \omega_n}$$

since the natural damped frequency of the galvanometer is obtained from

$$\omega_n^2 = \frac{4ac - b^2}{4a^2} = \frac{c}{a} - \frac{1}{4} \left(\frac{b}{a} \right)^2 = \omega_o^2 - \frac{1}{4} \left(\frac{b}{a} \right)^2.$$

This shows that for the greatest sensitiveness, the *damping* b and the *natural frequency of the instrument must be low*; resonance then occurs when the frequency of the source $\omega_f = \omega_n = \omega_o$, since b is negligible.

Frequency tuning is easily studied in a general way. Introducing the undamped natural frequency and the critical damping gives $\omega_o = \sqrt{c/a}$, $b_c = 2\sqrt{ca}$; the amplitude then becomes

$$\theta_1 = \frac{\sqrt{2} G I}{c \sqrt{\left(1 - \frac{\omega^2}{\omega_o^2}\right)^2 + 4 \frac{b^2}{b_c^2} \cdot \frac{\omega^2}{\omega_o^2}}}.$$

When the frequency is very low, i.e. $\omega \rightarrow 0$, the amplitude is $\sqrt{2} G I / c$; then if

$$\gamma = \frac{\omega}{\omega_o} = \frac{\text{applied frequency}}{\text{undamped natural frequency}}$$

and $\delta = \frac{b}{b_c} = \frac{\text{damping constant}}{\text{critical damping constant}} = \text{degree of damping},$

$$\frac{\text{Amplitude}}{\text{Amplitude for zero frequency}} = \frac{1}{\sqrt{(1 - \gamma^2)^2 + 4\gamma^2\delta^2}}$$

$$\tan \psi = 2\gamma\delta / (1 - \gamma^2).$$

Fig. 45 shows these expressions plotted as a function of γ for various degrees of damping. When the damping is equal to or in excess of the critical value, gradually raising the applied frequency continually reduces the amplitude of motion. When the damping is less than critical, the amplitude attains a maximum for some value of frequency less than the undamped frequency of the moving system, diminishing again as the frequency is still further raised. The maximum amplitude is greater as the damping is reduced, and occurs more and more nearly at the value of applied frequency equal to the undamped frequency of the system. Hence the great importance in vibration galvanometers of reducing the damping as much as possible. At the same time these curves show the selective sensitiveness of the galvanometer for currents of the resonating frequency and its insensitiveness to all other applied frequencies, e.g. to harmonics in the deflecting current.

Referring to the curves illustrating the phase of the motion, it is clear that, whatever be the degree of damping, the deflecting couple and the resulting motion will be in phase when the frequency is very low. As the frequency is increased, the motion lags behind the deflecting couple until, when the applied frequency and the undamped

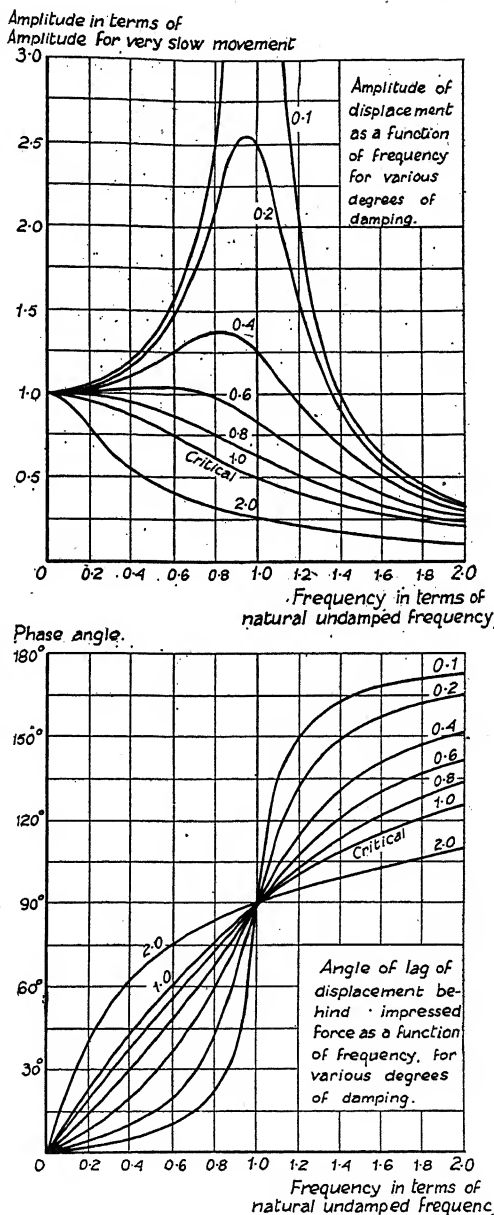


FIG. 45.—FREQUENCY TUNING OF A VIBRATING SYSTEM

natural frequency are equal, they are in quadrature. Thereafter the phase-angle increases until, whatever be the damping, the couple and the motion are in opposition of phase when the frequency is infinite. A careful study of these curves, which apply to all cases of forced oscillations in mechanical systems, is recommended to the student.*

CONTROL TUNING. In most bridges, tests are to be carried out at a definite frequency, so that ω is fixed. Hence the tuning is more usually done on the galvanometer itself, thereby enabling one instrument to cover a range of frequencies. It is not easy to produce a continuous variation of the moment of inertia a ; the damping b is seldom sufficiently adjustable to serve as a means of tuning. Thus, tuning is effected by alteration of c by variation of the length and tension of the coil suspension.

With ω , G , a and b fixed, θ_1 will be a maximum when $c = a\omega^2$; that is, when $\omega_0 = \omega$, the undamped natural frequency of the moving system being equal to the fixed applied frequency. The amplitude has the maximum value

$$\sqrt{2}GI/\omega b$$

and the motion is $\pi/2$ later than the applied current. For the greatest sensitivity the magnetic field should be strong, the damping slight, and the applied frequency low.

To illustrate the control tuning of a vibration galvanometer, curves are plotted in Fig. 46 for a Campbell† instrument in which $a = 6.9 \times 10^{-6}$, $b = 23.2 \times 10^{-6}$, $G = 585$, giving the amplitude in arbitrary units in terms of the control constant c when applied frequencies of 50 and 100 are used. Resonance occurs when the applied and undamped frequencies are equal, and the extreme sharpness of tuning is to be noted, showing the selective frequency property of the galvanometer due to its slight damping. For a frequency of 100, $c = 2.72$, so that $b_c = 2\sqrt{ac} = 8.68 \times 10^{-3}$; hence $b/b_c = 0.0027$.

To show how much more sensitive the galvanometer is to currents of the frequency to which it is tuned, consider the response of an instrument to the n th harmonic when tuned to the fundamental. The amplitude due to the fundamental is $\sqrt{2}GI/\omega b$, since ω_0 is tuned to equal ω . With ω_0 at this

* C. C. Hawkins, *The Dynamo*, pp. 116–126, 6th edition. A. Blondel, *loc. cit.*

† *Dictionary of Applied Physics*, Vol. 2, p. 974.

value, let ω be changed to $n\omega$ in the general equation for the amplitude, p. 165; then the amplitude due to a current of n times fundamental frequency is, for the same current strength, $\sqrt{2} GI / \{ \sqrt{a^2(\omega^2 - n^2\omega^2)^2 + n^2b^2\omega^2} \}$. Neglecting the damping term, this is $\sqrt{2} GI / \omega^2 a(n^2 - 1)$; hence

$$\begin{aligned} \text{Sensitivity to fundamental} / \text{sensitivity to } n\text{th harmonic} \\ = \omega a(n^2 - 1) / b. \end{aligned}$$

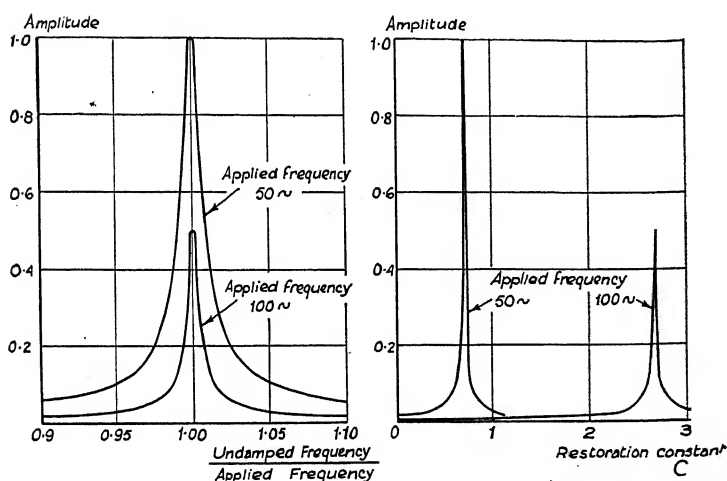


FIG. 46.—CONTROL TUNING OF A VIBRATION GALVANOMETER

In the case of the Campbell instrument treated above, when tuned to resonate at 100 cycles per second the ratio works out to be 1,495 when $n = 3$; i.e. the galvanometer is 1,495 times more sensitive to the fundamental than it is to the third harmonic, and the ratio is considerably greater for higher harmonics. Hence it is not necessary to supply absolutely pure wave forms of current to bridges in which a vibration galvanometer is used, since the sensitiveness of the instrument is so much greater for current of fundamental frequency than it is for the harmonics.

39. Vibration Galvanometers with One Degree of Freedom. Voltage Sensitivity. In its usual application to a.c. bridges, the vibration galvanometer is used to detect small differences of potential between the branch points of the bridge. Its

voltage sensitivity then becomes important, and will now be investigated.*

When the coil of the galvanometer is vibrating, it cuts the field issuing from the magnet, and therefore has an e.m.f. induced in it. The current passing through the coil is thus determined in part by the back e.m.f. appearing in the coil due to its vibratory motion in the field, this e.m.f. opposing the alternating p.d. applied to the circuit in which the galvanometer is placed.

If the plane of the moving coil makes an angle θ with the direction of the field H , the flux through it will be $HA \sin \theta$, and the linkages will be $HAN \sin \theta = G \sin \theta$. When θ is small the linkages are $G\theta$, so that the back e.m.f. is

$$e_b = -G \frac{d\theta}{dt}.$$

Since θ is a periodic function of ωt , e_b can be represented by a uniformly rotating vector $\mathbf{e}_b = -j\omega G\boldsymbol{\theta}$.

Suppose the galvanometer to be inserted in a circuit to which an electromotive force $e = e_1 \cos \omega t$ is applied. If the entire circuit, inclusive of the galvanometer, has resistance R and reactance X , its impedance operator will be $R + jX$. If \mathbf{e} be the vector of applied e.m.f., the current is given by

$$\mathbf{i} = (\mathbf{e} + \mathbf{e}_b)/(R + jX) = (\mathbf{e}/z) - (j\omega G\boldsymbol{\theta}/z) \quad (a)$$

From page 165, the symbolic equation of motion of the galvanometer is

$$\{(c - a\omega^2) + j\omega b\}\boldsymbol{\theta} = G\mathbf{i}; \quad (b)$$

substituting for the current from above expresses $\boldsymbol{\theta}$ in terms of \mathbf{e} ,

$$\left\{ (c - a\omega^2) + j\omega \left(b + \frac{G^2}{z} \right) \right\} \boldsymbol{\theta} = \frac{G}{z} \mathbf{e}$$

Inserting $z = R + jX$, and simplifying, gives

$$\{[R(c - a\omega^2) - \omega bX] + j[X(c - a\omega^2) + \omega(bR + G^2)]\}\boldsymbol{\theta} = G\mathbf{e};$$

so that

$$\theta = \theta_1 \cos(\omega t - \psi), \quad (c)$$

* F. Wenner, *loc. cit.*; S. Butterworth, *loc. cit.*; H. Zölllich, *loc. cit.* See also for a very full discussion, A. Blondel, "Sur l'analyse harmonique directe de l'onde des courants alternatifs par résonance mécanique ou électrique," *Annales de Phys.*, t. 10, pp. 195-334 (1918).

where the amplitude

$$\theta_1 = \frac{\sqrt{2GE}}{\sqrt{[R(c-a\omega^2)-\omega bX]^2 + [X(c-a\omega^2) + \omega(bR+G^2)]^2}},$$

and the phase

$$\tan \psi = \frac{X(c-a\omega^2) + \omega(bR+G^2)}{R(c-a\omega^2) - \omega bX},$$

E being the effective value of $e = e_1 \cos \omega t$.

Alternatively, elimination of θ gives the relation between e and i , thus,

$$zi = e - j\omega \cdot \frac{G^2 i}{(c-a\omega^2) + j\omega b}$$

$$\begin{aligned} \text{or } e &= \left[R + jX + j\omega G^2 \left\{ \frac{(c-a\omega^2) - j\omega b}{(c-a\omega^2)^2 + \omega^2 b^2} \right\} \right] i, \\ &= \left[R + \frac{\omega^2 G^2 b}{(c-a\omega^2)^2 + \omega^2 b^2} + j \left\{ X + \frac{\omega G^2 (c-a\omega^2)}{(c-a\omega^2)^2 + \omega^2 b^2} \right\} \right] i, \\ &= (R' + jX') i. \quad (d) \end{aligned}$$

so that

$$i = \frac{e_1}{\sqrt{R'^2 + X'^2}} \cos(\omega t - \phi), \text{ where } \tan \phi = X'/R'.$$

R' is called the effective resistance of the galvanometer circuit and X' the effective reactance.

CONTROL TUNING. The conditions for maximum sensitivity of a vibration galvanometer when used as a voltmeter are quite different from those applying to its current sensitivity. Referring to Equation (c) above, let a , b , and ω be fixed and, for the present, let G , R , and X be fixed also. Then let c be adjusted until the amplitude θ_1 has a maximum value. This will be found to occur when

$$(c-a\omega^2) = -\frac{\omega XG^2}{X^2 + R^2};$$

the value of the amplitude when the instrument is tuned is

$$\frac{\sqrt{2GE}}{\omega[b(R^2 + X^2)^{\frac{1}{2}} + G^2 R(R^2 + X^2)^{-\frac{1}{2}}]} \quad (e)$$

The phase of the motion is then $\tan \psi = R/-X$ or $\psi = \pi - \tan^{-1} \frac{R}{X}$.

When the circuit has zero reactance, e.g. as is very nearly the

case when the galvanometer alone is in circuit, then the amplitude is $\sqrt{2GE}/\omega(bR + G^2)$.

OPTIMUM SENSITIVITY. The maximum amplitude obtained by tuning the galvanometer in the above manner does not represent the greatest sensitiveness which is possible. The expression (e) can itself be made to attain a maximum value by adjustment of either of the quantities G or X . In both cases the optimum sensitiveness of the instrument as a voltmeter is secured when

$$b/R = G^2/(R^2 + X^2).$$

The optimum amplitude is then

$$\sqrt{2E}/2\omega\sqrt{bR}$$

When the galvanometer has been control tuned and has then been adjusted to the optimum condition by alteration of its magnet field, or by varying the circuit inductance, or by the use of an interbridge transformer, it is of interest to examine the resulting current and the back e.m.f. in the coil. Referring to Equation (d), let the conditions $(c - a\omega^2)/\omega X = -G^2/(R^2 + X^2) = -b/R$ be inserted; then the effective resistance $R' = 2R$ and the effective reactance $X' = 0$. Hence, $i = (e_1/2R) \cos \omega t$, or, in r.m.s. values, $I = E/2R$. Thus, when a vibration galvanometer is used as a voltmeter and is adjusted to optimum sensitiveness, the effective resistance of the galvanometer circuit is twice its d.c. resistance, and the current through it is in phase with the applied voltage.

When the galvanometer is so adjusted, it is easy to show that the back e.m.f. in the coil is given by $e_b/e = -\frac{1}{2}\left(1 - j\frac{X}{R}\right)$. If the reactance is negligible, as, for example, when the galvanometer alone is in circuit, then $e_b/e = -\frac{1}{2}$. Thus, when the instrument is used in a bridge and is adjusted to the optimum conditions, the back e.m.f. in the coil is half the applied voltage and is in opposition of phase therewith. Under these conditions, half the power supplied to the instrument is dissipated in heat and half in mechanical work, the latter maintaining the motion of the moving system as a synchronous motor. This condition is well known, and can be realized in practice by adjusting the strength of the

galvanometer field by the use of an electromagnet, as has been shown by Haworth* and later by Zöllich. After control tuning has been effected, the optimum is attained by adjusting the electromagnet; the effective resistance of the instrument is then $2R$, as is shown by the curves of Fig. 47, taken from a paper by Zöllich.

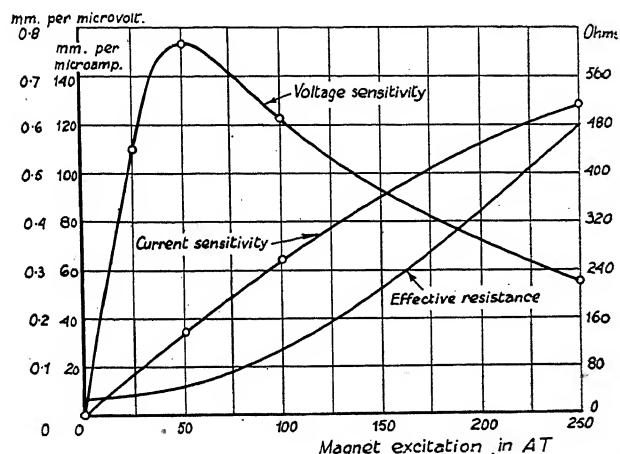


FIG. 47.—VOLTAGE SENSITIVITY, CURRENT SENSITIVITY, AND EFFECTIVE RESISTANCE OF A MOVING COIL VIBRATION GALVANOMETER PLOTTED AS FUNCTIONS OF THE STRENGTH OF THE MAGNET

An electromagnet has the disadvantage that it and its associated battery may produce large electrostatic capacity effects when the instrument is in an a.c. bridge. The battery supplying the magnet should be small and well insulated. The trouble can be completely overcome by using a permanent magnet of adequate strength provided with a magnetic shunt.

Another way of avoiding the troubles due to an electromagnet is to adapt the instrument impedance to that of the network by connecting it to the bridge through a transformer of suitable ratio. This method was described by Wenner and provides a simple means of attaining optimum sensitivity.

MOTIONAL RESISTANCE AND REACTANCE. Referring to Equation (d), suppose the galvanometer only to exist in the

* H. F. Haworth, "The maximum sensibility of a Duddell vibration galvanometer," *Proc. Phys. Soc.*, Vol. 24, pp. 230-237 (1912).

circuit, R being its resistance and X its reactance. Then the effective resistance R' is composed of the d.c. resistance of the instrument and a term, called its *motional resistance*, arising from the movement of the galvanometer coil. Similarly, the effective reactance X' consists of X together with the *motional reactance* of the coil. The quantities $R' - R$ and $X' - X$ have been investigated in a number of cases by Kennelly,* and are shown to have interesting physical properties.

In most galvanometers the inductance is very small, so that X is negligible; the effective reactance of the instrument is then equal to $\omega G^2(c - a\omega^2)/[(c - a\omega^2)^2 + \omega^2 b^2]$ its motional reactance. If the galvanometer is tuned so that $c/a = \omega_o^2 = \omega^2$, the reactance is zero and the galvanometer behaves as if it were a non-inductive resistance $R + (G^2/b)$. If the frequency be lower than this value, i.e. $\omega < \omega_o$, the instrument has positive reactance and behaves as if it were inductive. On the other hand, when $\omega > \omega_o$, the reactance is negative and the instrument acts as a condenser.†

40. Vibration Galvanometers with an Infinite Number of Degrees of Freedom. Asymmetric Systems. Galvanometers of the bifilar loop or Duddell pattern do not behave according to the theory just laid down for a moving coil instrument. The stretched wires carrying at their mid-point a load in the shape of a mirror, perform oscillations in the same way as the vibrating string of a monochord when centrally loaded. They have, therefore, an infinite number of normal modes of vibration, the relations between which are determined by the relative masses of the mirror and the wires.

Butterworth‡ has worked out the theory of such instruments and has shown how to find the infinite number of resonance frequencies. He points out that if the mirror be fairly heavy in comparison with the vibrating loop, the damping of the higher harmonics is large and the instrument acts as if it had only one degree of freedom. With too light a mirror, the frequency selectivity of the galvanometer is spoilt. The wires can then vibrate in some complex manner, and the spot of light, instead of moving in a straight line across the scale,

* A. E. Kennelly and G. W. Pierce, *Proc. Amer. Acad.*, Vol. 48, p. 113 (1912); A. E. Kennelly and H. O. Taylor, *Proc. Amer. Phil. Soc.*, Vol. 55, p. 415 (1916).

† Zölllich, *loc. cit.*; also see S. Butterworth, "On a null method of testing vibration galvanometers," *Proc. Phys. Soc.*, Vol. 26, pp. 264-273 (1914).

‡ S. Butterworth, *Proc. Phys. Soc.*, Vol. 24, pp. 75-94 (1912).

will describe closed curves resembling Lissajou's figures. Similar effects will occur if the two sides of the loop do not come into resonance together, due to inequality of tension or to unsymmetrical mounting of the mirror upon them.

It has recently been shown* that multiple resonance is not confined exclusively to instruments of the Duddell pattern. A vibration galvanometer of the moving magnet or moving coil type may exhibit more than one point of resonance, since the moving system may possess more than one degree of freedom arising from the imperfect lateral rigidity of the suspension and from the fact that the axis of suspension does not pass through the centre of mass of the moving system. Rosa and Grover† some years ago showed a resonance curve for an instrument of the Rubens type possessing two degrees of freedom, the curve having two well-defined resonance peaks separated by a region of low sensitiveness. Prof. Ll. Jones in the paper cited has satisfactorily worked out the theory of such double resonance and has shown how to deduce the mechanical constants from the resonance curve. The theory is analogous to that of coupled electric circuits.

41. Measurement of the Sensitivity of a Vibration Galvanometer. The vibration galvanometer possesses several characteristic properties which are used to compare its sensitiveness with other detecting instruments. Moreover, these quantities can be used to determine the value of the intrinsic constants in the equation of motion for use in design.

(i) *Alternating Current Sensitivity.* Referring to Fig. 44, let the galvanometer be connected in series with a high resistance, so that the back e.m.f. in the instrument can be neglected, and tapped across a known low resistance carrying alternating current which is accurately measured by a dynamometer. The instrument is tuned to be in resonance with the supply, which should be taken from a source of constant frequency. From the value of the current and the resistances, the current through the instrument is calculated; the total width of the resulting band of light (proportional to twice the amplitude) on the scale is observed; then a.c. sensitivity = width of band in mm. at 1 metre scale distance per microampère.

(ii) *Alternating Voltage Sensitivity.* The high resistance is

* R. Ll. Jones, "Vibration galvanometers with asymmetric moving systems," *Proc. Phys. Soc.*, Vol. 35, pp. 67-80 (1923).

† See *Bull. Bur. Stds.*, Vol. 1, p. 298 (1905).

cut out, the current from the source being much reduced. The p.d. across the instrument is then known, and the a.v. sensitivity = width of band in mm. at 1 metre scale distance per microvolt.

(iii) *Direct Current Sensitivity.* Reverting to the connections in the figure, let the alternator be replaced by a battery, and let the direct current through the instrument be calculated, the single deflection being observed. Then, in the usual way, d.c. sensitivity = single deflection in mm. at 1 metre per microampère. The ratio a.c. sensitivity/d.c. sensitivity is called the *resonance magnification* and should be as large as possible.

(iv) *Resonance Frequency.* The frequency of the source to which the instrument is tuned is measured by some suitable means.

(v) *Resistance.* The galvanometer resistance should be obtained in a Wheatstone bridge.

For the manner of deducing the intrinsic constants from these five quantities, and for typical values, see *Dictionary of Applied Physics*, Vol. 2, pp. 972-974.

CHAPTER IV

THE CLASSIFICATION OF BRIDGE NETWORKS

1. Introduction. The reader approaching the subject of alternating current bridge measurements for the first time will be impressed by the very large number of networks which have been proposed by various writers for use in practice; and he may find some difficulty in choosing from the profusion of methods at his disposal, the one best suited to the measurement of a given quantity. It is the object of this chapter to present a concise description and classification of bridge networks, together with a summary of the literature concerning them.

Since a given network may be of service for the measurement of more than one quantity, it is clear that confusion may arise by attempting to classify bridge methods according to the quantities which they are designed to measure. It is simpler, for the primary classification developed in this chapter, to consider the networks according to the arrangements of resistance, inductance, and capacitance which compose the balancing branches of the networks, and which enter into the balance conditions. It is proposed, therefore, to examine the construction of each bridge network and to classify it in the manner just suggested. The balance conditions can then be worked out, the vector diagram drawn, and the conditions for sensitivity deduced. The purposes to which each network can be put will be referred to, and, so far as possible, experimental results of the use of each bridge will be given.

The bridge networks having been classified in this chapter with respect to the way in which they are built up, it will be easy to show in Chapter V how to choose the network most suitable for a given measurement, and to point out the practical precautions which it is necessary to observe in order that reliable results may be secured.

In Chapter I the reader has been shown that modern alternating current bridge methods for the measurement of induction coefficients are the direct result of applying an interrupted current and a telephone to the old ballistic bridges. It will be of service in the present classification to make a few observations upon these old methods, from which so many of the present-day methods are derived.

In the use of a ballistic method a suitable network is arranged and

adjusted, so that the galvanometer in the bridge conductor remains undeflected when steady currents flow in the branches, i.e. the ordinary Wheatstone bridge relation among the branch resistances is first obtained. Then, without interfering with this condition, such adjustments are made in the constants of the branches that the galvanometer remains undisturbed when the source of current is applied to or removed from the network, i.e. the second condition for balance is that no ballistic effect is exerted on the galvanometer.

Now a ballistic galvanometer can remain undeflected under either of two conditions: (i) when the *aggregate quantity* of electricity passed through the instrument during the continuance of the transient condition in the network is zero; or (ii) when the *current* in the galvanometer is zero at every instant. Obviously, if condition (ii) be satisfied, the first condition is automatically fulfilled; the converse is not necessarily true. Assuming steady current balance to be first secured, balance for the transient state can be obtained by imposing in addition one or other of these conditions (i) and (ii). Ballistic bridges are thereby divided into three definite classes* which have *aggregate balance*, *continuous balance*, or *conditional continuous balance*; according as (i), (ii), or (ii) with further conditions is satisfied.

For example, Rimington's method (*see* p. 214) for comparison of a condenser with a self-inductance essentially has aggregate balance under condition (i), and cannot be balanced if (ii) be imposed. On the other hand, Maxwell's method (p. 180) of comparing two self-inductances can be balanced by means of condition (ii) and has continuous balance. Again, such a bridge as Carey Foster's method (p. 258) for the comparison of a condenser and a mutual inductance is primarily balanced under condition (i), but, by imposing a further condition, balance can be obtained under (ii), giving conditional continuous balance.

Now if such bridge networks be supplied with alternating current, the detecting instrument is essentially some form of current-measuring or detecting device, such as a telephone or a vibration galvanometer; hence, as has been shown in Chapter II, the condition for balance is that there shall be no current in the detector at any instant. It follows, therefore, that any ballistic bridge which has continuous balance can be used without modification with alternating current, provided that the battery be replaced by an alternator and that a telephone or vibration galvanometer be substituted for the ballistic galvanometer. By this means a large and important class of alternating current bridges is obtained.

It will be obvious that a ballistic bridge which has aggregate balance cannot be balanced by the use of an alternating current. If, however, the steady current or Wheatstone bridge condition be abandoned, it becomes possible to use such bridges with alternating current. Balance is usually obtainable only at one frequency, and the use of such networks is limited, owing to their practical inconvenience.

Ballistic bridges of the class which may be made to have conditional continuous balance can be adapted to alternating current at once by adopting the slight modification necessary to make the balance continuous.

* In this connection *see* a paper by A. O. Allen, "On measurements of inductance," *Phil. Mag.*, 6th series, Vol. 25, pp. 520-534 (1913).

In addition to these alternating current adaptations of old ballistic methods, there is a large class of methods specially developed as the result of alternating current practice. These methods are either modifications of the old ballistic bridges devised to simplify the practical procedure of securing balance; or they are entirely new methods developed in the course of research work for the purpose of some special measurement. One of the two balance conditions is not necessarily the ordinary Wheatstone bridge resistance relation, and a knowledge of the frequency of the alternating current may enter explicitly into the balance conditions.

As will be shown later in this chapter, the two conditions of balance deduced on page 44 for an alternating current bridge may be functions of the frequency of the current applied to the network. When these conditions are independent of frequency, either a telephone or a tuned detector may be employed, since the bridge is balanced for currents of all frequencies simultaneously. It is, moreover, not important that the wave-form of the applied potential difference be sinusoidal, since the adjustment balancing the fundamental of the wave balances simultaneously the harmonics. On the other hand, when the balance conditions involve the frequency of supply, this frequency must be independently measured, and the knowledge of it becomes, as it were, a third condition of quantitative measurement. Moreover, if in such a case the applied wave of potential difference be impure, the use of a telephone will be inadmissible, since, although the condition of balance appropriate to the fundamental of the wave be satisfied, balance for the harmonics is not simultaneously secured, and silence in the telephone will be impossible. In such cases the use of a tuned detector, the response of which is small to harmonics of the frequency to which it is tuned, is essential. This is usually a simpler expedient than that of purifying the wave-form in order that a telephone may be used.

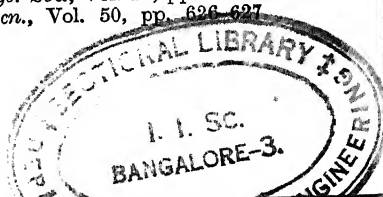
The first classified collection of a.c. bridge networks was published by Max Wien,* in 1891, a number of old ballistic bridges being adapted for use with alternating current and certain new networks introduced. H. Rowland,† in 1898, collected together the circuit diagrams and balance conditions of some 27 bridges, and added others in later papers. In 1908 A. Campbell‡ published a collection of bridges in which mutual

* Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 689-702 (1891). For a collection of the old ballistic methods, see W. E. Sumpner, *Journal S.T.E.*, Vol. 16, pp. 344-379 (1888).

† H. Rowland, "Electrical measurements by alternating currents," *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429-448 (1897); also *Phil. Mag.*, 5th series, Vol. 45, pp. 66-85 (1898).

H. Rowland and T. D. Penniman, "Electrical Measurements," *Amer. J. Sc.*, 4th series, Vol. 8, pp. 35-57 (1899).

‡ A. Campbell, "On the use of variable mutual inductances," *Phil. Mag.*, 6th series, Vol. 15, pp. 155-171 (1908); *Proc. Phys. Soc.*, Vol. 21, pp. 69-87 (1910). Also, "Inductance measurements," *Electn.*, Vol. 50, pp. 626-627 (1908).



inductance occurs, the theory of these having been given by Heaviside many years previously. Two important summaries of bridge methods have recently been published by C. E. Hay,* in 1912, and D.I. Cone, in 1920.† The latter paper gives very useful and concise diagrams for a number of bridges used in modern practice.

In the succeeding sections of this chapter, a large number of bridge networks are passed in review and classified according to the way in which they are constructed. It is found that the networks fall under the following headings—

- (i) Containing Resistance and Self-inductance.
- (ii) Containing Resistance and Capacitance.
- (iii) Containing Resistance, Self-inductance, and Capacitance.
- (iv) Containing Resistance, Self-inductance, and Mutual Inductance.
- (v) Containing Resistance, Self-inductance, Mutual Inductance, and Capacitance.

NETWORKS CONTAINING RESISTANCE AND SELF-INDUCTANCE

2. Maxwell's Method. In the second volume of his *Treatise on Electricity and Magnetism*,‡ Maxwell describes a simple method for comparing the self-inductances of two coils by the use of a ballistic galvanometer and a battery. Max Wien,§ in 1891, appreciating that the method possesses continuous balance, adapted it to alternating current.

In Fig. 48 (a), let L_1 and L_2 be the two self-inductances, the branches of the network in which they are connected having resistances P and R respectively. A resistance r is arranged so that it may be included at will in P or in R . The remaining branches are composed of resistances Q and S , frequently coils in a ratio box. The branch impedance operators are

$$z_1 = P + j\omega L_1, z_2 = Q, z_3 = S, z_4 = R + j\omega L_2,$$

* C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal P.O.E.E.*, Vol. 5, pp. 451-454 (1913); also *Professional Papers*, No. 53.

† D. I. Cone, "Bridge methods for alternating current measurements," *Journal Amer. I.E.E.*, Vol. 39, pp. 640-647 (1920).

‡ 1st Edn., p. 357 (1873). See also M. Brillouin, "Comparison des coefficients d'induction," *Ann. de l'Ecole normale*, tome 11, pp. 339-424 (1882).

§ Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 689-712 (1891).

where P or R includes r according to the needs of the experiment. Then, for a four-branch network, the balance condition is (p. 43),

$$z_1 z_3 = z_2 z_4,$$

or
$$S(P + j\omega L_1) = Q(R + j\omega L_2);$$

whence, by separating the two components,

$$\frac{L_1}{L_2} = \frac{P}{R} = \frac{Q}{S}$$

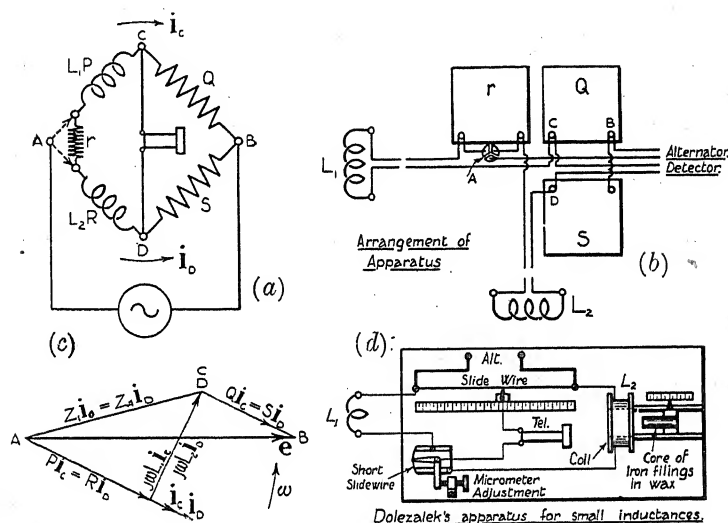


FIG. 48.—MAXWELL'S METHOD FOR COMPARING TWO SELF-INDUCTANCES

are the two conditions which must be satisfied if no current flows in the detector. Since the conditions do not involve ω , the bridge can be balanced even when the wave form of applied potential difference is impure, and a telephone can be satisfactorily employed to indicate balance.

In practical working, the following procedure will be found convenient. Adjustment of the bridge will be most easily attained if the standard inductance, L_2 , be of the type in which the inductance can be continuously varied without alternation of resistance.* As a preliminary, the resistance of the unknown

* For example, the Ayrton-Perry Inductance or other forms shown on pp. 94-98.

inductance L_1 should be found by means of a Wheatstone bridge, the value obtained acting as a guide in deciding where to connect the resistance r . A suitable arrangement of the apparatus is shown diagrammatically in Fig. 48 (b). The inductances L_1 and L_2 are arranged at some distance from one another, so that the mutual inductance between them may be negligible; they are connected to the remainder of the network by bifilar leads, so that errors due to inductance of the leads may be a minimum. The branches Q and S are shown as separate resistance boxes, but it is often convenient to combine them in some form of ratio box. The connections from the source of current and to the detector are preferably of twisted wire; in general, care should be taken to avoid loops of any considerable area in the connections used in the bridge by arranging that the leads are grouped in pairs. By such means, stray inductance errors are minimized as far as possible. A three-point plug serves to connect the alternator terminal A to one or other end of the resistance r ; this resistance may be composed of a dial or plug box in combination with a fine-adjustment rheostat* or slide wire for the purpose of securing accurate balance.

Assuming the value of L_1 to be entirely unknown, balance may be found by a process of methodical trial and error. Start by making $Q/S = 1$, setting the resistance r in AC or in AD , so that P/R is about 1 also. Then, by successive adjustment of L_2 and r , endeavour to obtain balance. Clearly, if $L_1 =$ or $< L_2$, balance may be at once secured, but if it be not within the range of the standard inductance, a second trial is necessary. Observe at which end of the range of L_2 the indication of the detector is least; then alter the ratio Q/S to bring the balance point within the values obtainable in L_2 . Re-adjust r to make P/R about equal to the new ratio, and then obtain balance by alteration of r and L_2 in successive steps.

Should it be found that the alteration of L_2 over its whole range makes no appreciable difference to the indication of the detector when $Q = S$, a considerable alteration in the value of Q/S is requisite, say to 10 or to $1/10$. An attempt to balance should then be made by the procedure just described. It is always best, wherever the available standards render it possible, to work with $Q = S$, and to extend the range of a

* Of constant or calculable inductance, pp. 83-85.

small variable inductance by means of additional fixed inductance standards connected in series with it. Such extra coils should obviously be put at some distance from the variable inductance and from the coil under test, so that there shall be no errors due to mutual actions between them. By using $Q = S$, any residual errors and slight inequalities which they may possess can be eliminated by re-balancing the bridge when Q and S are interchanged and averaging the two results.*

If only fixed inductance standards are available, balance must be secured by successive adjustment of Q/S and r . After a preliminary trial has indicated the order of magnitude of L_1 , a standard L_2 of about equal magnitude should be chosen. The resistance boxes Q and S are then preferably connected by a slide wire, the point B being the sliding contact thereon; the final precise adjustment of Q/S can then be easily obtained.

It is worth while to notice at this stage two artifices which may be of service in adapting the bridge to work with available standards. Properly, these artifices constitute separate methods and will be considered in detail in their proper place. Suppose that L_1 is much larger than any available standard L_2 . Connect in the branch AC a condenser of capacitance C ; then the apparent inductance of the branch is $L_1 - \frac{1}{\omega^2 C}$, which by suitable choice of C may be brought within the range of the standard L_2 . It should be noticed that the frequency must be constant and known. Again, suppose that L_1 is much smaller than the standard L_2 . Connect a resistance R_2 in parallel with the branch AD . Then, referring to the table of operators given in Fig. 10 and to page 91, the apparent inductance of the branch is

$$\frac{L_2 R_2^2}{(R + R_2)^2 + \omega^2 L_2^2}$$

which can be made as small as is desired by suitable choice of R_2 . This in effect is the Wien-Dolezalek method described on p. 191. In the use of both these artifices such magnitudes of C or of R_2 should be chosen as will convert the bridge into one with equal ratios, in order that residual errors may be reduced to a minimum.

Experimental Examples. The following typical examples will serve to illustrate the balancing procedure described above—

(i) The various branches were made up as follows. AC , a coil L_1 of nominally 40 millihenrys inductance and 5 ohms resistance in series with a decade resistance box and constant inductance rheostat (r , in Fig. 48(a)). AD , an Ayrton-Perry variable self-inductance L_2 of 43 millihenrys maximum value, and $R = 10.69$ ohms. Q and S were equal coils in a ratio box, and were set at 10 ohms each. The source was a valve oscillator working at a frequency of 459.3; the detector a tuned

* See Chap. V for further precautions in precise work.

Duddell vibration galvanometer. Balance was secured by successive alterations of r and L_2 , the balancing values being 5.50 ohms and 40.72₀ millihenrys respectively. Hence $L_1 = 40.72_0$ millihenrys and the effective resistance of L_1 is $10.69 - 5.50 = 5.19$ ohms.

(ii) In a second test made on a coil of about 0.6 henry, Q was set at 1,500 ohms and S at 100 ohms, these now being coils in separate decade boxes. r was included in the branch with the unknown coil. The settings of r and L_2 at balance were 65.15 ohms and 40.40₀ millihenrys respectively, so that $L_1 = 15 \times 0.04040_0 = 0.6060_0$ henry, and its effective resistance is $15 \times 10.69 - 65.15 = 95.20$ ohms. The frequency in this test was 407.1.

3. The Vector Diagram for Maxwell's Method. The vector diagram for the balanced bridge is drawn in Fig. 48 (c). Since the points C and D are always at the same potential, and the branches CB and DB are pure resistances, the currents passing the branch points C and D must be in phase. Then if AB be the vector of potential difference, e , applied between the terminals A, B , the remaining vectors are easily constructed as shown. Relative to A , the common potentials of C and D are represented by coincident points C, D , i.e. $z_1 i_c = z_4 i_d$. To these must be added $Q i_c = S i_d$ to give the vector e . From the geometry of the figure it is easily seen that $P i_c = R i_d$, $L_1 i_c = L_2 i_d$, and $Q i_c = S i_d$; hence $L_1/L_2 = P/R = Q/S$.

4. Sources of Error in Maxwell's Method. The method described above in its simplest form is, in practice, subject to various sources of error, which will now be examined.

Eddy Current Effects. It is to be noted that although the balance conditions given in the above do not involve the frequency, it does not follow that balance obtained at one frequency will be retained at any other. This is due to the fact that the quantities involved are effective inductances and resistances, the values of which are different with alternating current from the values with direct current, owing to the effects of eddy currents in the resistances and the standard coils. With properly constructed resistance boxes and well stranded inductances, the effect of eddy currents can be reduced to a very small amount, the alternating and direct current values being practically identical. Thus, if three branches be made up of standard apparatus, as free from eddy current effects as possible, the bridge serves to find the inductance and the effective resistance of a coil at a given frequency. It has been applied by Dolezalek* in this connection to measure the additional resistance in alternating current apparatus due to the effect of eddy current losses; this investigator has also described self-contained bridges capable of measuring an inductance of 10^{-7} henry with an accuracy of 1 or 2 per cent (Fig. 48 (d)).

* F. Dolezalek, "Messeinrichtung zur Bestimmung der Induktionskonstanten und des Energieverlustes von Wechselstromapparaten," *Zeits. f. Inst.*, 23 Jahrgang, pp. 240-248 (1903).

Mutual Inductance. In arranging the apparatus for the simple bridge just described, the two coils have been placed so that the mutual inductance between them is very small. This can be secured by putting them at some distance apart, with their planes perpendicular. However, in precise work, the mutual inductance effect, though small, may not be negligible, and it becomes necessary to examine the way in which it modifies the balance conditions.

Let M be the mutual inductance between the coils in branches 1 and 4. Then, in the expression given on page 52, put all the mutual operators zero except $m_{14} = j\omega M$, giving $\alpha = 0$, $\beta = 0$, $\gamma = j\omega M$, $\delta = 2j\omega M$. Then

$$(SP - QR) + j\omega\{S(L_1 + M) - Q(L_2 + M)\} = 0;$$

whence balance occurs if

$$\frac{L_1 \pm M}{L_2 \pm M} = \frac{P}{R} = \frac{Q}{S},$$

remembering that M may be positive or negative.

If the coils to be compared are equal, i.e. if $L_1 = L_2$, then the above equation is independent of M . Then $P = R$ and $Q = S$. Thus, *in an equal ratio bridge balance is unaffected by the mutual inductance between the coils*, so that it is not necessary in this case to go to any trouble to reduce mutual action to a minimum.

Residual Errors in the Ratio Branches. It has been assumed that the resistances Q and S are perfect, i.e. they contain no residual inductance or capacity. Now, although this is very nearly true in the case of resistances specially constructed for alternating current work, the small residuals may have an important effect, especially in bridges where the ratio is such that Q and S are very unequal. If these coils are of low value the residual inductance may preponderate; if of high value, the residual self capacity may be important. In any case, the effect can be represented by writing $Q + j\omega\lambda$ for Q and $S + j\omega\mu$ for S where λ and μ are the residuals (*see* p. 59).

The balance condition for the bridge is then

$$(P + j\omega L_1)(S + j\omega\mu) = (R + j\omega L_2)(Q + j\omega\lambda),$$

which, on separation of the two components, gives

$$\frac{L_1}{L_2} = \frac{Q}{S} - \left(\frac{P\mu - R\lambda}{L_2 S} \right) = \frac{Q}{S} - k_1$$

$$\text{and} \quad \frac{P}{R} = \frac{Q}{S} + \omega^2 \left(\frac{L_1\mu - L_2\lambda}{RS} \right) = \frac{Q}{S} + k_2$$

as found by Giebe.*

To show the importance even of small residuals, consider a bridge† in which $L_1 = 0.1$ henry, $P = 20$ ohms, $L_2 = 0.01$ henry, $R = 2$ ohms, $S = 5$ ohms. Then if Q and S are free from residual effects, Q must be 50 ohms for balance. Now let $\lambda = 0$ and $\mu = 0.25$ microhenry, then the second equation gives $Q = 45.57$ ohms when the frequency is

* E. Giebe, "Messung induktiver Widerstände mit hochfrequenten Wechselströmen. Methode zur Messung kleiner Selbstinduktionskoeffizienten," *Ann. der Phys.*, Bd. 24, pp. 941-959 (1907).

† From an example given by E. Orlich, *Kapazität und Induktivität*, p. 236.

3,000 cycles per second. Hence, the value of Q requisite for balance is reduced by 4.43 ohms or 8.86 per cent. If the residual inductance had been neglected and L_1/L_2 calculated from the observed Q/S , the result would have been in error by 8.86 per cent, which is a considerable amount.

Error due to these residual effects can be avoided in three ways: (a) By making the bridge symmetrical, so that $L_1 = L_2$, $Q = S$, and $P = R$. Any slight difference between Q and S is eliminated in the mean of the values obtained for balance when their positions are interchanged. Or by means of a small auxiliary inductance in Q or S , make $Q/S = \lambda/\mu$, so that the residuals of these branches are proportional to their resistances. Then $L_1/L_2 = Q/S = P/R$ as if residuals were not present. With an equal ratio bridge, let balance be secured in the ordinary way. Then alter the small auxiliary inductance and the other adjustments of the network until, on reversing the positions of Q and S , balance is not disturbed. (b) By a substitution method. The bridge is balanced with the unknown coil in position, and then re-balanced when the coil is replaced by one of the same resistance and calculable inductance. The adjustable rheostats in the branches should be of constant inductance, and the substituted coil should be as nearly equal to the unknown coil as possible. This procedure (since the balanced bridge is only very little disturbed from its initial state) entirely eliminates residual errors, and determines with high precision the difference between the unknown and the standard. Grover and Curtis* have employed it to measure the inductance of low value resistance coils. (c) By the use of a specially constructed bifilar bridge in which the residuals are calculable.

This third method has been developed by Giebe† for the precise comparison of two inductances on a bridge of large ratio. His arrangement is shown diagrammatically in Fig. 49 (a) for a bridge in which the ratio is 10, and forms an excellent illustration of the care which must be exercised when precision is desired. The two coils to be compared were wound with carefully stranded wire, their inductances and resistances being $L_2 = 10^7$ cm., $R = 2.84$ ohm, $L_1 = 10^8$ cm., $P_1 = 18.77$ ohm. They are connected to the rest of the bridge by bifilar leads about 1 metre long, so that mutual inductance between the coils and the bridge may be neglected.

The four branch points A , B , C , D are arranged to be as close together as possible, the telephone or galvanometer being connected to CD and the alternator to AB , each by a pair of bifilar leads. The points A , C , D are represented by pins fixed in a piece of ebonite, soldered connection being made at them to the other branches of the network. The point B is represented by a knife-edge contact moving on a manganin wire about 2 cm. long and 0.36 mm. diameter for fine adjustment of the ratios.‡

The ratio branches Q and S are resistances designed so that their

* F. W. Grover and H. L. Curtis, "The measurement of the inductances of resistance coils," *Bull. Bur. Stds.*, Vol. 8, p. 463 (1913).

† E. Giebe, *loc. cit.*; also "Präzisionsmessungen an Selbstinduktionsnormalen," *Zeits. f. Inst.*, 31 Jahrgang, pp. 6-20, 33-52 (1911).

‡ For detailed drawings of a bifilar bridge, see W. Hüter, "Kapazitätsmessungen an Spulen," *Ann. der Phys.*, Bd. 39, pp. 1350-1380 (1912).

residuals may be readily calculated.* Each is composed of a pair of parallel, bare manganin wires stretched out on a suitable board, and bridged at the ends remote from the branch points by the short-circuiting pieces K_2, K_3 . If l be the length of the wires, a the distance between their axes, r their radius, the self-inductance of such a parallel wire resistance is $L = \left[4 \log \left(\frac{a}{r} \right) + 1 \right] l$ cm. The dimensions and approximate constants of these resistances are

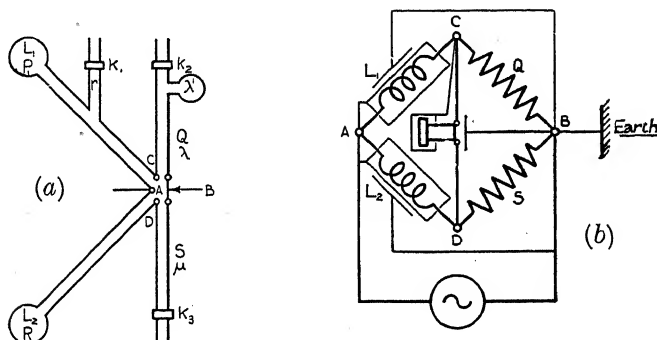


FIG. 49.—GIEBE'S BIFILAR BRIDGE AND ELECTROSTATIC SCREENING ARRANGEMENTS

	Q	S
r mm.	0.025	0.075
a mm.	1.3	1.3
l cm.	102.4	87.1
Resistance	390 ohms	39 ohms
Inductance	1,720 cm. (λ_1)	1,080 cm. (μ)

Now, referring to the balance conditions, if Q and S are such that $Q/S = \lambda/\mu$, then the equations reduce to $L_1/L_2 = Q/S = P/R$ just as if there were no residuals present. Hence, to ensure that this condition is approximately fulfilled, a coil of inductance $\lambda' = 9,870$ cm. is connected in series with S . Thus, $\lambda = \lambda_1 + \lambda' = 1,720 + 9,870 = 11,590$ cm., so that $\mu/\lambda = 1,080/11,590 \approx 1/10$. The resistance of λ' is negligible, it being a copper coil. It should be noticed that the correction term k_2 can be kept small by keeping the radius of the wire of which S is composed as small as possible; as the radius is reduced, S increases at a greater rate than μ , so that k_2 is diminished.

A bifilar resistance r is included in series with L_1 for purpose of adjustment. Balance is secured by successive alterations of r and of the slider B . The movement of the latter allows of a variation of ratio of about 2 per cent; further adjustment can be obtained by movement of the bridges K_2, K_3 . The following table shows how

* See p. 80.

accurately, with such precautions, a bridge of ratio 10 can be balanced at various frequencies when the residuals are allowed for

$$k_1 = 0.000006$$

Frequency.	Q/S	r	P ₁	P	R	k ₂	Observed P/R	Calculated P/R = $\frac{Q}{S} + k_2$
50	10.000	9.6 ₃	18.77	28.4 ₀	2.84	0	10.00	10.00
2,040	10.026	9.6 ₃	18.77	28.4 ₈	2.84	-0.01 ₂	10.02	10.01 ₄
3,035	10.053	9.5 ₀	18.77	28.2 ₇	2.84	-0.02 ₆	9.97	10.02 ₇

It will be seen that the values of P/R directly observed and those calculated from the corrected value of Q/S are in very close agreement. The discrepancies are discussed by Giebe.

Self capacitance of L_1 . Self capacitance* in the coil under test, especially if it has a great number of turns, will introduce an error at higher audio frequencies. It has been shown on page 90 that the effect can be represented by a condenser C in parallel with the coil, the combined impedance operator being $(P_1 + j\omega L_1)/\{(1 - \omega^2 C L_1) + j\omega C P_1\}$, so that the effect of self capacitance is to increase the resistance of the coil to

$$P'_1 = (1 + 2\omega^2 L_1 C) P_1,$$

and its inductance to

$$L'_1 = (1 + \omega^2 L_1 C) L_1.$$

These values must replace P_1 and L_1 in all the preceding expressions.

The value of C can be found by measuring L'_1 at two frequencies, and its effect can be allowed for. For example, in Giebe's bifilar bridge described above, the value of L_1 is 10^8 cm., and C is 0.0001, microfarad. The values of P'_1 and L'_1 at two frequencies are

Frequency	2,040 cycles/second	3,035 cycles/second
P'_1	18.86 ohm	18.97 ohm
L'_1	1.0025×10^8 cm.	1.0055×10^8 cm.

from which an idea of the magnitude of the effect can be obtained, remembering that $P_1 = 18.77$ ohm at low frequency.

Earth Capacities. Considerable error may arise in measurements at high frequencies when the earth capacities are neglected, and without making some compensation for them, balance may be difficult or impossible to obtain. The general question of earth capacities will be discussed on page 285, but it will be well to mention at this stage the general nature of the effects involved.

Every branch of the network has electrostatic capacity with respect (a) to surrounding earthed objects, and (b) to every other branch of the network. To a first approximation the earth capacities (a) may be represented by condensers connected between the branches and earth; and in a similar way the intercapacities (b) may be considered as a set of condensers joining the several branches. These condensers constitute, as it were, branches additional to those forming the original

* See E. Giebe, *loc. cit.*; also Max Wien, "Messung der Induktionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, p. 711 (1891), and F. Dolezalek, "Ueber Präzisionsnormale der Selbstinduktion, *Ann. der Phys.*, Bd. 12, pp. 1142-1152 (1903).

network, and by their presence the balance conditions may be profoundly affected. Naturally, the effects will be more pronounced at high frequencies than at low, since the reactances of the capacity paths connecting the branches to one another and to earth become less as the frequency is raised.

In order to make the effect of earth and intercapacities as small and as definite as possible, it is necessary to screen the several branches from one another by enclosing them in metal boxes or shields, the potentials of the latter being maintained at definite values with respect to earth and to the branch points of the network.

Fig. 49 (b) shows diagrammatically the screening arrangements adopted by Giebe for the bifilar bridge. The inductive coils L_1 , L_2 are enclosed in metal boxes maintained at the potential of A . The bifilar ratio resistances Q , S are earthed at the branch-point B , screens connected to B rendering the capacitance of the boxes definite with respect to earth.

A further important capacitance effect when a telephone is used as a detector is the capacitance between the observer and the telephone. Though at balance the potentials of the points CD are identical, it does not follow that their common potential will be the same as that of the observer. A capacity current will flow from the branch points through the telephone to earth, *via* the observer, and silence can not be secured unless the capacitance effect be eliminated. Several devices for securing this result are described in the next chapter. Giebe's arrangement is to enclose the telephone in a metal case connected to the branch-point C , an earthed screen making the capacitance definite. The observer applies his ear to a tube of insulating material projecting through the walls of the metal case.

5. Giebe's Modification of Maxwell's Method for Small Inductances.

Giebe* has shown that Maxwell's method can, by means of a simple modification, be used for the measurement of small inductances which have a large resistance; as, for example, resistance coils. Briefly, the principle is to use as the ratio branches coils of high inductance and low resistance; it is then easy to show that residual errors in the bridge, when used to compare two small inductances of comparatively high resistance, are very small and can be allowed for experimentally. By this means Giebe was able to measure inductances of the order of 500 cm. with time constants as low as 10^{-8} second.

Referring to Figs. 48 and 49, the following procedure is adopted. L_1 and L_2 are known large inductances of relatively low resistances. In Giebe's arrangement, L_1 is nominally 0.1 henry and L_2 0.01 henry; self capacitance is important in the case of L_1 , altering its inductance to $L'_1 = (1 + \omega^2 L_1 C) L_1$ and its resistance to $P'_1 = (1 + 2\omega^2 L_1 C) P_1$ at frequency $\omega/2\pi$. S is a high resistance of which the small inductance, μ , is to be measured; Q is a parallel wire resistance of which the residual inductance may be calculated. Let a bifilar bridge be set up, as in Fig. 49, balance being secured by adjustment of the sliders K_1 , K_2 , and B . Then, from the balance equations on p. 185, balance is attained when

$$\frac{P'_1 + r}{R} = \frac{Q}{S} + \omega^2 \frac{(L'_1 \mu - L_2 \lambda)}{RS}.$$

* *Loc. cit*

Now re-balance the bridge with direct current by adjustment of r only, the balance value being r' ; then

$$(P_1 + r')/R = Q/S,$$

assuming Q , R , and S do not vary appreciably with frequency. Subtracting the two equations and solving for μ , gives

$$\mu = \frac{S[(P'_1 - P_1) + (r - r')]}{\omega^2 L'_1} + \frac{L_2}{L'_1} \lambda.$$

Substituting for P'_1 gives,

$$\mu = \frac{S[r - r' + 2\omega^2 L_1 C P_1]}{\omega^2 L'_1} + \frac{L_2}{L'_1} \lambda,$$

all the quantities on the right-hand side being known.

To show the degree of precision attainable, Giebe compares two parallel wire resistances of calculable inductances. The observed values for the two balances were as follows, $L_1 = 0.1$ henry, $L_2 = 0.01$ henry, $C = 0.00015$ microfarad, $L'_1 = 0.10017$ henry, $P_1 = 18.77$ ohm, $r = 11.95$ ohm, $r' = 9.59$ ohm, $\omega = 2\pi \times 2,016$, $S = 2.01$ ohms. The first term on the right-hand side is then equal to 303×10^{-9} henry. Hence,

$$\text{Measured value of } \mu - \frac{L_2}{L'_1} \lambda = 303 \text{ cm.}$$

The calculated values of μ and λ were 380 cm. and 744 cm. respectively; so that the calculated value of the above quantity is $380 - \frac{0.01}{0.10017} \times 744$ i.e., 306 cm; agreeing with the observed value to within 1 per cent.

6. Wien's Methods. Max Wien* has described a modification of Maxwell's method by means of which a self-inductance can be found in terms of resistance and frequency. Two arrangements are shown in Fig. 50. The first is that described by Wien in 1891, L_1 and L_2 being unknown inductances, P_2 , P_3 , and r rheostats, and Q , S the ratio branches joined by a slide wire. The second gives Wien's† bridge of 1896, as used by him for the measurement of standard inductances of 0.001, 0.01, and 0.1 henry respectively. This arrangement differs from the former in the omission of the rheostat P_3 , and in this form has been used by Dolezalek‡ to investigate the influence of stranding on the errors in standard inductances. A. Campbell,|| in 1905, has described the use

* Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 689-712 (1891).

† Max Wien, "Einheitsrollen der Selbstinduction," *Ann. der Phys.*, Bd. 58, pp. 553-563 (1896).

‡ F. Dolezalek, "Ueber Prazisionsnormale der Selbstinduktion," *Ann. der Phys.*, Bd. 12, pp. 1142-1152 (1903).

|| A. Campbell, *Proc. Phys. Soc.*, Vol. 19, pp. 171-172 (1905).

of the method to measure inductances of the order of 100 microhenrys.

In practical working, L_1 is the unknown inductance ; L_2 is an adjustable inductance which need not be known. Balance is attained by first fixing P_2 and L_2 , and then adjusting successively the ratio Q/S and the rheostats P_3 and r . In using the second method of connection, it may be necessary to include a rheostat in P_1 .

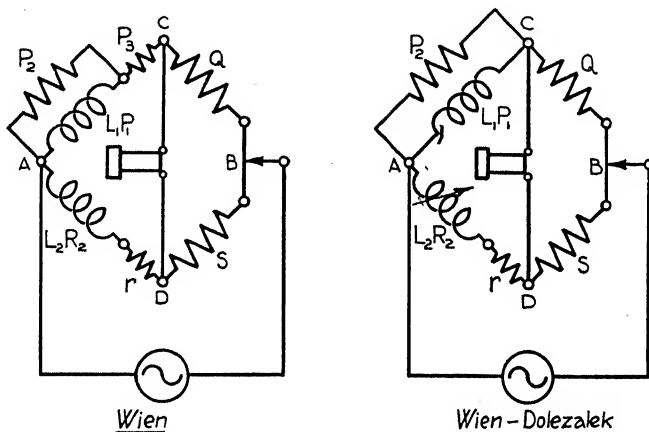


FIG. 50.—WIEN'S METHODS FOR THE DETERMINATION OF SELF-INDUCTANCE

Referring to the more general arrangement, put $R_2 + r = R$; then the impedance operators for the branches are

$$z_1 = P_3 + \frac{P_2(P_1 + j\omega L_1)}{P_1 + P_2 + j\omega L_1}, z_2 = Q, z_3 = S, z_4 = R + j\omega L_2.$$

Substituting in the balance equation

$$z_1 z_3 = z_2 z_4$$

and collecting terms

$$S\{P_3(P_1 + P_2) + P_1P_2\} + Q\{\omega^2 L_1 L_2 - R(P_1 + P_2)\} \\ + j\omega[L_1\{S(P_2 + P_3) - QR\} - L_2Q(P_1 + P_2)] = 0.$$

Now the total resistance of the branch AC is $P = P_3 + \frac{P_1 P_2}{P_1 + P_2}$; substituting in the above equation and putting the two components separately equal to zero, gives

$$\omega^2 L_1 L_2 = (P_1 + P_2) (QR - SP) / Q,$$

and

$$\frac{L_1}{L_2} = \frac{Q(P_1 + P_2)}{S(P_2 + P_3) - QR}$$

for the balance conditions, from which L_1 or L_2 can be calculated independently.

In the case of the more commonly used Wien-Dolezalek bridge, shown in the right-hand diagram, these conditions reduce to a simple form. Put $P_3 = 0$ in the above equations, then the balance conditions reduce to*

$$\left. \begin{aligned} \omega^2 L_1 L_2 &= R(P_1 + P_2) - P_1 P_2 \frac{S}{Q} \\ \text{and} \quad \frac{L_1}{L_2} &= \frac{Q(P_1 + P_2)}{SP_2 - QR} \end{aligned} \right\} \quad (a)$$

Solving these equations for the value of L_1 , gives

$$\omega^2 L_1^2 = (P_1 + P_2)^2 \frac{R - \frac{P_1 P_2 S}{(P_1 + P_2)Q}}{P_2 \frac{S}{Q} - R} \quad (b)$$

from which L_1 can be found in terms of resistance and frequency. Wien has shown how, in practice, the knowledge of all these separate resistances can be avoided by making two additional measurements with direct current. After balance has been secured with alternating current, apply direct current and a reflecting galvanometer to the bridge. Then the resistance r must be adjusted so that the branch AD falls in resistance by an amount

$$r' = R - \frac{P_1 P_2}{(P_1 + P_2)} \cdot \frac{S}{Q}.$$

Then remove the inductive coil L_1 ; the resistance r must then be adjusted so that the resistance of AD increases by

$$r'' = P_2 \frac{S}{Q} - R.$$

Putting these observed alterations of r in the above equation makes

$$L_1 = \frac{S}{Q} \cdot \frac{P_2^2}{\omega} \sqrt{\frac{r'}{r''}} \cdot \frac{1}{r' + r''} \quad (c)$$

It is now necessary to determine the best arrangement of the network. Referring to Equation (b), it will be seen that the numerator and denominator both involve the difference between terms. Accuracy

* The first of these conditions is incorrectly given in the paper by Dolezalek quoted above, the factor $P_1 + P_2$ being omitted.

can only be expected, therefore, if P_2 be chosen so that these differences have reasonably large values. Following the usual practice, put $Q/S = 1$ in Equation (a) and find L_2 , R in terms of L_1 , P_1 and P_2 ; the resulting expressions are

$$\left. \begin{aligned} R &= P_2 \frac{\omega^2 L_1^2 + P_1(P_1 + P_2)}{\omega^2 L_1^2 + (P_1 + P_2)^2} \\ \text{and } L_2 &= L_1 \frac{P_2^2}{\omega^2 L_1^2 + (P_1 + P_2)^2} \end{aligned} \right\} \dots \dots \dots (d)$$

Now put these values in the numerator of Equation (b); then if P_1 is small compared with ωL_1 , it is easy to show that the numerator is a maximum if

$$P_2 = \omega L_1.$$

In a similar manner, insertion of these values in the denominator will show that the latter increases with P_2 , attaining neither a maximum nor a minimum; it has, however, a sufficiently large value if $P_2 = \omega L_1$.

In practice, therefore, the following procedure may be conveniently adopted—

- (i) Measure approximately the values of L_1 and P_1 .
- (ii) Set up the bridge, making $Q = S$ and P_2 about equal to ωL_1 .
- (iii) Calculate from Equations (d) the values of R and L_2 , setting the rheostat r and the variable inductance in AD to these values.
- (iv) Attain balance by adjustments of the slider B and L_2 , noting their positions.
- (v) Measure the frequency, $f = \omega/2\pi$.
- (vi) Make the two measurements with direct current corresponding to Equation (c).

When the above procedure is adopted, accuracy of about 1 part in 1,000 is obtainable.

The bridge should be used only at relatively low frequencies, say 400 to 500 cycles per second, since, on account of the skin-effect at high frequencies, the two direct current balances will not bear any simple relation to the values which the resistances will have with alternating current. Moreover, owing to these eddy effects the bridge current will no longer be sinusoidal, and the use of the telephone becomes inadmissible. It then becomes necessary to use the vibration galvanometer, the sensitivity of which is greater at lower frequencies than at high (see p. 166).

Experimental Example. The following figures were obtained in a test at 459.3 cycles per second on a coil, L_1 , of about 40 millihenrys. Using the above procedure, P_2 was set at 100 ohms, which is approximately the value of ωL_1 . To enable the d.c. balances to be made, a resistance of 20 ohms was included in series with L_1 , thus making P_1

about 25 ohms. Using these values in Equations (d) makes $R \cong 51$ ohms and $L_2 \cong 15.6$ millihenrys; the latter was an Ayrton-Perry variable inductance of resistance $R_2 = 10.69$ ohms. The resistance r was a decade box in series with a constant inductance rheostat, and in the a.c. tests was set at 40.0 ohms to make R of about the required value. Q and S were decade resistances, the former being in series with a low reading rheostat. The a.c. balances were obtained by adjustments of Q and L_2 for various fixed values of S , using a Duddell vibration galvanometer and a valve oscillator. The d.c. balances were made by reading the changes in the setting of r , using a cell and reflecting moving coil galvanometer.

S ohms.	Q ohms.	P_2 ohms.	r' ohms.	r'' ohms.	L_1 mH.
30	34.03	100	32.86	37.55	40.59
60	68.10	100	32.86	37.55	40.56
90	102.10	100	32.86	37.55	40.58
				Average	40.58 mH.

NETWORKS CONTAINING RESISTANCE AND CAPACITANCE

7. De Sauty's Method. The simplest way to compare two condensers is to make use of the method introduced by Mr. De Sauty,* of the Eastern Telegraph Co., and first applied to alternating current measurement by Max Wien,† in 1891.

Referring to Fig. 51, C_1 is the condenser to be tested, C_2 a suitable standard condenser; Q and S are non-inductive resistances. The branch impedance operators are $z_1 = 1/j\omega C_1$, $z_2 = Q$, $z_3 = S$, $z_4 = 1/j\omega C_2$; so that balance occurs when

$$S/j\omega C_1 = Q/j\omega C_2,$$

or

$$C_1/C_2 = S/Q.$$

The usual two balance conditions thus coalesce and become

* Latimer Clark and Robert Sabine, *Electrical Tables and Formulae*, p. 62 (1871). For the ballistic bridge, see W. E. Sumpner, *loc. cit.* (1888).

† Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telefon,'" *Ann. der Phys.*, Bd. 44, p. 697 (1891). J. Hanauer, "Ueber die Abhängigkeit der Capacität eines Condensators von der Frequenz der benutzen Wechselströme," *Ann. der Phys.*, Bd. 65, pp. 789-814 (1898). The following investigations were undertaken with the aid of an electro-dynamometer as detector, W. Stroud and J. H. Oates. "On the application of alternating current to the calibration of capacity boxes, and to the comparison of capacities and inductances," *Phil. Mag.*, 6th series, Vol. 6, pp. 707-720 (1903); E. M. Terry, "On the variation of a capacity with temperature," *Phys. Rev.*, Vol. 21, pp. 193-197 (1905).

one. In practice, Q is fixed and S is adjusted until the detector indicates zero.

The vector diagram for the balanced bridge is easily drawn, and is practically self-explanatory.

It is interesting to note the effect of residual inductance in the resistances Q and S upon the accuracy of the measurements. If λ

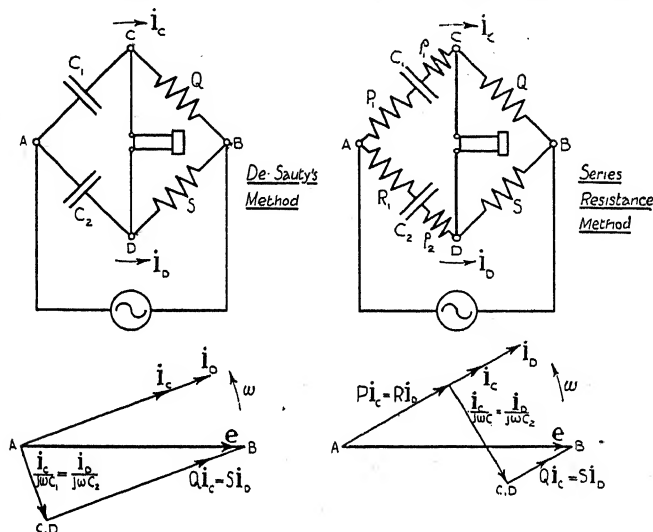


FIG. 51.—DE SAUTY'S METHOD AND THE SERIES RESISTANCE METHOD FOR COMPARING TWO CONDENSERS

and μ be the residuals, the operators for Q and S become $Q + j\omega\lambda$ and $S + j\omega\mu$ respectively. The conditions for balance are then

$$C_1/C_2 = S/Q = \mu/\lambda.$$

Hence not only the resistances but the residuals also must be in the ratio C_1/C_2 . It may be necessary in this case to include in series with S or Q a small variable self-inductance in order that the second condition may be fulfilled. The bridge then becomes a variant of Grover's series inductance method discussed on page 231.

It is pointed out on page 54 that De Sauty's bridge has the greatest sensitivity when $C_1 = C_2$ and, therefore, $Q = S$. With a vibration galvanometer as detector, the frequency should be low for great sensitiveness; hence, if C_1 and C_2 are small, the impedances of the branches in which they lie will be very large. Hence, when condensers are tested at low frequencies (say, less than 0.1 microfarad at 100 cycles per

second or lower), the ratio branches should have a high resistance, and both source and detector should be of high impedance. (It is often convenient in such a case to connect the source and detector to the bridge through step-up transformers.) This rule is in agreement with Glazebrook's* analysis of the bridge when used ballistically.

Experimental Example. In a test at 1,500 cycles per second, C_1 and C_2 were variable air condensers whose maximum values were to be compared. Q was fixed at 10,000 ohms, S being adjusted for balance. To allow for the effects of earth capacitance, each condenser was provided with a screen, that of C_1 being joined to the point C and that of C_2 to D . In addition the Wagner earthing device (see Fig. 73) was applied, z_A being a variable air condenser (also screened) and z_B a variable resistance. Successive adjustment of the main bridge (by varying S) and of the auxiliary bridge (by varying z_A and z_B) gave balance. The detector was a high resistance telephone. Then with $C_2 = 1.240 \text{ m}\mu\text{F}$, S was found to be 7,770 ohms, so that the apparent value of C_1 is $0.964 \text{ m}\mu\text{F}$. Removing the lead joining C_1 to the branch point A , balance was again obtained by setting $C_2 = 0.160 \text{ m}\mu\text{F}$, S and the auxiliary bridge being successively adjusted as before, giving $S = 2,000$ ohms. Hence the capacitance of the leads connected to C_1 is $0.032 \text{ m}\mu\text{F}$, so that $C_1 = 0.932 \text{ m}\mu\text{F}$.

8. Series Resistance Method. Although De Sauty's method is apparently so simple, it will be found difficult in practice to secure a sharp balance, except when air condensers are being tested. If the condensers have solid dielectrics there will inevitably be absorption effects therein, and the condensers can no longer be considered perfect. What is done in ordinary rough tests is to adjust Q or S until the indication of the detector is a minimum, and when the two condensers are of similar quality a reasonably good balance can be obtained. If the absorption be very different in the two condensers, the minimum is by no means small and is very badly defined; hence accurate settings cannot be obtained.

The phase displacement between the voltage applied to an imperfect condenser and the current through it is less than $\pi/2$ by an angle θ , called the phase difference of the condenser. Then, as shown on page 116, the imperfect condenser can be replaced by a perfect condenser C , in series with a resistance ρ to account for the losses and to give the same phase-difference, $\theta = \tan^{-1} \omega C \rho$.

* R. T. Glazebrook, "On a method of comparing the electrical capacities of two condensers," *Proc. Phys. Soc.*, Vol. 4, pp. 207-214 (1881). Also see F. E. Kester, "The bridge method for comparison of condensers," *Phys. Rev.*, Vol. 24, pp. 120-121 (1907).

The unsuitability of the above-mentioned minimum balance in accurate work is best realized by the examination of a numerical example.* Suppose C_1 is a paper condenser of 1 microfarad having $\theta_1 = 30'$ corresponding to $\rho_1 = 14.545$ ohms. A mica condenser $C_2 = 1.05$ microfarad having $\theta_2 = 1'$ and $\rho_2 = 0.462$ ohm is used as a standard. S is fixed at 1,000 ohms. A potential difference of 100 volts is applied to the points AB , the frequency being $600/2\pi$. The detector has a resistance of 200 ohms. If the condensers had been perfect, sharp balance and zero current in the detector would have been secured when $Q = 1,050$ ohms. Making use of these figures in the equations given on page 42, the current in the detector is found to have the following values as this ideal value of Q is approached, passed through and exceeded.

Q ohms.	Detector current in microampères
1045.0	20.184
1049.0	14.506
1049.75	14.336
1049.9	14.325
1050.0	14.323
1050.1	14.324
1050.25	14.333
1051.0	14.502
1055.0	18.870

From this table it is seen that the current in the detector is never zero, but the value of Q corresponding to minimum current is the same as if the condensers were perfect. However, the minimum is so flat that, with a telephone as detector (sensitive, say, to 1 microampère), settings could hardly be made closer than to the nearest ohm, i.e. to about 1 part in 1,000. With a vibration galvanometer the state of affairs is little better, a precision of about 5 in 10,000 being possible.

If, however, the phase differences of the condenser branches can be adjusted while, at the same time, the ratio of Q to S is varied, the points C and D of De Sauty's bridge can be brought to the same potential and phase. This can be secured in a variety of ways, one of the simplest being to join in series with the condenser which has the lesser absorption an adjustable resistance of known value. The bridge is then referred to as a *Series Resistance Method* (Fig. 51).

Since it may not be known beforehand which of the two condensers has the smaller absorption, it is usual, in practice, to join a resistance in series with each condenser. Thus, C_1 , ρ_1 , and C_2 , ρ_2 , are the two imperfect condensers to be compared, adjustable resistances P_1 and R_1 being connected in series with them as shown. If Q and S be non-inductive

* F. W. Grover, "Simultaneous measurement of capacity and power factor of condensers," *Bull. Bur. Stds.*, Vol. 3, pp. 375-376 (1907).

resistances, $P = P_1 + \rho_1$ and $R = R_1 + \rho_2$, balance will be secured when

$$S \left(P + \frac{1}{j\omega C_1} \right) = Q \left(R + \frac{1}{j\omega C_2} \right)$$

i.e.

$$C_1/C_2 = S/Q = R/P.$$

The practical process is to make successive adjustments of Q or S and P_1 or R_1 until true balance is attained.

If the phase-difference of the condenser C_2 be known, this bridge provides a ready means of finding the phase-difference of C_1 . From the balance condition,

$$C_1(P_1 + \rho_1) = C_2(R_1 + \rho_2),$$

whence $\omega C_1 \rho_1 - \omega C_2 \rho_2 = \omega C_2 R_1 - \omega C_1 P_1$

i.e. $\tan \theta_1 - \tan \theta_2 = \omega C_2 \left(R_1 - \frac{S}{Q} P_1 \right).$

Since the phase-differences are usually small, this equation is, very approximately,

$$\tan (\theta_1 - \theta_2) = \omega C_2 \left(R_1 - \frac{S}{Q} P_1 \right),$$

from which θ_1 can be found.

Experimental Example. The following results were obtained at a frequency of 407.1 cycles per second, the detector being a Duddell vibration galvanometer. C_1 was a paper condenser whose capacitance and series resistance ρ_1 were to be determined; the resistance P_1 was omitted. The branch AD contained a mica condenser of capacitance $C_2 = 0.334 \mu\text{F}$, having a series resistance of 0.30 ohms; R_1 was a low reading variable resistance. Q was a resistance fixed at various values, balance being obtained by successive adjustments of S and R_1 , as shown in the table.

Q ohms.	S ohms.	R ohms.	C_1 μF .	ρ_1 ohms.
4000	4785	$5.9_0 + 0.3_0$	0.400_3	5.1_7
2000	2392	$5.9_0 + 0.3_0$	0.400_2	5.1_7
1000	1196	$5.9_0 + 0.3_0$	0.400_2	5.1_7
500	598.4	$5.9_0 + 0.3_0$	0.400_4	5.1_7
200	239.4	$5.9_0 + 0.3_0$	0.400_4	5.1_7
		Average	$0.400_3 \mu\text{F}$.	5.1_7 ohms

For the standard condenser the loss angle is given by

$$\omega C_2 \rho_2 = \tan \theta_2 = 0.000257 \approx \theta_2 \approx 53'';$$

hence for the condenser under test,

$$\tan \theta_1 = \omega C_2 R_1 + \tan \theta_2 = 0.0052, \approx \theta_1 \approx 18'.$$

As in the case of De Sauty's bridge, and following the usual principle, it is best to arrange for an equal ratio network; possibility of error is thereby minimized. The method is subject to several sources of error,* which must now be briefly noticed, as it is particularly necessary to take precautions to eliminate errors when tests are being made on condensers in which θ is very small and also in the case of capacitance measurements on condensers less than about 0.1 microfarad.

Residual Inductance in Q and S . If λ and μ be the residuals in Q and S , it is easy to show that the balance conditions become

$$\frac{C_1}{C_2} = \frac{S}{Q} \left[1 + \frac{\omega^2 \mu C_1 P}{S} - \frac{\omega^2 \lambda C_2 R}{Q} \right].$$

$$\text{and} \quad \tan(\theta_1 - \theta_2) = \omega C_2 R_1 - \omega C_1 P_1 + \omega \left(\frac{\lambda}{Q} - \frac{\mu}{S} \right).$$

Grover works out a case in which $C_1 = C_2 = 1$ microfarad, $\rho_1 = 49$ ohms, $\rho_2 = 5$ ohms, $Q = S = 1,000$ ohms. λ is taken as -0.00004 henry and μ as -0.0015 henry, i.e. residual capacitance is the preponderant residual. With $P_1 = 1$ ohm and $R_1 = 45$ ohms, balance is attained at a frequency of 100 cycles per second. With these figures the correction to be applied to the capacitance ratio is only 3 parts in 10^5 . On the other hand, $\tan(\theta_1 - \theta_2) = 0.0276 - 0.00092$, an error of over 3 per cent.

This example serves to show that not only should an equal ratio bridge be arranged, but the branches Q , S should be similar in make-up, so that they have similar residuals. If then a second setting be made with them reversed in the bridge, the mean of the two balances will be little affected by residual errors. The coils should be well adjusted so that error in the ratio of Q and S is inappreciable.

Residuals in P_1 and R_1 . Assuming an equal ratio bridge, the above equation shows that for a given value of $\theta_1 - \theta_2$ the difference between R_1 and P_1 will be larger the smaller the value of the capacitance becomes. Since in small condensers quite large values of θ are found, this may necessitate the inclusion of high resistances in series with the condensers. It has been shown on page 59 that such high resistances may have a not inconsiderable equivalent capacitance, represented by a condenser joined in parallel with the resistance. The introduction of these capacitances into the condenser branches will, therefore, introduce an error in the capacitance ratio and in the determination of phase-differences.

* For a full discussion of the sources of error and for applications of the bridge to tests on condensers, see F. W. Grover, *loc. cit.*, pp. 378-389 (1907); H. L. Curtis, "Mica condensers as standards of capacity," *Bull. Bur. Stds.*, Vol. 6, pp. 431-488 (1910); F. W. Grover, "The capacity and phase difference of paraffined paper condensers as functions of temperature and frequency," *Bull. Bur. Stds.*, Vol. 7, pp. 495-578 (1911); K. W. Wagner, "Zur Messung dielektrischer Verluste mit der Wechselstrombrücke," *Elekt. Zeits.*, 32 Jahrgang, pp. 1001-1002 (1911); "Die Messung der dielektrischen Ableitung und Kapazitäten mehradrigen Kabel mit Wechselstrom," *Elekt. Zeits.*, 33 Jahrgang, pp. 635-637 (1912).

Grover (*loc. cit.*) shows that the correction due to the capacitance of the series resistances is negligible when the condensers under comparison are large, and, in general, will be unimportant whatever the value of the condensers, provided that they have nearly equal capacitances and phase differences. With small condensers (< 0.001 microfarad) of quite different power factors the corrections may become extremely important. It is with the object of overcoming this source of error that the Series Inductance method (p. 231) was suggested by the late Prof. E. B. Rosa.

Earth Capacities. Particularly when testing small condensers, the electrostatic capacity of the branches to earth may prove troublesome. Assuming that balance can be secured (in this connection, *see* p. 286), the method of substitution removes almost completely any troubles due to this cause.

Neglecting residuals, etc., the process is as follows: Let balance be secured with the unknown condenser in the bridge. Then $C_1/C_2 = S/Q$ and $\tan \theta_1 - \tan \theta_2 = \omega C_2 \left(R_1 - \frac{S}{Q} P_1 \right)$, the two condensers being as nearly equal as possible, so that a symmetrical network is obtained. Now replace the unknown by a standard condenser C'_1 of nearly equal value and phase difference θ'_1 , re-balancing the bridge by adjustments of P_1 to P'_1 and Q to Q' , all else being undisturbed. Then $C'_1/C_2 = S/Q'$ and $\tan \theta'_1 - \tan \theta_2 = \omega C_2 \left(R_1 - \frac{S}{Q'} P'_1 \right)$. Subtracting the tangent formulae and dividing the capacitance ratios gives

$$C_1 = C'_1 Q'/Q \text{ and } \tan(\theta_1 - \theta'_1) = \omega C'_1 \left(P'_1 - \frac{Q'}{Q} P_1 \right),$$

so that C_2 need not be known. Since the arrangement of the bridge is not disturbed between the taking of the two balances—except for the small alterations of P_1 and Q —earth capacity effects are eliminated. Moreover, the effects of residuals in the branches are also eliminated by the process, so that by using such a “dummy balance” on an equal ratio bridge, settings of very high precision may be obtained.

Wagner and Wertheimer* have also used the series resistance method for the purpose of measuring the time-constants of medium value or high resistance coils. As the time-constant is usually a very small quantity, of the order of 10^{-8} second in coils constructed for bridge work, it is necessary to set up the network in a rather special manner. The condensers C_1 and C_2 are air condensers, so that $\rho_1 = \rho_2 = 0$; C_1 is adjustable in value. P_1 and R_1 are resistance boxes. Q is the coil whose time-constant T_Q is to be found, and S is a standard resistance of a value about equal to the resistance of Q . With the bridge arranged in this way, balance is secured by adjustments of C_1 and P_1 . The resistance Q is then removed and replaced by a standard of equal resistance whose time-constant can be calculated from its dimensions, T , say. Re-balance by adjustment of P_1 and C_1 to values P'_1 , C'_1 . Then it is easy to show that $T_Q = T + PC_1 - P'C'$. It is essential

* K. W. Wagner and A. Wertheimer, “Über Präzisionswiderstände für hoch frequenten Wechselstrom,” *Elekt. Zeits.*, 34 Jahrgang, pp. 613–616 (1913). K.W. Wagner, *ibid.*, 36 Jahrgang, pp. 606–609 (1915).

to make allowance for earth capacities in measurements on such small quantities (see p. 286 for the method adopted).

9. Parallel Resistance Method. The parallel resistance method illustrated in Fig. 52 is sometimes of service for the comparison of fairly large condensers. Wien* was the first to describe alternating current measurements with the bridge. The two condensers C_1 , C_2 are shunted by resistances P , R ;

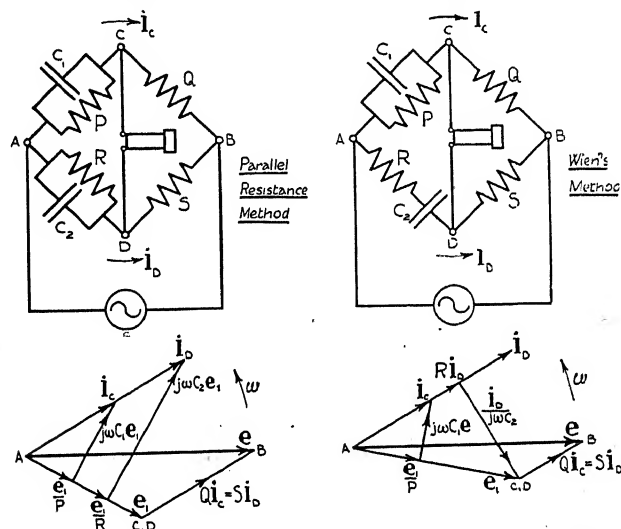


FIG. 52.—PARALLEL RESISTANCE METHOD AND WIEN'S METHOD FOR COMPARING TWO CONDENSERS

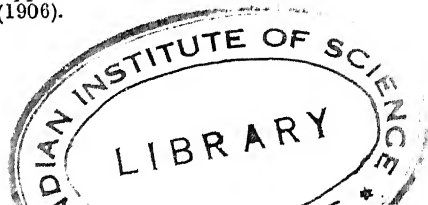
Q and S are, as before, the ratio branches. The branch impedance operators are $z_1 = 1/\left(\frac{1}{P} + j\omega C_1\right)$, $z_2 = Q$, $z_3 = S$, $z_4 = 1/\left(\frac{1}{R} + j\omega C_2\right)$; and balance is obtained when

$$S/\left(\frac{1}{P} + j\omega C_1\right) = Q/\left(\frac{1}{R} + j\omega C_2\right),$$

i.e.

$$C_1/C_2 = S/Q = R/P.$$

* Max Wien, *loc. cit.*, pp. 696–697 (1891). See also J. Hanauer, *loc. cit.* (1898); and J. Cauro, “Sur la capacité électrostatique des bobines, et son influence dans la mesure des coefficients d’induction par le point de Wheatstone,” *Comptes Rendus*, tome 120, pp. 308–311 (1895), for examples of the use of the method. For the parent ballistic bridge, see W. E. Sumpner, *loc. cit.* (1888), and E. C. Rimington, “Note on comparing capacities,” *Proc. Phys. Soc.*, Vol. 9, pp. 60–67 (1888). For an application of the bridge, see A. Campbell, *Proc. Roy. Soc.*, Vol. 78, p. 196 (1906).



The resistances Q or S and P or R are successively adjusted until the indication of the detector is reduced to zero.

The construction of the vector diagram is obvious. When the bridge is balanced the currents passing the branch points C and D are clearly in phase with one another and lead on the voltage e applied to the points A, B . Subtracting from e the vector $Qi_c = Si_d$ gives the vector e_1 , which represents the voltage across the branches AC and AD . The current through P is e_1/P and through the condenser C_1 is $j\omega C_1 e_1$, their sum being the current i_c . Similarly, the sum of e_1/R and $j\omega C_2 e_1$ is equal to i_d . It is easy, from a comparison of the sides of the vector triangles, to verify the above balance conditions.

From the vector diagram, if ωC_1 and ωC_2 are small compared with $1/P$ and $1/R$, then the condensers contribute only a small component to the currents passing the branch points, i_c and i_d , coming more nearly in phase with e . Hence if ω , P , and R be fixed, C_1 and C_2 must be large for the method to be sensitive to their presence in the network.

It has been shown on page 116 that the effects of absorption in an imperfect condenser may be represented by a perfect condenser in series with a resistance. Alternatively, it may be represented by a condenser in parallel with a resistance, the phase-difference in the two cases being the same. Grover* has shown how the parallel resistance method may be used to find the equivalent capacitance and shunt resistance of an imperfect condenser. In general, the fictitious resistance in parallel with a condenser in which there are dielectric losses will have a very high value; hence, the balancing resistance in parallel with the standard may be inconveniently large. To avoid the use of such large resistances, an artifice due to Wien may be employed. Let C_1, C_2 be the unknown and standard capacitances, their effective shunt resistances being W_1 and W_2 . In parallel with C_1 put a resistance P_1 of a value relatively low compared with W_1 , say 100,000 ohms or so, according to the circumstances. An adjustable resistance box, R_1 , in parallel with C_2 , will then be of a reasonable value. The values of C_1 and C_2 should be approximately equal, and the bridge balanced by adjustment of R_1 and Q/S .

Using this notation, $P = 1/\left(\frac{1}{P_1} + \frac{1}{W_1}\right)$ and $R = 1/\left(\frac{1}{R_1} + \frac{1}{W_2}\right)$

* *Loc. cit.*, pp. 393-396 (1907).

From the balance conditions

$$C_1/C_2 = R/P = \left(\frac{1}{P_1} + \frac{1}{W_1}\right) / \left(\frac{1}{R_1} + \frac{1}{W_2}\right),$$

or
$$\frac{1}{P_1 C_1} + \frac{1}{W_1 C_1} = \frac{1}{R_1 C_2} + \frac{1}{W_2 C_2}.$$

Multiplying by $1/\omega$, and remembering that the phase-difference of a condenser C in parallel with a resistance W is given by $\tan \theta = 1/\omega CW$, the relation becomes

$$\tan \theta_1 - \tan \theta_2 = \frac{1}{\omega C_2 R_1} - \frac{1}{\omega C_1 P_1}.$$

Since the resistances P_1 , R_1 are usually large, they will have self-capacitances c_1 and c_2 in parallel with them, and hence the second balance condition is

$$(C_1 + c_1)/(C_2 + c_2) = S/Q.$$

When the unknown and standard condensers are similar, c_1 and c_2 will be nearly equal, and will be small, since P_1 and R_1 will be nearly equal. Error due to them can then be neglected.

The method is subject to the same sources of error as the series resistance method, and accurate settings can be obtained by the use of the process of substitution.

Experimental Example. An experiment was made at 407.1 cycles per second on the condensers previously tested by the series resistance method (*see* p. 198). C_2 had a capacitance of 0.334_μF and an effective shunt resistance of $W_2 = 4.56$ megohm. The condensers C_1 and C_2 were shunted by resistances P_1 and R_1 respectively, P_1 being fixed at 1,000 ohms. Balance was secured by successive adjustments of Q , S , and R_1 , the values obtained being $Q = 795.0$ ohms, $S = 951.5$ ohms, $R_1 = 1190.5$ ohms. From these results

$$R = 1 / \left(\frac{1}{R_1} + \frac{1}{W_2} \right) = 1190.2 \text{ ohms,}$$

hence
$$P = QR/S = 994.5 \text{ ohms.}$$

Remembering that $P = 1 / \left(\frac{1}{P_1} + \frac{1}{W_1} \right)$, gives $W_1 = 0.181$ megohm.

Again, $C_1 = SC_2/Q = 0.400_\mu\text{F}$. From the equations on page 117, the series loss resistance ρ_1 equivalent to W_1 is $\rho_1 = W_1 / (1 + \omega^2 C_1^2 W_1^2)$ or 5.2_μohms , which should be compared with the value found by the series resistance bridge.

10. Wien's Method. An important method for comparing two condensers was introduced by Wien,* and is illustrated in

* *Loc. cit.*, pp. 704-707 (1891).

Fig. 52. The unknown condenser C_1 is joined in parallel with a resistance P ; the standard C_2 is provided with a series rheostat R , Q and S being ratio coils. The operators are $z_1 = 1/(\frac{1}{P} + j\omega C_1)$, $z_2 = Q$, $z_3 = S$, $z_4 = R + \frac{1}{j\omega C_2}$ and the balance equation is

$$S/(\frac{1}{P} + j\omega C_1) = Q(R + \frac{1}{j\omega C_2}).$$

Separating components,

$$\frac{C_1}{C_2} = \frac{S}{Q} - \frac{R}{P},$$

and

$$C_1 C_2 = 1/\omega^2 P R.$$

The vector diagram combines the features of the series resistance method with those of the parallel resistance method. The current in the branches ADB , i_d leads on the voltage e applied to the points A and B . The vector difference between e and $S i_d$ is the voltage $e_1 = (R + \frac{1}{j\omega C_2}) i_d$, which also acts on the branch AC (since C and D are at the same potential). The current i_c is in phase with i_d and is compounded of the vectors of current e_1/P and $j\omega C_1 e_1$.

The principal use of Wien's bridge is to determine the equivalent capacitance C_1 and shunt resistance W_1 of an imperfect condenser, such as a sample of insulation or a length of cable. The resistance P in the diagram is then equal to W_1 ; C_2 is an air condenser in series with an adjustable resistance R ; Q and S are resistance boxes. Balance is obtained by successive alterations of R and Q or S . Solving the above equations, and putting $P = W_1$, gives

$$C_1 = \frac{S}{Q} \cdot \frac{C_2}{(1 + \omega^2 C_2^2 R^2)}$$

and

$$W_1 = \frac{Q}{S} \cdot \frac{(1 + \omega^2 C_2^2 R^2)}{\omega^2 C_2^2 R}$$

Now W_1 is usually a large resistance and C_2 is an air condenser of relatively small capacitance. Hence, the ratio of Q to S will tend to become large. The network will have very unequal branches and will be very susceptible to error, due to unsymmetrically distributed earth capacities. The presence of such error can be detected by reversing the connections to the

alternator and noticing if the balance is disturbed. If the change in the adjustments then necessary to restore balance is small, taking the mean of the two results will largely eliminate the error. It is better, however, to eliminate earth capacity troubles by the Wagner earthing device described on page 286.

When, as is frequently the case in the measurements to which the method is commonly applied, C_1 is small and becomes comparable with the air condenser C_2 , a more symmetrical bridge may be secured by shunting C_1 with a resistance P_1 of a value small compared with W_1 . This resistance will have a known self capacity c_1 . Referring to the balance conditions, write $C_1 + c_1$ for C_1 and $1/P = (1/W_1) + (1/P_1)$. Solving for C_1 , W_1 gives

$$C_1 = \frac{S}{Q} \cdot \frac{C_2}{(1 + \omega^2 C_2^2 R^2)} - c_1,$$

and
$$\frac{1}{W_1} = \frac{S}{Q} \cdot \frac{\omega^2 C_2^2 R}{(1 + \omega^2 C_2^2 R^2)} - \frac{1}{P_1}.$$

Experimental Example. The condensers compared by the series resistance method (see p. 198) and by the parallel resistance method (see p. 203) were again tested by the Wien bridge. C_1 is the paper condenser whose shunt loss resistance W_1 is to be determined; it is joined in parallel with a resistance $P_1 = 1000$ ohms. C_2 is the mica standard of 0.334_μF. ; R is composed of the series loss resistance $\rho_2 = 0.30$ ohms and a resistance box R_1 , so that $R = R_1 + 0.30$. Q is fixed at 400 ohms, balance being found (at 407.1 cycles per second) by successive adjustments of S to 940 ohms and of R_1 to 1147.5 ohms. A Duddell vibration galvanometer was used. Then, from above,

$$P = \frac{Q}{S} \cdot \frac{(1 + \omega^2 C_2^2 R^2)}{\omega^2 C_2^2 R} = 994.57 = 1 / \left(\frac{1}{W_1} + \frac{1}{P_1} \right),$$

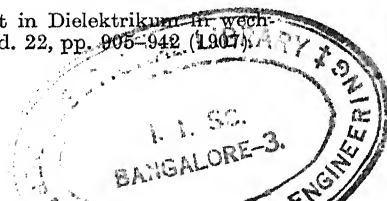
whence $W_1 = 0.18_\text{megohm}$. Also, neglecting the capacitance of P_1 , $C_1 = 0.400_\mu\text{F.}$ The series resistance ρ_1 corresponding to W_1 is, from the equation on page 117, 5.2_ohm , which checks with the values previously found.

In a second trial with $P_1 = 10,000$ ohms, $Q = 500$ ohms, $S = 604.3$ ohms, and $R_1 = 119.9$ ohms. From these values $W_1 = 0.18_\text{megohm}$, $\rho_1 = 5.0_\text{ohms}$, and $C_1 = 0.400_\mu\text{F.}$, which checks with the above test.

This method has been used in some important researches on the properties of imperfect condensers, particularly to find the loss-angle for the dielectric of a high voltage cable. Hanauer,* in 1898, measured the change in C_1 and W_1 with frequency for a number of solid and liquid dielectrics. Monasch,† in 1907, carried out an important and elaborate

* *Loc. cit.*, 1898.

† Bruno Monasch, "Ueber den Energieverlust in Dielektrikum für wechselnden elektrischen Feldern," *Ann. der Phys.*, Bd. 22, pp. 905-942 (1907).



investigation of the law connecting the dielectric loss in a cable with the applied voltage. High voltages, ranging from 1,000 to 9,000 volts, were used on the bridge, the detector being an optical telephone. Tests were made on quite small samples of cable, having capacitances of the order of 0.001 microfarad; the standard was a cylindrical air condenser specially constructed to withstand the high voltages used, its maximum capacitance being about $230 \mu\mu\text{F}$. Allowance was made for the capacitance of the resistances and for that of the bridge with respect to its surroundings. The point B was earthed. Wien's method has also been used by Campbell and Eckersley,* in 1910, to test the variation of W_1 with frequency for some common dielectrics.

Rosen† has recently made an investigation of the losses in the dielectric of single core and multicore cables at voltages up to 30 kilovolts using a modified form of Wien's bridge. He uses the conjugate arrangement with the alternator across CD and the detector across AB . By choice of a suitably large value for Q/S , the voltage across C_2 is reduced and this condenser can now be an ordinary variable mica standard instead of being a special air condenser suitable for extra high tension. It is now necessary to construct the ratio resistances to withstand the high testing voltage; as these resistances are of large value, some error may be introduced by distributed self capacitance in them. Earth capacity effects at the detector branch points are eliminated by use of the Wagner earthing device (see p. 286).

11. Hay's Method. The late Mr. C. E. Hay‡ suggested the following modification of Anderson's bridge (see p. 215) for the purpose of measuring the small residual capacity of resistance coils. Q is the resistance whose effective capacity C_1 is to be found; C_2 is a standard condenser. The rest of the branches are composed of non-reactive resistances, as in Fig. 53.

Comparing this diagram with Fig. 15, it will be seen that $z_1 = P$, $z_2 = 1/(\frac{1}{Q} + j\omega C_1)$, $z_3 = S$, $z_4 = R$, $z_6 = r$, and $z_7 = 1/j\omega C_2$. Now, from p. 46, the balance equation for the network is

$$z_7(z_1 z_3 - z_2 z_4) = z_2(z_6(z_3 + z_4) + z_3 z_4),$$

i.e.

$$SP\left(\frac{1}{Q} + j\omega C_1\right) - R = j\omega C_2[r(R + S) + RS].$$

Separating components,

$$SP = QR,$$

$$C_1 = \frac{C_2}{SP} [r(R + S) + RS].$$

* A. Campbell and J. L. Eckersley, "On the insulation of inductive coils," *Elecn.* Vol. 64, pp. 350-352 (1910).

† A. Rosen, "The use of the Wien bridge for the measurement of the losses in dielectrics at high voltages, with special reference to electric cables." Read before the *Physical Society of London* in May, 1923, but not published at the time of going to press.

‡ C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal, P.O.E.E.*, Vol. 5, pp. 451-454 (1913); also *Professional Papers*, No. 53, pp. 19-21, 36-37.

In practical work it is best to make $R = S$ and hence $P = Q$. Balance is secured by adjustment of P and r , the equation for C_1 reducing to

$$C_1 = \frac{C_2}{P} [2r + R].$$

By re-balancing with R and S interchanged and taking the mean result, small differences between them are eliminated. The effect of residuals in the remaining branches and of electrostatic capacity of the bridge to earth may be eliminated by the process of substitution or "dummy" balance. After balancing with the unknown in position, a new balance is obtained when the unknown is replaced by a standard equal in resistance to Q and having calculable residual capacity. The difference between the unknown and the standard is thus found with a high degree of precision. For further details, reference should be made to the discussion given on page 223 of a similar process applied to resistance coils which have residual inductance.

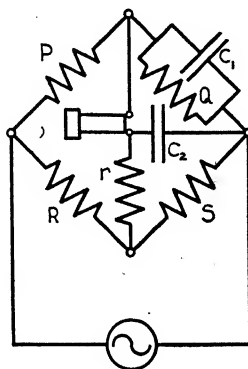


FIG. 53.—HAY'S METHOD FOR MEASURING THE RESIDUAL CAPACITANCE OF A RESISTANCE COIL

12. Schering and Semm's Method.

In modern high voltage technology it is commonly required to measure the dielectric losses in cables and in insulators when subjected to the working voltage. On page 206 methods have been described which are applicable to such tests. More recently, Semm* has devised the following method, which is intended for use at extra high voltages.

Referring to Fig. 54, C_1 is the effective capacitance and ρ_1 the equivalent series resistance of the imperfect condenser to be tested. C_2 is a special cylindrical air condenser having a capacitance of about 50 cm. and capable of withstanding voltages up to 100 kilovolts. This condenser is provided with guard rings, which are earthed. C_3 is a mica condenser of adjustable value lying between 1 and 2 microfarads. Q and S are non-reactive resistances, the former being fixed at a value of 200 ohms. Balance is secured by adjustment of S and C_3 .

* A. Semm, "Verlustmessungen bei Hochspannung," *Arch. f. Elekt.*, Bd. 9, pp. 30-34 (1921); "Tätigkeitsbericht der Physikalische Technische Reichsanstalt," *Zeits. f. Inst.*, 40 Jahrgang, p. 124-125 (1920). See also H. Schering, *ibid.*, p. 124 (1920), for the original suggestion of the method.

Writing down the branch impedance operators $z_1 = \rho_1 + \frac{1}{j\omega C_1}$, $z_2 = Q$, $z_3 = 1/\left(\frac{1}{S} + j\omega C_3\right)$, $z_4 = 1/j\omega C_2$, the balance eqn. is

$$\left(\rho_1 - \frac{j}{\omega C_1}\right) = Q\left(\frac{C_3}{C_2} - \frac{j}{\omega C_2 S}\right);$$

$$\text{whence } \rho_1 = \frac{C_3}{C_2} Q,$$

$$\text{and } C_1 = \frac{S}{Q} C_2.$$

From this the phase-difference θ_1 of the tested condenser is

$$\tan \theta_1 = \omega C_1 \rho_1 = \omega S C_3,$$

from which the power factor may be easily calculated.

The bridge has recently been used* in an extensive and precise research on the losses in condensers at ordinary laboratory voltages. Standard mica, fixed and rotary air condensers were tested by a process of substitution in a bridge sensitive enough to enable a phase-difference so small as 1" to be detected. The bridge is arranged in a symmetrical way with bifilar leads and suitable screening devices.

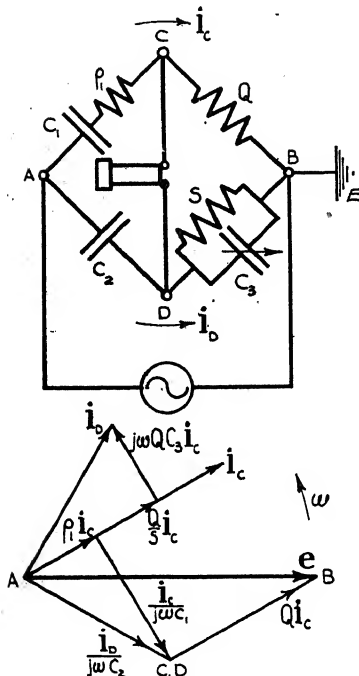


FIG. 54.—SEMM'S METHOD FOR TESTS ON DIELECTRICS AT HIGH VOLTAGES

13. Fleming and Dyke's Method. On page 196 it has been pointed out that for sensitivity the ratio branches of a simple condenser bridge must have high resistance, since the impedance of the condenser branches is high, especially when small condensers are to be tested at low frequencies. For example, suppose C_1 and C_2 in Fig. 51 are each of the order of 0.001 microfarad, and the frequency is about 1,000 cycles per second. Then the impedance, $1/\omega C$, is of the order of 150,000 ohms. For a sensitive arrangement, therefore,

* E. Giebe and G. Zickner, "Verlustmessungen an Kondensatoren," *Arch. f. Elek.*, Bd. 11, pp. 109-129 (1922).

Q and S must each be of the order of 100,000 ohms or so. Now it is far from easy to produce resistances of this magnitude which will be free from the effects of residual inductance or capacitance, so that the theory of the bridge becomes complicated by the consideration of the residuals in the ratio branches. However, it is quite easy to make variable air condensers of small capacitance (about 0.002 microfarad), and as these are free from absorption, they may be used as high impedance ratio branches in a condenser bridge without introducing any residual error.

Fleming and Dyke* have applied this artifice to the Wien bridge of Fig. 52 in a research on the alternating current conductivity of dielectrics, their arrangement being drawn in Fig. 55. The branch AC contains the imperfect condenser which is to be tested, C_1 being its effective capacitance and P its effective shunt resistance. The branch AD contains a variable air condenser in series with a small adjustable resistance R . C_2 and C_3 are variable air condensers to act as ratio branches.

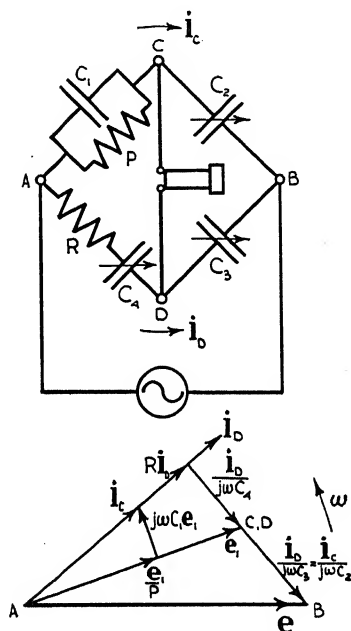


FIG. 55.—FLEMING AND DYKE'S FOUR CONDENSER BRIDGE

The impedance operators are $z_1 = 1 / \left(\frac{1}{P} + j\omega C_1 \right)$, $z_2 = 1 / j\omega C_2$, $z_3 = 1 / j\omega C_3$, $z_4 = R + \frac{1}{j\omega C_4}$; and the balance equation

$$C_2 / C_3 = \left(R - \frac{j}{\omega C_4} \right) \left(\frac{1}{P} + j\omega C_1 \right).$$

* J. A. Fleming and G. B. Dyke, "On the power factor and conductivity of dielectrics when tested with alternating currents of telephonic frequency at various temperatures," *Journal, I. E. E.*, Vol. 49, pp. 323-431 (1912). The idea of a four condenser bridge seems to be due to Nernst, *Ann. der Phys.*, Bd. 60, p. 600 (1897). For its use at radio-frequencies, see *Dictionary of Applied Physics*, Vol. 2, p. 132.

Separating components,

$$\frac{C_1}{C_4} = \frac{C_2}{C_3} - \frac{R}{P},$$

and $C_1 C_4 = 1/\omega^2 P R$.

The vector diagram follows at once from that for the Wien bridge, as a comparison of Figs. 52 and 55 will show.

Solving these equations for C_1 and P gives,

$$C_1 = \frac{C_2}{C_3} \cdot \frac{C_4}{1 + \omega^2 C_4^2 R^2}$$

$$P = \frac{C_3}{C_2} \cdot \frac{1 + \omega^2 C_4^2 R^2}{\omega^2 C_4^2 R}.$$

Hence the frequency must be known and constant, and the wave form must be pure if telephones are used to detect balance.

In practical working, the condensers C_1 , C_2 , C_3 , C_4 are arranged to be as nearly equal as possible. A suitable value of C_4 being chosen, balance is secured by successive adjustments of C_2/C_3 and R . The condensers should not be too close together or mutual capacitances will exist between them and vitiate the results. Nor must the body of the observer be brought too near the condensers when balance is nearly obtained. All connections should be made of fine wire, so that capacitance in them is reduced to a negligible amount.

Experimental Example. Fleming and Dyke describe the following illustrative experiment carried out at 4,400 cycles per second, a high resistance telephone being used to find the balance point. An air condenser C_1 of 430 $\mu\mu\text{F.}$ was shunted with a resistance $P = 2.15$ megohm. At balance, $C_4 = 476$ $\mu\mu\text{F.}$, $C_2 = 1130$ $\mu\mu\text{F.}$, $C_3 = 1216$ $\mu\mu\text{F.}$, and $R = 2975$ ohms. Using these values, $C_1 = 443$ $\mu\mu\text{F.}$, and $P = 2.10$ megohm, which are in good agreement with the actual values. A large number of tests on small imperfect condensers are described in their paper, to which the student is referred for further details.

If a symmetrical bridge be arranged, as suggested, the effects of earth capacities between the branches and earth will be small. After balance has been obtained, the connections to the alternator should be reversed. Balance will, in general, be slightly disturbed and should be restored by small adjustments of C_2/C_3 and R . The mean of the two settings may be taken as the correct result.

In accurate tests, C_2 , C_3 , and C_4 should be provided with

screens. The resistance R should then be connected in the branch AD on the detector side of C_4 , not on the alternator side as shown in the figure; the screen of C_4 should be joined to one terminal of the resistance R , whose other terminal is joined to D . The screens of C_2 and C_3 should be connected to C and D respectively. C_1 should, if possible, be provided with a screen joined to the branch point C . Earth capacitance effects at the points CD are eliminated by the Wagner earthing device (p. 286). These precautions of screening should be taken in the case of any bridge in which small air condensers are used. The detector should have a high impedance or should be connected through a step-up transformer to the bridge (see p. 151 and p. 173).

Fleming and Dyke carried out an extensive research by this method. Their source of current was a 900 cycle Crompton alternator with a very impure wave form. Pure waves were obtained for use on the bridge by means of a "wave filter" which resonated a chosen harmonic in the alternator wave. By this means a range of 900 to 5,000 cycles per second was covered. As the tests had reference to telephonic work, low voltages were used on the bridge (4 to 5 volts). The detector was a high resistance telephone. Very consistent results were obtained, their paper being well worthy of study by those intending to make power factor measurements on small condensers, especially at high frequencies.

NETWORKS CONTAINING RESISTANCE, SELF-INDUCTANCE, AND CAPACITANCE

14. Maxwell's Method. A large class of networks, in which self-inductance and capacitance are compared, is developed from a ballistic method introduced by Maxwell.* Fig. 56 shows Wien's† arrangement of the Maxwell bridge for use with alternating current.

The branch impedance operators are $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S/(1 + j\omega CS)$, $z_4 = R$, so that balance occurs when

$$S \frac{(P + j\omega L)}{(1 + j\omega CS)} - QR = 0;$$

i.e. when

$$SP = QR = L/C.$$

* *A Treatise on Electricity and Magnetism*, 1st Edn., Vol. 2, pp. 377-379 (1873). For further descriptions of this ballistic method, see E. C. Rimington, "Self-induction and its measurement," *Tel. J.*, Vol. 21, (1887). W. E. Sumpner, "The variation of the coefficients of induction," *Proc. Phys. Soc.*, Vol. 9, pp. 235-259 (1888). A. Russell, "Measuring coefficients of induction," *Electn.*, Vol. 33, pp. 5-6 (1894).

† Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 689-712 (1891).

These balance conditions are illustrated by the vector diagram, the construction of which is self-evident.

In practice, Q , R , and S are non-inductive resistances. If L and C are both fixed in value, balance must be secured by successive adjustments of Q/S and R . As these two adjustments interfere, balancing the bridge is often a tedious

process. Considerable simplification results if the branch AC contains a variable inductance or if the condenser be subdivided. It is then possible to fix two of these resistances and to balance by successive adjustment of the third resistance and either the inductometer or the condenser, as the case may be.

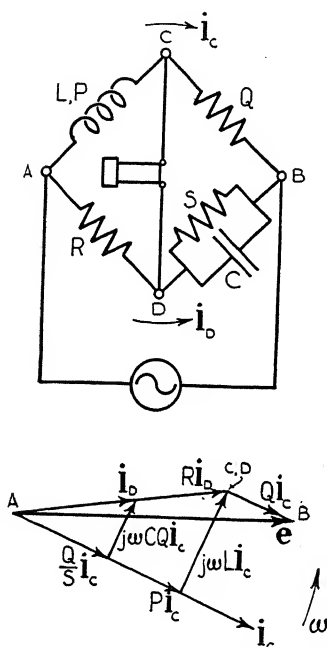


FIG. 56.—MAXWELL'S METHOD FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE

Experimental Example. The following results were obtained in a test at 407.1 cycles per second in which a mica condenser C of $0.334 \mu\text{F}$ was compared with a coil of about 40 mH. A resistance of 350 ohms was joined in series with the coil, balance being obtained by successive adjustments of Q , S , and R , using a Duddell vibration galvanometer as detector. The values found were $Q = 346.7$ ohms, $S = 342.3$ ohms, and $R = 350.5$ ohms, whence $L = 40.66$ mH. Also, $P = 354.97$ ohms, so that the effective resistance of the coil is 4.97 ohms.

C. E. Hay,* in England, and H. Curtis and F. Grover,† in America, have described the use of Maxwell's method to measure the small residual inductances of resistance coils. It is

necessary to make allowance for residuals in the branches Q , R , and S , P being the resistance whose residual inductance L is to be measured. Hay's procedure is then as follows. Let R and S be ratio coils having equal resistances and residuals, and suppose Q to be a constant inductance rheostat. With L , P in the bridge, balance by adjustment of C and Q . Then replace L , P by a standard of nearly equal resistance and

* C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal P.O.E.E.*, Vol. 5, pp. 451-454 (1913); also *Professional Papers*, No. 53, pp. 19-22.

† F. W. Grover and H. L. Curtis, "The measurement of the inductances of resistance coils," *Bull. Bur. Stds.*, Vol. 8, p. 462 (1913).

calculable inductance L' , e.g. a parallel wire resistance, re-balancing by adjustment of C and slight alteration of Q to a value Q' . Then it is easy to prove, since the inductance of the branch CB remains unchanged, that $L = L' + R(CQ - C'Q')$; or assuming the standard to have a resistance exactly equal to that of the coil under test, $L = L' + QR(C - C')$, to a very high degree of approximation. Residual effects are thus very nearly, if not entirely, eliminated.

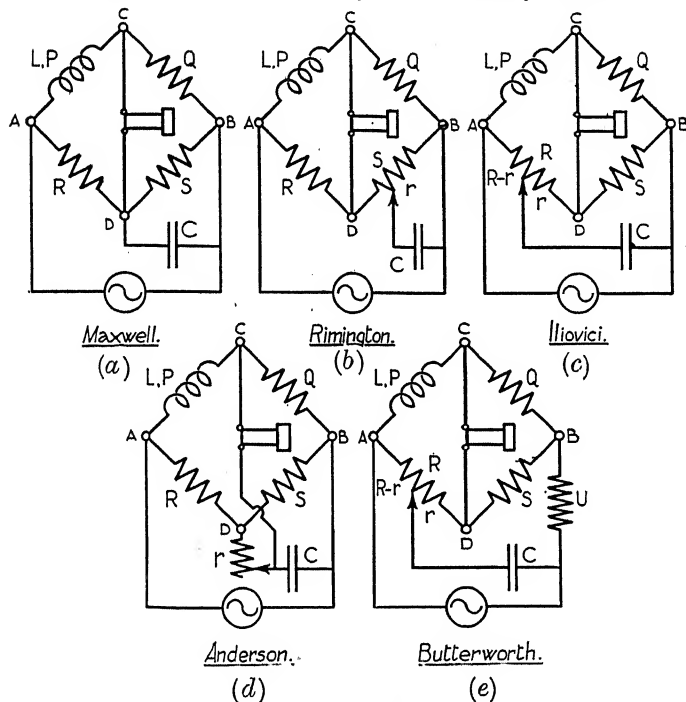


FIG. 57.—MODIFICATIONS OF MAXWELL'S METHOD FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE

Grover and Curtis slightly modify this procedure by fixing the resistances Q , R , and S . A rheostat is then included in the branch AC , the bridge being balanced by successive adjustments of this rheostat and the condenser. If ΔC be the change of capacity necessary to secure balance when the test coil is substituted for the standard, and Δl be the change of inductance corresponding to the adjustment of the rheostat in the branch AC , then $L = L' + QR \cdot \Delta C - \Delta l$.

15. Modifications of Maxwell's Method. The apparatus required for the measurement of a self-inductance by Maxwell's method is simple and would be found in any laboratory. If, therefore, some means could be devised to remove the troublesome interference of the balance

adjustments, the method would have great practical convenience. In Fig. 57 several modifications of Maxwell's bridge are shown, having for their common object the simplification of the balancing procedure. It is proposed in succeeding sections to describe each of these bridges in detail, and to point out their particular advantages.

16. Rimington's Method. The arrangement shown in Fig. 57 (b) was introduced independently by E. C. Rimington* and C. Niven,† in 1887, as a modification of Maxwell's ballistic bridge. Their procedure is to balance the bridge for steady current, so that $SP = QR$, and then to adjust the tapping on the resistance S until the bridge is also balanced for make and break. When this ballistic balance is secured, then $L = CPr^2/S$. Rimington has shown that the balance obtained in this way is for aggregate zero quantity through the galvanometer, so that the method is not primarily suitable for alternating current.

It has recently been shown by Dalton‡ that Rimington's method can be used with alternating current, provided that the initial steady current balance condition be abandoned. Let P , Q , and S be fixed, balance being obtained by successive adjustments of R and r . Then, writing $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = (S - r) + [r/(1 + j\omega Cr)]$, $z_4 = R$, the balance equation is

$$(P + j\omega L) \left\{ (S - r) + \frac{r}{1 + j\omega Cr} \right\} = QR$$

Separating the components gives for the balance conditions,

$$LC = \frac{SP - QR}{\omega^2 r(S - r)},$$

$$L/C = \frac{r}{S} \{ Pr - (SP - QR) \},$$

whence L may be found in terms of the resistances and ω . A knowledge of the frequency becomes necessary, and it should be observed that when L , C , and all the resistances are fixed, balance is only possible at one frequency. The method is not convenient in practice.

It is easily seen from the above equations why the condition $SP = QR$ is not admissible when the bridge is used with alternating current. From the first equation, if $SP = QR$ then $\omega^2 = 0$; so that balance is only possible for very slow alternations, i.e. ballistically. The second equation then gives $L/C = r^2 P/S$. If, in addition, $S = r$, the first equation is satisfied whatever be the value of ω , provided $SP = QR$. The method is then Maxwell's, in which continuous balance is always possible, and $L/C = SP$.

17. Nivici's Method. A modification of Maxwell's method, having some resemblance to Rimington's arrangement, is

* E. C. Rimington, *loc. cit.*; also "On a modification of a method of Maxwell's for measuring the coefficient of self-induction," *Proc. Phys. Soc.*, Vol. 9, pp. 26-32 (1888).

† C. Niven, "On some methods of determining and comparing coefficients of mutual induction," *Phil. Mag.*, 5th series, Vol. 24, pp. 225-238 (1887).

‡ J. P. Dalton, "On a new continuous balance method of comparing an inductance with a capacity," *Phil. Mag.*, 6th series, Vol. 27, pp. 37-44 (1914).

shown in Fig. 57 (c). This network has been introduced independently by Iliovici,* Butterworth,† Orlich,‡ and Dalton|| at various dates between 1904 and 1914.

Comparing this diagram with Fig. 15, Iliovici's method is seen to be a six-branch network for which the balance equation is given on page 46 as

$$z_7(z_1z_3 - z_2z_4) = z_2\{z_6(z_3 + z_4) + z_3z_4\}.$$

Substituting $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = 1/j\omega C$, $z_4 = R - r$, $z_6 = r$, $z_7 = S$, gives

$$S[P + j\omega\{L - QC(R - r)\}] = Q\{R + j\omega Cr(R - r)\}.$$

Separating components,

$$SP = QR,$$

$$\text{and} \quad L = \frac{CQ}{S}(R - r)(S + r)$$

are the balance conditions. These do not interfere with one another and are free from the objections to those of Rimington's method.

In practice, it is best to use an equal ratio bridge, setting $P = Q$ and $S = R$. Balance is obtained by adjustment of the condenser tapping r and small alteration of S or Q . For rapid work, the resistance R should be a slide wire, upon which the position of tapping r can be conveniently adjusted.

Experimental Example. An experiment was made using the coil and condenser compared by Maxwell's method (see p. 212), the detector and frequency remaining the same. The branch AD was made up of two decade resistances joined by a short slide wire, the total value of R being 2,000 ohms. Q was fixed at 1,000 ohms and a resistance of 910 ohms was joined in series with the coil L . At balance, $r = 1935.6$ ohms and $S = 2185.3$ ohms, so that $L = 40.63$ mH. Also, $P = 915.2$ ohms, which makes the effective resistance of the coil equal to 5.2 ohms.

18. Anderson's Method. Of all the modifications of Maxwell's method that introduced in 1891 by A. Anderson§ is the

* M. Iliovici, "Sur une méthode propre à mesurer les coefficients de self induction," *Comptes Rendus*, tome 138, pp. 1411-1413 (1904).

† S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75-94 (1912).

‡ E. Orlich, *Kapazität und Induktivität*, p. 260.

|| J. A. Dalton, *loc. cit.*

§ A. Anderson, "On coefficients of induction," *Phil. Mag.*, 5th series, Vol. 31, pp. 329-337 (1891).

most important and the most useful. Originally used ballistically, H. Rowland,* in 1898, adapted the method for use with alternating current, the arrangement of the network being shown in Figs. 57 (d) and 58.

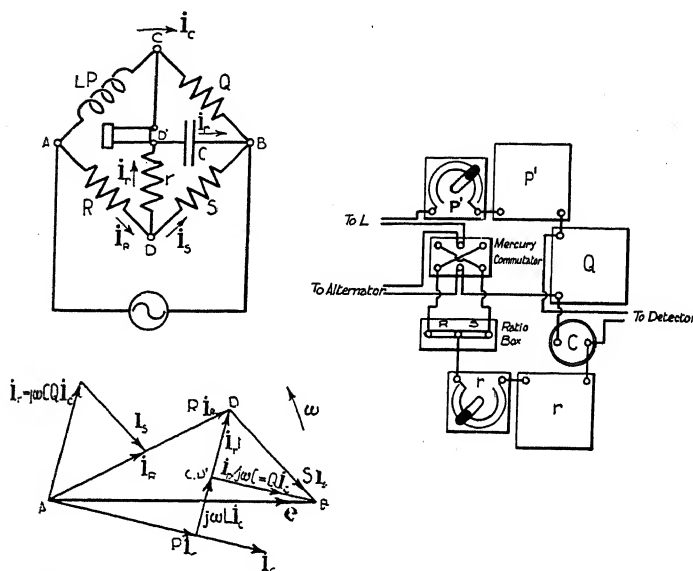


FIG. 58.—ANDERSON'S METHOD FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE

Anderson's method appears to have been first used for the accurate measurement of inductance by Prof. J. A. Fleming and W. C. Clinton,† in 1903. These investigators employed a commutator arrangement of the secohmmeter type, supplying the bridge with interrupted current and using a ballistic galvanometer to detect balance. Prof. Fleming,‡ in 1905, described experiments in which inductances of the order of 20 millihenrys were measured, the source of current being an electromagnetic buzzer and the detector a telephone. In both these uses of the bridge the supply of current is of an irregular wave form and is not a true alternating current. As mentioned above, H. Rowland, in 1898, had used a regular alternating source of supply, his detector

* H. Rowland, "Electrical measurement by alternating currents," *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429-448 (1897); *Phil. Mag.*, 5th series, Vol. 45, pp. 66-85 (1898).

† J. A. Fleming and W. C. Clinton, "On the measurement of small capacities and inductances," *Proc. Phys. Soc.*, Vol. 18, pp. 386-409 (1903).

‡ J. A. Fleming, "Note on the measurement of small inductances and capacities, and on a standard of small inductance," *Proc. Phys. Soc.*, Vol. 19, pp. 160-172 (1905).

being an electro-dynamometer. W. Stroud and J. H. Oates,* in 1903, using similar apparatus, recorded measurements of inductance lying between a few millihenrys and about a henry. The full advantages of Anderson's bridge were not realized, however, until 1905, when E. Rosa and F. W. Grover,† of the U.S. Bureau of Standards, applied the vibration galvanometer to the network, thereby converting it into a method for measuring inductance with very high precision. The discussion of the bridge which follows is largely adapted from their paper on the subject and from other publications of the Bureau.

Referring to Fig. 58, let the impedance operators of the various branches be $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$, $z_6 = r$, $z_7 = 1/j\omega C$. Putting these values in the balance equation on p. 46 gives, as the symbolic condition for balance

$$SP - QR + j\omega LS = j\omega CQ\{r(S + R) + SR\}.$$

Separating the components, there will be no current in the detector when

$$SP = QR,$$

$$\text{and} \quad L = CQ \left[r \left(1 + \frac{R}{S} \right) + R \right]. \quad (a)$$

Making use of the first of these conditions in the second, the inductance is given by

$$L = C[r(Q + P) + QR], \quad (b)$$

when the condenser is perfect and the resistances are free from residuals. The bridge can be balanced by independent adjustment of one of the branches (P , Q , R , or S) and r . Moreover, the range of L covered by a given bridge and condenser is very considerable, since r may have any desired value.

The vector diagram for the balanced bridge is easily constructed. In Fig. 58 let the vector AB denote the voltage \mathbf{e} , across the points A and B . In the branch ACB a current \mathbf{i}_c is flowing, of magnitude and phase given by $\mathbf{i}_c = \mathbf{e}/z_{AB}$, so that $\mathbf{e} = (P + j\omega L)\mathbf{i}_c + Q\mathbf{i}_c$. The potential of the point C is thus represented by the point C on the diagram of vectors; and since no current flows in the detector, C is also the potential of the point D' . The potential difference across the condenser is

* W. Stroud and J. H. Oates, "On the application of alternating currents to the calibration of capacity boxes, and to the comparison of capacities and inductances," *Phil. Mag.*, 6th series, Vol. 6, pp. 707-720 (1903).

† E. B. Rosa and F. W. Grover, "Measurement of inductance by Anderson's method, using alternating currents and a vibration galvanometer," *Bull. Bur. Sds.*, Vol. 1, pp. 291-336 (1905).

Qi_c , so that the current flowing through it must be $i_r = j\omega CQi_c$. This current also flows in the resistance r , producing a drop of potential ri_r ; the vector difference between this and the vector $Qi_c = i_r/j\omega C$ being the potential difference across the resistance S . Thus the magnitude and phase of the current i_s is determined, and the rest of the diagram may be easily completed.

In setting up Anderson's bridge, it is desirable to arrange the various resistances so that the greatest sensitivity is attained. Assuming the bridge to be adjusted so that it is nearly balanced, and that a vibration galvanometer is used, Mr. Butterworth* has shown that, for the greatest sensitivity to a change in r , the resistances should be arranged so that $Q = P$, $S = R = \frac{1}{2}P$ and $L/C = 2P^2$. At the same time, the frequency should be low to allow the galvanometer to work at the most sensitive part of its range. Thus, the conditions for best sensitivity of Anderson's bridge make it an equal ratio method, which is precisely what is required to reduce residual errors to a minimum.

A convenient practical arrangement of the necessary apparatus is shown in Fig. 58. The branches R and S are equal coils in a ratio box, and are connected to the bridge through a reversing switch, so that, by taking two balances with their positions interchanged, any small differences of the residual inductances of R and S may be eliminated in the mean. The resistance Q is composed of a non-inductive box, and is set to a fixed value; the rheostat r consists of a resistance box and a fine adjustment rheostat. The condenser may be a suitable mica standard and, as will be shown later, need only be fairly good, provided that its calibration with alternating current be known. The remaining branch contains the coil of which the inductance is to be determined, together with a resistance box similar to Q and a fine adjustment rheostat (P'). The following is then the best procedure to adopt—

- (i) Choose a reasonable value of C , so that L/C makes $P = Q$ of reasonable magnitude. Set Q to this value.
- (ii) Make $S = R = \frac{1}{2}P = \frac{1}{2}Q$.
- (iii) Adjust successively the resistances P' in the inductance branch and r until balance is attained.

* S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, p. 83 (1912).

(iv) Calculate L . Since $P = Q$, Equation (b) gives

$$L = CQ[R + 2r].$$

(v) Repeat with the positions of R and S interchanged; re-calculate L , and take the mean of the two values as the true inductance of the coil.

It may be found in practical working that the resistance r has a fairly large value. As this resistance lies in the detector circuit, it may, when large, considerably reduce the sensitivity of setting. In such a case the sensitiveness can be increased by interchanging the source and detector and increasing the voltage applied to the bridge in order to allow for the effect of r , which will then lie in series with the source. This modification is virtually that introduced by Stroud and Oates.*

Experimental Example. The coil previously tested by Maxwell's and by Iliovici's methods was compared with a mica condenser of $0.334_6 \mu\text{F}$. capacitance at a frequency of 407.1 cycles per second, using a Duddell vibration galvanometer and an arrangement of apparatus similar to that shown in Fig. 58. Since L is about 40 mH, $Q = \sqrt{L/2C}$ must be about 240 ohms to satisfy the sensitivity condition. Hence, Q was set to values of 200, 250, and 300 ohms; $R = S$ were kept at 100 ohms each. For each value of Q balance was found by adjusting successively the resistance r and the resistances P' in series with L . After each balance the ratio coils were reversed in the bridge, but it was not found that any change in balance was caused thereby. The observations are given in the table—

Q ohms.	r ohms.	P' ohms.	L mH.	Eff. res. of Coil ohms.
200	253.9 ₀	194.8 ₇	40.67 ₅	5.1 ₃
250	193.1 ₀	244.9 ₂	40.67 ₁	5.0 ₈
300	152.6 ₇	294.9 ₁	40.68 ₈	5.0 ₉
		<i>Average</i>	40.67 ₈ mH.	5.1 ₀ ohms

19. Sources of Error in Anderson's Method. Anderson's method is particularly applicable to precise measurements of inductance; as the method is largely used for such measurements, it is important to examine the various possible sources of error and to determine their effects upon observations obtained from the bridge.

Residual Inductance or Capacitance in the Resistances. The various resistances used to make up the bridge may have slight residual inductance or capacitance, and error in the measurement of L may be introduced

* *Loc. cit.*

thereby. If R and S be made equal, slight differences in their resistances or their residuals can be eliminated in the mean of two balances obtained by reversing their positions in the bridge. However, differences between P and Q can not be so eliminated, since these branches are not made up in a similar way; P is composed of the resistance of the coil to be measured and a resistance box, whereas Q is merely a resistance box. Hence only the non-inductive part of P can balance the residuals of a corresponding number of similar coils in Q . Moreover, the resistance of L is that of a copper coil, whereas the rheostats in P and Q are usually of manganin; hence there may be a drift in the balance due to temperature differences. Butterworth* has overcome the difficulty of balancing the residuals of P and Q , and also the effects of temperature by making up the branch Q of a rheostat exactly similar to that in P , together with a non-inductive copper coil equal in resistance to the inductance L . The latter and the balancing coil are enclosed in a wool-lined box so that they may have the same temperature.

The residual inductance of a resistance may be positive or negative, according to whether inductance or capacitance effects predominate. If l_p , l_q , l_R , l_S , and l_r be the residual inductances (plus or minus) of P , Q , R , S , and r , it is easy to prove† that

$$L = CQ \left[r \left(1 + \frac{R}{S} \right) + R \right] + \frac{1}{S} [l_R Q - l_P S - l_S P + l_Q R] \quad . \quad . \quad . \quad (c)$$

$$- \frac{\omega^2 C}{S} [r l_Q (l_R + l_S) + Q (l_R l_S + l_S l_r + l_r l_R) + S l_Q (l_R + l_r) + R l_Q (l_S + l_r)]$$

Comparing the first term with Equation (a) it will be seen that it represents the value obtained for the inductance when all the resistances are assumed perfect; call it L_0 . Then, if α and β be written for the corrections introduced by the residuals,

$$L = L_0 + \alpha - \beta.$$

It is found in practice that β is negligible, except at the highest frequencies; hence, since β alone contains l_r , a small residual in r does not introduce any important error. The correction α includes two terms, namely, $(l_Q R - l_P S)$ and $(l_R Q - l_S P)$. If R and S be made equal and reversed, the second term is eliminated from the mean, leaving a correction $\alpha = l_Q - l_P$. Hence the make-up of Q and P should be as nearly the same as possible, so that α is a very small quantity.

Rosa and Grover have calculated the magnitude of the errors to be expected, the results being tabulated on opposite page.

The preceding theory shows that—

- (i) Residuals in R and S are eliminated by making them equal and reversing them in the bridge.
- (ii) Small residual inductance in r produces an inappreciable error.

* S. Butterworth, "On the use of Anderson's bridge for the measurement of the variations of the capacity and effective resistance of a condenser with frequency," *Proc. Phys. Soc.*, Vol. 34, pp. 1-7 (1921).

† See Rosa and Grover, *loc. cit.*, pp. 304-311. The student should deduce it from the equations on p. 46.

(iii) The difference between the residuals of P and Q should be made as small as possible—especially when measuring low inductances—by making up these branches of similar coils.

Inductance to be Measured.	C	$R=S$	L_R	L_S	$P=Q$	L_P	L_Q	r	α
Milli- henrys.	Micro- farads.	Ohms.	Micro- henrys.	Micro- henrys.	Ohms.	Micro- henrys.	Micro- henrys.	Ohms.	Milli- henrys.
100	1.0	250	+ 2	- 2	250	- 2	+ 2	75	0.008
10	0.4	100	+ 1	- 1.0	100	- 1	+ 1	75	0.004
1	0.1	50	+ 0.5	- 0.5	100	- 1	+ 1	25	0.004
0.1	0.05	20	+ 0.5	- 0.5	50	- 0.5	+ 0.5	10	0.0045
0.01	0.02	20	+ 0.5	- 0.5	20	- 0.5	+ 0.5	2.5	0.002

Experiments are also given to verify the results predicted by the theory. From this table it is clear that large errors may be introduced into the measurement of small inductances, unless the resistances are wound so that they have the smallest possible residuals. If the residuals of the rheostat in series with the inductive coil be known, accuracy can be attained by balancing the bridge with Q , S , and R fixed in value, (i) when the inductance is in place, and (ii) when it is removed and its resistance compensated for by a coil of known residual inductance. The difference then gives the inductance of the coil to be measured.

Errors Due to the Leads. The inductance or capacitance of the leads connecting L to the bridge will be included in the measured value of the inductance. In precise work, it is necessary to allow for the effect of the leads and to arrange them so that they introduce only a very small error (see p. 284).

Self Capacitance of L . As shown on p. 188, the self capacitance of the inductance under test will affect the measured value by an amount depending on the true value of the inductance, its self capacitance, and the square of the frequency. In accurate work, it is necessary to make some allowance for the effect, more especially at high frequencies. For example, in a certain coil which has a true inductance of 1 henry and a capacitance of 10^{-10} farad, the measured value at 112 cycles per second will be 1.00005 henry, which is an inappreciable correction. But if the frequency is 1,120, then the measured value becomes 1.005 henry, which is an important correction.

Imperfection of the Condenser C . In the above discussion, the condenser has been assumed perfect, i.e. its insulation resistance has been taken as infinite, and the current through the condenser has been assumed to lead on the applied potential difference by exactly a quarter of a period. Now, in practice, the insulation resistance of a condenser, though usually very high, is certainly not infinitely great. Moreover, in condensers which have solid dielectrics there is always a certain amount of energy absorption, so that the phase displacement is less than $\pi/2$. It is necessary, therefore, to examine the errors which these two factors will introduce.

Taking first the effect of insulation resistance, a leakage across the

condenser can be represented by a resistance, R_l , in parallel with the condenser; R_l is seldom less than 1,000 megohms in a good mica condenser. In the analysis preceding Equation (a) substitute for $z_1 = 1/j\omega C$ the term $R_l/(1 + j\omega CR_l)$. Then the balance equation is

$$R_l\{SP - QR + j\omega LS\} = Q\{r(S + R) + SR\} \cdot \{1 + j\omega CR_l\}.$$

Separating components gives

$$SP = QR + \frac{Q}{R_l}\{r(S + R) + SR\} = QR + \frac{L_0 S}{CR_l}$$

$$\text{and} \quad L = CQ \left[r \left(1 + \frac{R}{S} \right) + R \right] = L_0.$$

The second balance condition is identical with that obtained when the condenser is perfect; hence *leakage has no effect upon the measured value of inductance, and only a small effect on the measured value of P .*

The phase displacement due to the absorption of energy in an imperfect dielectric can be accounted for by a resistance ρ in series with the condenser. In a good mica condenser the phase displacement will fall short of 90 degrees by a very small angle, so that ρ is a small resistance. In the analysis before Equation (a), put $z_1 = \rho + \frac{1}{j\omega C}$; the balance equation is

$$(1 + j\omega C\rho)[SP - QR + j\omega LS] = j\omega CQ[r(S + R) + SR].$$

Separating the components makes

$$SP - QR = \omega^2 LCS\rho$$

$$\text{and} \quad L = CQ \left[r \left(1 + \frac{R}{S} \right) + R \right] - C\rho(SP - QR)$$

In the second equation, the first term is the value of the measured inductance when the condenser is perfect, namely, L_0 . Substituting from the first equation in the last term of the second, gives

$$L = L_0 - \omega^2 C^2 \rho^2 L.$$

Now the angle by which the phase displacement falls short of $\pi/2$ is $\theta = \tan^{-1} \omega C\rho$ (see p. 116) so that

$$L_0 = L(1 + \tan^2 \theta).$$

The angle θ is usually small, so that very nearly

$$L = L_0(1 - \tan^2 \theta).$$

If in a good mica condenser θ were half a minute of arc, the correction would only amount to 2 parts in 10^8 ; it would be allowable for θ to attain a value of 5 or 6 minutes without producing any marked error. Hence, *absorption in the dielectric has usually a negligible effect.*

20. Special Uses of Anderson's Method. As described above, Anderson's method has been treated as a means of measuring self-inductance in terms of capacitance. As such, it has a very large range, from the smallest to the largest inductance, depending entirely upon the choice of the various branches of the network.

By a suitable arrangement of Anderson's bridge, the very small inductance of resistance coils can be measured.* It is essential in this case to make some allowance for the residual effects in the bridge itself, these being of a magnitude comparable with that of the inductance under test. The procedure is as follows. A standard is first prepared having a resistance as nearly as possible equal to that of the coil to be tested; this standard is made in a form such that its residuals may be calculated (*see* p. 79). Inserting this standard in AC (Fig. 58) in series with a rheostat, and, if necessary, an auxiliary inductance (so that r may have a reasonable value), let balance be obtained by adjustment of P and r . Substituting the unknown for the standard, re-balance by adjustments of P and r as before. If Δr be the change of r and Δl the alteration of inductance of the rheostat in AC , it is easy to show that

$$\Delta L = 2 C Q \Delta r - \Delta l, \dagger$$

where ΔL is the difference between the inductance of the standard and the unknown, the bridge having equal ratio branches, $R = S$. Residual errors are thus entirely eliminated.

If a standard self-inductance is available, Anderson's method becomes a very convenient way to measure a condenser with high precision. Butterworth \ddagger has recently shown that the variation of capacitance and effective series resistance of a condenser with frequency can be determined from tests made in a specially arranged Anderson bridge.

21. Butterworth's Method.§ A bridge which is very suitable for the measurements of small inductances is shown in Fig. 57 (e), embodying the features of Iliovici's method with those of the conjugate to Anderson's method. The trouble experienced in Iliovici's bridge is that the tapping for the resistance r , by which inductive balance is secured, is made upon one of the resistances entering into the resistance balance, namely, R . Hence R is preferably a slide wire if rapid work is to be done. The use of a slide wire can be avoided if the Anderson principle be adapted to the bridge, so that R may be an ordinary resistance box.

By putting cyclic currents in the four meshes of the network

* F. W. Grover and H. L. Curtis, "The measurement of the inductance of resistance coils," *Bull. Bur. Stds.*, Vol. 8, pp. 461-462 (1913). *See also* A. H. Taylor and E. H. Williams, "Distributed capacity in resistance boxes," *Phys. Rev.*, Vol. 26, pp. 417-423 (1908).

† The student should verify this result by reference to Equation (c), p. 220.

‡ S. Butterworth, "On the use of Anderson's bridge for the measurement of the variations of the capacity and effective series resistance of a condenser with frequency," *Proc. Phys. Soc.*, Vol. 34, pp. 1-7 (1921). *See also* B. V. Hill, "The variation of apparent capacity of a condenser with the time of discharge and the variation of capacity with frequency in alternating current measurements," *Phys. Rev.*, Vol. 26, pp. 400-405 (1908).

§ S. Butterworth, "A method of measuring small inductances," *Proc. Phys. Soc.*, Vol. 24, pp. 210-214 (1912).

and solving the resulting equations, no current will flow in the detector when

$$SP = QR,$$

$$\text{and } L = \frac{(R-r)}{S} [(Q+S)U + Q(S+r)]C.$$

Thus, by choice of $(R-r)$, inductances of any value can be measured by means of a single condenser; and also inductive balance is secured independently of resistance balance by adjustment of U .

To determine the best conditions for working the bridge, let V be the total resistance of the source between the branch points A and B , and G that of the detector, reactances being neglected. Then Butterworth has shown that the condenser has little influence on the sensitivity, and that the best values of the branch resistances are approximately

$$S = \sqrt{VG}, \quad Q = \sqrt{PG \cdot \frac{P+V}{P+G}} \text{ and } R = \sqrt{PV \cdot \frac{P+G}{P+V}},$$

where U is included in V and, for greatest sensitivity, should be kept small (see page 54).

In practice, R and S should be set to the values for maximum sensitivity, suitable values of P , C , and r being chosen. Balance should then be obtained by successive adjustments of Q and U . If a vibration galvanometer be used, the frequency should be low.

Experimental Example. The following results were obtained at a frequency of 407.1 cycles per second, using a Duddell vibration galvanometer. The coil L was joined in series with a resistance box, but in three tests P consisted of the resistance of the coil alone. S was fixed at 1,000 ohms throughout; R consisted of a 4.08₁ ohm slide wire in series with a resistance box set to some definite value. The position of the slider was set to give a desired value of r . Balance was found by successive alterations of Q and U , using a condenser of 0.1010 μF . The results are tabulated below—

R ohms.	r ohms.	P ohms.	Q ohms.	U ohms.	L $\mu\text{H.}$
4.08 ₁	2.72 ₁	0.64 ₈	157	188	51.4 ₄
4.08 ₁	3.49 ₈	0.64 ₈	157	615	51.1 ₅
4.08 ₁	3.49 ₈	1.64 ₈	403	332	51.2 ₆
14.08 ₁	12.72 ₁	0.64 ₈	45.9	315	51.5 ₈
				Average	51.4 ₈ $\mu\text{H.}$

By this method, Butterworth has been able to measure a coil of 20 microhenrys with an agreement among several observations within 1 per cent of the mean at a frequency of 100, C being 0.1 microfarad. A Duddell vibration galvanometer for which $G \approx 200$ ohms was used.

22. Hay's Method for Large Inductances. When the time-constant of a coil is large it becomes a matter of difficulty to measure the inductance and effective resistance of the coil in an ordinary inductance bridge. For example, certain coils, called "loading coils," are used in telephony for the purpose of improving speech transmission. These loading coils are constructed so that they have a large inductance, although the amount of wire in them, and therefore the resistance, is maintained small. In a typical case, a coil which had an inductance of about 140 millihenrys had a resistance of only 6 ohms, the time-constant being about $1/40$ second. Coils of much higher inductance, sometimes up to 40 henrys, are constructed for use in cable telegraphy, and have even larger time-constants.*

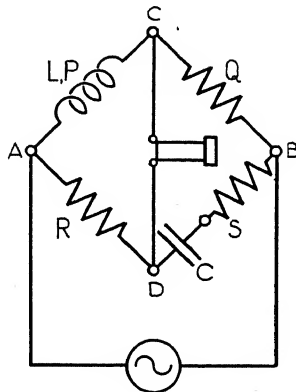


FIG. 59.—HAY'S METHOD FOR MEASURING LARGE INDUCTANCES

C. E. Hay† has devised a simple method for the determination of the inductance and effective resistance of loading coils, the arrangement of the bridge being shown in Fig. 59. The branch AC contains the coil to be tested, CB and AD being composed of non-inductive resistances. The resistance S should be capable of fine adjustment, as also should the condenser C . The latter should be composed of mica condensers with a continuously variable air condenser in parallel with them for attaining final balance.

* In contrast, it should be noted that reactance coils used in power stations have time-constants as high as 1 second.

† C. E. Hay, "Measurement of self-inductance and effective resistance at high frequencies," *Elec. Rev.*, Vol. 67, pp. 965-966 (1910).

Taking the branch impedances as $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S + \frac{1}{j\omega C}$, and $z_4 = R$, the balance condition will be

$$(P + j\omega L) \left(S + \frac{1}{j\omega C} \right) = QR;$$

or

$$L/C = (QR - SP),$$

$$LC = P/\omega^2 C.$$

Solving these equations for L and P gives

$$L = QR \frac{C}{(1 + \omega^2 C^2 S^2)}, \quad P = QR \frac{\omega^2 C^2 S}{(1 + \omega^2 C^2 S^2)}, \quad L/P = 1/\omega^2 CS.$$

In practice, Q is fixed; R , S , and, if necessary, C also, are varied until balance is secured. Since the results involve ω , the frequency must be maintained constant and be accurately measured. If telephones be used to detect balance, the wave form must be pure. Insulation of the whole apparatus should be very good; and capacitance errors should be avoided by suitable screening of the branches, and by taking the bridge current from a well-insulated transformer with an earthed screen between the primary and secondary windings.

Experimental Example. Two sets of tests were made, (i) on a 40 mH. coil of d.c. resistance 5.1, ohm at 15° C., and (ii) on a short thick coil of 0.6 henry and 54.1 ohm d.c. resistance at 15° C. In both cases a Duddell vibration galvanometer was used, current being supplied to the bridge at 407.1 cycles per second from a valve oscillator and screened transformer. L , P is the coil under test, Q a fixed resistance; S and R are each composed of a resistance box in combination with a 1 ohm rheostat. C is a mica condenser of 0.334, μ F. capacitance; its series loss resistance of 0.30 ohm is included in the tabulated values of S .

(i) 40 mH. COIL

Q ohms.	R ohms.	S ohms.	L mH.	P ohms.
400	304.6 ₅	58.4 ₅	40.67 ₄	5.2 ₀
500	243.7 ₃	58.4 ₅	40.67 ₅	5.2 ₀
600	203.0 ₈	58.4 ₅	40.67 ₁	5.2 ₀
700	174.0 ₈	58.4 ₅	40.66 ₇	5.2 ₀
		Average	40.67 ₂ mH.	5.2 ₀ ohms.

The time-constant is thus 0.0078 second.

(ii) 0.6 HENRY COIL

Q ohms.	R ohms.	S ohms.	L henry.	P ohms.
700	2599	71.5 ₀	0.606 ₅	94.9 ₃
800	2274	71.5 ₀	0.606 ₄	94.9 ₂
900	2021	71.5 ₀	0.606 ₃	94.9 ₁
1000	1819	71.5 ₀	0.606 ₂	94.9 ₁
		Average	0.606 ₄ henry	94.9 ₂ ohms

The time-constant of the coil is 0.0064 second. The very great difference between the effective resistance to a.c. and the d.c. resistance of this coil is particularly to be noticed.

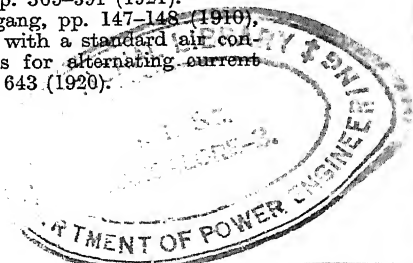
23. Series Condenser Methods. In the methods described above the inductance and condenser which are to be compared are situated in *different* branches of the network. A few bridges of minor importance will now be noticed, in which the inductance and condenser are put in series in the *same* branch of the network.

Haworth* has introduced a bridge for the measurement of the equivalent capacitance and series resistance of an electrolytic cell (Fig. 60). In practical working, C_x , P_x are respectively the equivalent capacitance and resistance to be determined; L_1 is a variable standard of self-inductance of the Ayrton-Perry type; Q , S , and r are non-inductive resistances, Q and S being made equal. To balance, the electrolytic cell is first short-circuited; the inductance L_2 is inserted in AD to balance the minimum reading of L_1 , and r is adjusted until the detector indicates zero. The cell is now inserted in AC , and balance is restored by adjustment of L_1 and r . Assuming $Q = S$, it is obvious that P_x is equal to the change in r , while $C_x = 1/\omega^2 L$, L being the change in value of L_1 . It is clear that the method is applicable to the measurement of the capacitance and series resistance of a condenser by using the same procedure.

Haworth's bridge is really a modification of the well-known resonance bridge† (Fig. 60). In this, three of the branches are non-inductive resistances. The fourth consists of a condenser and inductance in series. When L , C , or ω are adjusted so that the condenser and inductance are in resonance

* H. F. Haworth, "The measurement of electrolytic resistance using alternating currents," *Trans. Far. Soc.*, Vol. 16, pp. 365-391 (1921).

† Grüneisen and Giebe, *Zeits. f. Inst.*, 30 Jahrgang, pp. 147-148 (1910), have used the method to compare an inductance with a standard air condenser. See also D. I. Cone, "Bridge methods for alternating current measurements," *Journal Amer. I.E.E.*, Vol. 39, p. 643 (1920).



$\omega L = 1/\omega C$ and the branch AC is non-reactive. If then $SP = QR$ the bridge is balanced. The method can be used to measure with considerable accuracy and sensitiveness the effective resistance of a coil or condenser. In practice, Q and S are equal ratio coils and R is a fixed resistance. The branch AC contains the inductive coil L_1 , P_1 , in series with an adjustable self-inductance l of the Ayrton-Perry type, a

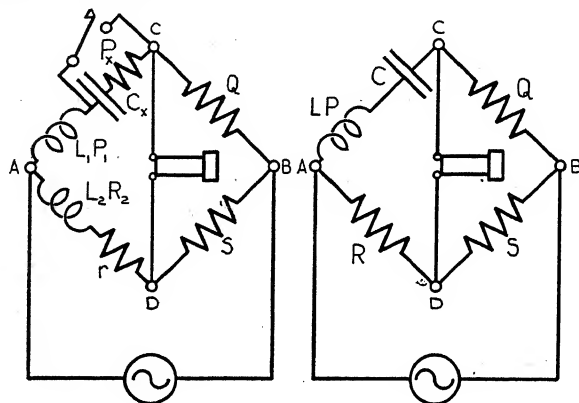
Haworth's Method.Resonance bridge.

FIG. 60.—SERIES CONDENSER METHODS FOR COMPARING SELF-INDUCTANCE AND CAPACITANCE; HAWORTH'S METHOD AND THE RESONANCE BRIDGE

resistance box r and the condenser C , the series loss resistance of the latter being ρ . With L_1 and C short-circuited, l and r are adjusted to values l_0 and r_0 to secure balance. (If l does not go down to sufficiently low values it may be necessary to insert a small balancing coil in AD , as in Haworth's modification.) L_1 and C are now unshorted and balance is restored by alteration of the adjustments to values l_1 , r_1 . Then

$$P_1 + \rho = r_0 - r_1$$

$$\text{and } (L_1 + l_1 - l_0)C\omega^2 = 1.$$

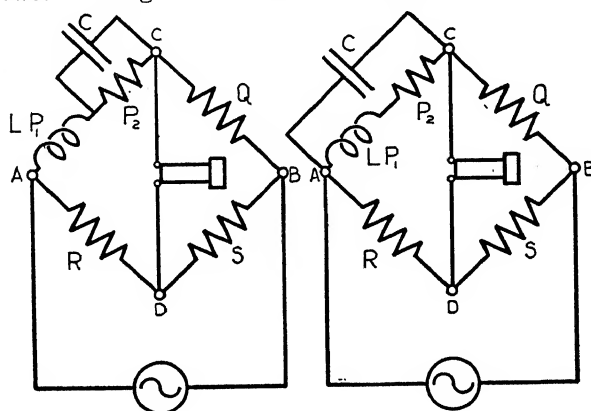
Thus if P_1 be known, then ρ can be found, and conversely.

In both these methods* it is essential that the frequency be

* For an additional modification applied to this bridge for tests on electrolytic cells, see A. H. Taylor, "A method for the determination of electrolytic resistance and capacity, using alternating currents," *Phys. Rev.*, Vol. 24, pp. 402-406 (1907). Balance is secured independently of frequency. Also see H. V. Higgitt, *Elec.*, Vol. 90, p. 114 (1923).

constant and known. A vibration galvanometer forms the best detector, though telephones can be used if the wave form of current in the bridge be pure.

Experimental Example. Following the procedure just described, the bridge was used to compare a coil and condenser at 407.1 cycles per second using a Duddell vibration galvanometer to detect balance. As the lowest reading of the variable inductance was 3 mH., a coil of



Pirani's Method.

Wien's Method.

FIG. 61.—PARALLEL CONDENSER METHODS FOR COMPARING SELF-INDUCTANCE AND CAPACITANCE; PIRANI'S AND WIEN'S METHODS

about 4 mH. was included in AD . Q and S were 100 ohms each, R was set at 101.2₁ ohms, and C was 2.103₈ μ F. The following readings were obtained. $r_0 = 90.33$ ohms, $l_0 = 4.35$ mH., $r_1 = 83.83$ ohms, $l_1 = 36.35$ μ H. Thus $L_1 + l_1 - l_0 = 1/C\omega_2 = 72.65$ mH., whence $L_1 = 40.6$ mH. Also $P_1 + \rho = 6.50$ ohms; since ρ was known to be 1.31 ohms, $P_1 = 5.1$ ohms.

24. Parallel Condenser Methods. The resonance principle can be applied to a bridge which has three non-inductive branches by a parallel connection of the inductance and condenser in the fourth branch.

One of the oldest methods in this category is Pirani's bridge (Fig. 61), originally devised for ballistic use. In Pirani's method* the inductance

* O. de A. Silva, "Sur la méthode de Pirani," *Écl. Élect.*, tome 50, pp. 113-116 (1907); C. H. Lees, "On Vaschy's or Pirani's method of comparing the self-inductance of a coil with the capacity of a condenser," *Phil. Mag.*, 6th series, Vol. 18, pp. 432-436 (1909); J. P. Kuenen, "On Pirani's method of measuring self-inductance," *Phil. Mag.*, 6th series, Vol. 19, pp. 439-441 (1910); E. C. Snow, "On Pirani's method of measuring the self-inductance of a coil," *Proc. Phys. Soc.*, Vol. 21, pp. 634-640 (1910).

is connected in series with a shunted condenser. When the constants of the various branches and the frequency are adjusted to values which will make the branch AC non-reactive and at the same time balance* the bridge, then

$$\frac{L}{C} = \frac{P_2^2}{(1 + \omega^2 C^2 P_2^2)} \text{ and } S \left[P_1 + \frac{P_2}{1 + \omega^2 C^2 P_2^2} \right] = QR.$$

The frequency must be accurately known when L and C are under comparison. With a known inductance and capacitance, the bridge can be used for frequency measurement. As in the case of Rimington's method (p. 214), it will only be possible to use the bridge ballistically if the resistance balance be obtained with direct current, i.e. if $S(P_1 + P_2) = QR$ be enforced, then $\omega = 0$ of necessity.

In a method introduced by Niven,† and used with alternating current by Max Wien,‡ the condenser is shunted across the branch AC as in Fig. 61. If $P = P_1 + P_2$ be the total resistance of AC , it is easy to prove that balance will occur when

$$LC = \frac{1}{\omega^2} \left(1 - \frac{SP}{QR} \right),$$

and

$$L/C = PQR/S.$$

From these equations L and C may be independently found in terms of resistance and frequency.‖ Balance is attained by adjustment of the rheostats in the four branches, such values being chosen as will make the bridge approximately of unity ratio, i.e. $P \approx R$, and $Q \approx S$.

In a practical example, Wien compares an inductance, nominally of 0.2 henry, with a condenser of about 1 microfarad. The frequency being 249 cycles per second, balance was secured when $P = 331.9$ ohms, $Q = 570$ ohms, $R = 23.503$ ohms, $S = 20.887$ ohms. Solving the above equation, it is found that $L = 0.1993$ henry and $C = 0.992$ microfarad.

Dongier§ has shown how the measurement of L in terms of C can be made by a combination of Pirani's and Wien's methods in which frequency is eliminated. Let Pirani's bridge be balanced, so that $L = CP_2^2/(1 + \omega^2 C^2 P_2^2)$, by adjustment of P_2 , R and S . Now change the connections of C so that it shunts the whole of the branch AC , as in Wien's method. Adjust P_2 , R , and S until balance is secured; then, if P be the resistance of AC , the above equations give $L = C(P^2 + \omega^2 L^2)$. Eliminating ω^2 gives $L = CPP_2$.

* J. P. Dalton, "On a new continuous balance method of comparing an inductance with a capacity," *Phil. Mag.*, 6th series, Vol. 27, pp. 37-44 (1914).

† C. Niven, "On some methods of determining and comparing coefficients of mutual induction," *Phil. Mag.*, 5th series, Vol. 24, pp. 225-238 (1887).

‡ Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 707-708 (1891); "Über einen Apparat zum variiren der Selbstinduction," *Ann. der Phys.*, Bd. 57, p. 257 (1896).

‖ For a modification of this method whereby balance is secured independently of frequency, see W. E. Forsythe, "On a method of comparing inductance and capacity," *Phys. Rev.*, series 2, Vol. 1, pp. 463-465 (1913).

§ R. Dongier, "Sur la mesure des coefficients de self-induction au moyen du téléphone," *Comptes Rendus*, tome 137, pp. 115-117 (1903).

Experimental Example. The following results were obtained at 407.1 cycles per second, using a Duddell vibration galvanometer and $C = 0.334_6 \mu\text{F}$. In *Pirani's method*, with $Q = 100$ ohms, balance was found by successive adjustments of P_2 , R , and S to the values $P_2 = 365.2_0$ ohms, $R = 337.8_8$ ohms, and $S = 99.9_0$ ohms. From these, $L = 40.65_4$ mH. and $P_1 = 5.2_1$ ohms. In *Wien's method*, balance was secured when $Q = 100$ ohms, $P_2 = 327.5_8$ ohms, $R = 363.0_8$ ohms, and $S = 99.4_4$ ohms. Taking $P_1 = 5.2_1$ ohms, $P = 332.7_0$ ohms, and hence $L = 40.65_6$ mH. Using this value of P and $P_2 = 365.2_0$ ohms in *Dongier's method* gives $L = 40.66_8$ mH.

25. Grover's and Owen's Methods. F. W. Grover* has described the method shown in Fig. 62, originally suggested by E. B. Rosa for the purpose of finding the power factor of a small imperfect condenser.† As shown on page 196, it is not possible to balance a simple De Sauty bridge when the condensers are imperfect—i.e. have dielectric losses—unless some means be provided to bring the potentials of the detector branch points into phase. Grover's method effects this by means of inductances in place of the resistances of the series resistance method. Since the condensers which are to be compared stand alone in the branches CD and BD , and balance is secured by adjustment of the remaining branches, the process of balancing does not affect the capacitance of the condenser branches. Hence the method is suitable for tests on small condensers.

Referring to Fig. 62 (*a*), L_1 and L_2 are standard inductances, either or both being of the variable pattern. If necessary, they are each connected in series with a suitable non-inductive rheostat. C_1 and C_2 are the unknown and standard condensers, ρ_1 and ρ_2 being their equivalent series resistances. Let balance be secured by adjustment of P or R and L_1 or L_2 . The branch impedance operators being $z_1 = P + j\omega L_1$, $z_2 = \rho_1 - \frac{j}{\omega C_1}$, $z_3 = \rho_2 - \frac{j}{\omega C_2}$, $z_4 = R + j\omega L_2$, balance will be secured when

$$(P + j\omega L_1) \left(\rho_2 - \frac{j}{\omega C_2} \right) = (R + j\omega L_2) \left(\rho_1 - \frac{j}{\omega C_1} \right).$$

* F. W. Grover, "Simultaneous measurement of the capacity and power-factor of condensers," *Bull. Bur. Stds.*, Vol. 3, pp. 389-393 (1907). H. L. Curtis, "Mica condensers as standards of capacity," *Bull. Bur. Stds.*, Vol. 6, pp. 436-438 (1910). F. W. Grover, "The capacity and phase difference of paraffined paper condensers as functions of temperature and frequency," *Bull. Bur. Stds.*, Vol. 7, pp. 498-499 (1911).

† For application of this method at low frequency and at voltages up to 10,000, see C. A. Butman, "Flexible and accurate method for dielectric tests," *Elec. World*, Vol. 71, pp. 502-506 (1918).

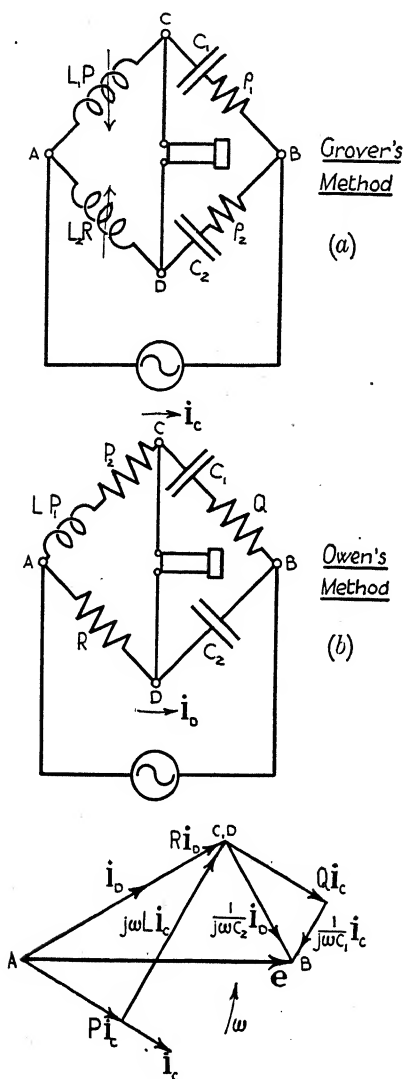


FIG. 62.—GROVER'S METHOD FOR COMPARING TWO CONDENSERS, AND OWEN'S METHOD FOR THE MEASUREMENT OF SELF-INDUCTANCE

Separating components, the two conditions for balance are

$$\frac{C_1}{C_2} = \frac{R}{P} + \frac{\omega^2 C_1}{P} (L_1 P_2 - L_2 P_1)$$

$$\text{and } R P_1 - P P_2 = \frac{L_1}{C_2} - \frac{L_2}{C_1}$$

It is possible to show that if L_1 , L_2 are not unduly large, the correction due to the second term in the first of these equations is very small, so that $C_1/C_2 = R/P$ to a high degree of approximation. Substituting this condition in the second equation, and remembering that the phase difference of an imperfect condenser is $\omega C P$ (see p. 116), gives

$$\tan \theta_1 - \tan \theta_2 = \omega \left(\frac{L_1}{P} - \frac{L_2}{R} \right)$$

Since θ_1 and θ_2 are small, this can be written as

$$\tan (\theta_1 - \theta_2) = \tan \phi_P - \tan \phi_R,$$

ϕ_P and ϕ_R being the phase angles of the branches AC and AD . Hence the phase-difference of the unknown condenser is given in terms of the known phase-difference of the standard.

This method is most conveniently used as a substitution method. The bridge is balanced with the unknown condenser C_1 and a standard C_2 ; the unknown being removed, an auxiliary standard C'_1 is

inserted in its place and balance restored by adjustment of L_1 and P , all else remaining unaltered. If θ'_1 be the phase difference of the auxiliary standard, L'_1 and P'_1 the balance values, then

$$\tan \theta_1 - \tan \theta'_1 = \omega \left(\frac{L_1}{P} - \frac{L'_1}{P'} \right) = \tan (\theta_1 - \theta'_1).$$

By this process, residual errors in the various branches and errors due to earth capacities from the bridge to earth are eliminated. Settings of great precision can be attained.

The method is most suitable for tests on condensers smaller than 0.01 microfarad at frequencies over 100 cycles per second, the difference between the unknown and the auxiliary condenser, $\theta_1 - \theta'_1$, being less than 1 degree.

D. Owen* has recently shown that Grover's bridge can be modified in such a way that it becomes a very useful means of measuring self-inductance in terms of standard condensers. Referring to Fig. 62 (b), the inductance to be tested is put in the branch AC in series with an adjustable non-inductive resistance P_2 , the total resistance of the branch being $P = P_1 + P_2$. The branch AD consists of a second non-inductive resistance, R ; CB contains a condenser and a resistance box. The fourth branch is composed of a standard condenser C_2 , assumed to be perfect. Comparing this diagram with that for Grover's bridge, it will be seen that they are very similar; in fact, $P = P_1 + P_2$, $\rho_1 = Q$, $\rho_2 = 0$, $L_2 = 0$ in Grover's network will convert it into that of Owen. Putting these values in the balance equations gives

$$C_1(P_1 + P_2) = C_2R,$$

$$\text{and} \quad L_1 = C_2QR$$

for balance.

The practical procedure is as follows: C_1 , C_2 , and R being fixed, balance is attained by successive adjustments of P_2 and Q . The process is easy, since the two conditions of balance are independent and do not depend on frequency or wave form. An Experimental Example is given on page 234.

The vector diagram for the balanced bridge is very simply constructed. The current i_b in the branches ADB leads on the voltage applied to AB by an angle $\tan^{-1}(1/\omega C_2 R)$, the two components of e being Ri_b and $(1/j\omega C_2)i_b$. The vector Ri_b is also equal to the potential difference between A and C , namely

* D. Owen, "A bridge for the measurement of self-induction in terms of capacity and resistance," *Proc. Phys. Soc.*, Vol. 27, pp. 39-55 (1915).

$(P + j\omega L)\mathbf{i}_c$, where \mathbf{i}_c is displaced from \mathbf{e} by an angle $\tan^{-1} \left[\left(\omega L - \frac{1}{\omega C_1} \right) / (P + Q) \right]$. Similarly, the vector $(1/j\omega C_2)\mathbf{i}_d$ is equal to $\left(Q - \frac{j}{\omega C_1} \right)\mathbf{i}_c$. The above balance conditions then immediately follow from the geometry of the triangles in the vector diagram.

Experimental Example. In a test at 407.1 cycles per second, the following values were obtained. The condensers had values $C_2 = 0.334_\mu\text{F}$. (mica standard) and $C_1 = 0.400_\mu\text{F}$. (paper). The latter had a series loss resistance of 5.20 ohms, which is included in the tabulated values of Q . A Duddell vibration galvanometer was used.

R ohms.	P_2 ohms.	Q ohms.	L mH.	P_1 ohms.
100	78.3 ₉	1216.2	40.69 ₄	5.1 ₈
200	161.9 ₆	608.0	40.68 ₇	5.1 ₇
300	245.5 ₂	405.3	40.68 ₅	5.1 ₉
		<i>Average</i>	40.68 ₉ mH.	5.1 ₈ ohms

There are a variety of residual effects which may cause error in tests of high precision, especially on small inductances. Such errors can be very nearly eliminated by making use of an auxiliary balance with the coil L removed, as will now be shown.

The sources of error are, briefly—

(i) In AC ; residual inductance in P_2 and in the leads joining L to the bridge. These modify the operator for the branch to the value $z_1 = P + j\omega(L + l_p)$ where l_p is the residual.

(ii) In CB ; residual inductance l_q in Q and in the leads joining C_1 to the bridge, also loss resistance ρ_1 in the condenser. The operator is $z_2 = Q + \rho_1 + j \left(\omega l_q - \frac{1}{\omega C_1} \right)$.

(iii) In BD ; residual inductance of the leads joining C_2 to the bridge, l_2 . Resistance of these leads and absorption in the condenser represented together by ρ_2 , make the operator $z_3 = \rho_2 + j \left(\omega l_2 - \frac{1}{\omega C_2} \right)$.

(iv) In DA ; residual inductance l_R , making $z_4 = R + j\omega l_R$.

Using these values to calculate the balance conditions, it is easy to show that

$$\begin{aligned} R/C_1 &= (P/C_2) + \omega^2[l_R(Q + \rho_1) + l_q R - (L + l_p)\rho_2 - l_2 P] \\ C_2 Q R &= L + l_p + C_2(\rho_2 P - \rho_1 R) - l_R(C_2/C_1) + \omega^2 C_2[l_q l_R - l_2(L + l_p)] \end{aligned}$$

in general.

Balance is obtained by adjustment of P_2 and Q . Let the coil L be short-circuited. Let the values of P and Q requisite for balance be P' and Q_0 , l'_P and l'_Q being the new residuals. Then, from the second condition,

$$C_2 Q_0 R = l'_P + C_2(\rho_2 P' - \rho_1 R) - l_R(C_2/C_1) + \omega^2 C_2[l'_Q l_R - l_2 l'_P].$$

Subtracting this from the equation obtained with L in place,

$$C_2 R(Q - Q_0) = L + (l_P - l'_P) + C_2 \rho_2(P - P') + \omega^2 C_2[l_R(l_Q - l'_Q) - l_2(l_P - l'_P)].$$

Now in most cases the third and fourth terms on the right-hand side are of negligible value, so that

$$L = C_2 R(Q - Q_0) - (l_P - l'_P).$$

The correction term is easily allowed for if P_2 be a rheostat of calculable inductance. If P_2 be of constant inductance, then the correction is zero. In any case, if the time constant of L be $> 10^{-4}$ the correction is negligible. It is essential, however, both to make this correction and also to make the auxiliary balance when small inductances are being measured.

In practice, Owen has shown that the range of the bridge is very wide even with limited apparatus. Thus, with C_1 and C_2 each equal to $\frac{1}{2}$ microfarad, and values of R ranging from 1 to 200 ohms, a range of inductance between 2 microhenrys and 0.5 henry can be satisfactorily covered. P_2 and Q should each contain fine adjustment—preferably constant inductance—rheostats. The bridge is extremely convenient to use, Turner* having devised a self-contained set for rapid measurements of inductance between 1/10 microhenry and 1/10 henry.

NETWORKS CONTAINING RESISTANCE, SELF-INDUCTANCE, AND MUTUAL INDUCTANCE

26. Felici's Method for Comparison of Two Mutual Inductances. The most direct way to compare two mutual inductances is by means of the arrangement shown in Fig. 63 (a). This circuit was used by R. Felici,† in 1852, as a null method for demonstration of the laws of mutual inductance, and is mentioned in this connection by Maxwell.‡ Heaviside,§ in 1886, drew attention to the excellence of the arrangement as a mutual inductance balance, using a telephone and interrupted

* L. B. Turner, "Everyday measurements of inductance and capacity in the wireless laboratory," *Rad. Rev.*, Vol. 1, pp. 585-590 (1920). The apparatus is made by Messrs. Tinsley & Co. and was devised for use in the Services.

† R. Felici, "Mémoire sur l'induction électrodynamique," *Annales de Ch. et Ph.*, tome 34, 3rd series, pp. 64-77 (1852). "Nota sopra una osservazione del sig. A. De La Rive ad una esperienze fondamentali della teoria del l'induzione elettro-dinamica," *Nuovo Cimento*, tome 9, pp. 345-347 (1859).

‡ *Treatise*, 1st Edn., Vol. 2, Sec. 536, pp. 168-170 (1873).

§ *Electrical Papers*, Vol. 2, p. 110 (1892).

current. Finally, A. Campbell,* in 1908, adapted the circuit to alternating current and a vibration galvanometer.

Let M_2 be a mutual inductance standard which can be continuously adjusted; and let M_1 be the mutual inductance to be tested, its value being not greater than the maximum

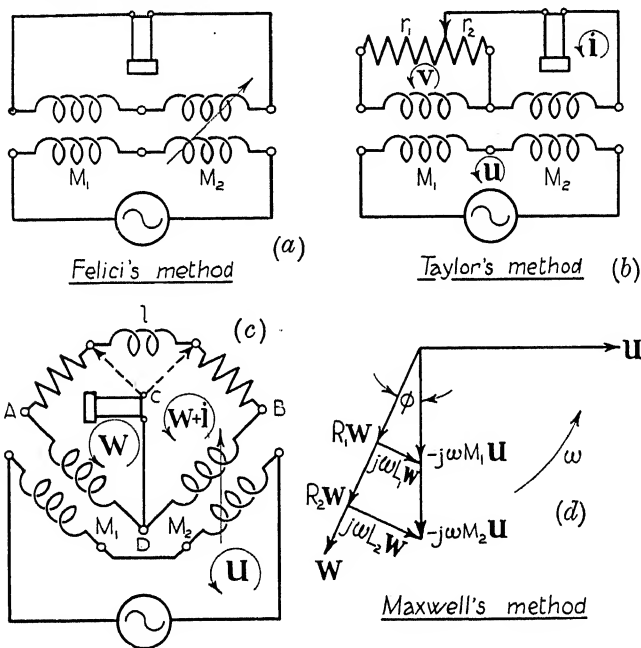


FIG. 63.—METHODS FOR THE COMPARISON OF TWO MUTUAL INDUCTANCES

obtainable in M_2 . Connect the primaries of the two inductances in series to a source of alternating current, and join the secondaries, with their windings in opposition, in series with a telephone or vibration galvanometer. Then, obviously, when no current flows through the detector,

$$M_1 = M_2$$

numerically, M_2 being adjusted until the indication of the detector is zero.

* A. Campbell, "On the use of variable mutual inductometers," *Proc. Phys. Soc.*, Vol. 21, pp. 74-75 (1910); *Phil. Mag.*, Vol. 15, 6th series, pp. 155-171 (1908).

Experimental Example. In an experiment at 407.1 cycles per second with a Duddell vibration galvanometer as detector, M_1 was a fixed mutual inductance of 0.01 henry nominal value. M_2 was a Campbell mutual inductometer of 11 mH. range. The reading at balance was 984₈ microhenrys, so that $M_1 = 0.00984_8$ henry.

This method has the advantage of quickness, one adjustment serving to secure balance; it is, moreover, not necessary to know the values either of the self-inductances or the resistances of the coils. The value of M_1 which can be measured is, however, limited by the range of the standard M_2 .

At high frequencies, the method is sometimes complicated by the effects of self capacitance and eddy currents in one or both of the mutual inductances being compared. Campbell* has shown that self capacitance of the secondary coil of the unknown inductance may be represented by a condenser C_1 connected across it, and can be balanced by joining an adjustable condenser C_2 in parallel with the secondary of M_2 . By successive adjustment of M_2 and C_2 balance is secured. If L_1, R_1, L_2, R_2 be the inductance and resistance of the secondary coils of the unknown and the inductometer, balance will occur when

$$M_1/M_2 = R_1 C_1 / R_2 C_2 = (1 - L_1 C_1 \omega^2) / (1 - L_2 C_2 \omega^2)$$

$$\text{or very nearly} \quad M_1 = M_2 \left\{ 1 - \omega^2 C_2 \left(\frac{L_1 R_2}{R_1} - L_2 \right) \right\}$$

Campbell also shows how to deal with the impurity effect due to eddy currents.

27. Taylor's† Modification of Felici's Method. If the mutual inductance M_1 be greater than that of the standard M_2 , a modification of the above method, shown in Fig. 63 (b), may be used. The standard, M_2 , may now be fixed in value.

Let a resistance be connected across the coil M_1 , the point of connection of the detector dividing the resistance into two parts, r_1 and r_2 . Let z_1 be the impedance operator of the secondary of M_1 and z_2 of M_2 , z_g being the operator for the detector; then the mesh equations are

$$(z_2 + z_g + r_2)\mathbf{i} - r_2\mathbf{v} = -j\omega M_2\mathbf{u},$$

$$-r_2\mathbf{i} + (z_1 + r_1 + r_2)\mathbf{v} = j\omega M_1\mathbf{u},$$

remembering that the secondaries are in opposition. If there is to be no current in the detector, then

$$\begin{vmatrix} -M_2 & -r_2 \\ M_1 & z_1 + r_1 + r_2 \end{vmatrix} = 0,$$

so that

$$M_1 r_2 - M_2 (z_1 + r_1 + r_2) = 0.$$

Now $z_1 = R_1 + j\omega L_1$, giving for the balance conditions

$$M_2 L_1 = 0,$$

$$M_1 r_2 = M_2 (R_1 + r_1 + r_2).$$

* *Dictionary of Applied Physics*, Vol. 2, pp. 423-424 (1922). Also *Proc. Roy. Soc. A.*, Vol. 87, p. 397 (1912).

† A. H. Taylor, "On the comparison of mutual inductances," *Phys. Rev.*, Vol. 20, pp. 393 (1905).

The first condition implies that $L_1 = 0$; hence balance cannot be secured unless the self-inductance of the secondary coil of M_1 is very small. The second condition is

$$M_1 = \frac{(R_1 + r_1 + r_2)}{r_2} M_2.$$

Taylor's arrangement consists of a high resistance, say 5,000 to 100,000 ohms, depending on the value of R_1 , connected across the secondary of M_1 . Assuming L_1 to be negligible, and that R_1 is small compared to $r_1 + r_2$, balance can be very nearly secured when

$$M_1 = \frac{r_1 + r_2}{r_2} M_2,$$

which is Taylor's expression.

In a similar way, the method can be used to reduce the effect of a fixed standard mutual inductance of too great a value.

Experimental Example. The mutual inductance M_1 was that tested by Felici's method (p. 237), the frequency and detector remaining the same. M_2 was a 1.1 mH. Campbell mutual inductometer. The value of $r_1 + r_2$ was fixed at 2,000 ohms; as the resistance and inductance of the coil shunted by it was very small, a good balance was secured with $r_2 = 200$ ohms and $M_2 = 985 \mu\text{H.}$, so that $M_1 = 0.00985$ henry. In a second trial with $r_1 + r_2 = 4,000$ ohms, $r_2 = 400$ ohms, M_2 was again $985 \mu\text{H.}$ leading to the same value for M_1 as before.

28. Maxwell's Method for Comparison of Two Mutual Inductances. The method described on page 236 is limited in its range to the maximum value of the available standard mutual inductance; that discussed on page 237, while effectively extending the range of the standard, is only capable of giving approximate balance. Maxwell* has introduced a ballistic method for comparing two mutual inductances which is free from both these defects. The network, as arranged for use with alternating current by A. Campbell,† in 1908, is shown in Fig. 63 (c).

The secondaries of the two mutual inductances are connected in series with resistances as shown, and, in addition, an adjustable self-inductance l is arranged so that it can be included in ACD or BCD at will. Care should be taken in arranging the network that all these inductive coils are placed

* *Treatise*, 1st Edn., Vol. 2, Sec. 755, p. 354 (1873). M. Brillouin, "Sur les méthodes de comparaison des coefficients d'induction," *Comptes Rendus*, tome 93, pp. 1010-1014 (1881); "Comparaison des coefficients d'induction," *Ann. de l'Ecole normale*, tome 11, pp. 339-424 (1882).

† A. Campbell, *loc. cit.*, pp. 73-74, "Inductance measurements," *Elecn.*, Vol. 60, pp. 626-627 (1908). Also see H. V. Carpenter, *Phys. Rev.*, Vol. 10, p. 52 (1900), for the adaptation of the conjugate to this method for the measurement of self-inductances.

well apart, so that there can be no interference between them. Suppose R_1 and L_1 are the total resistance and self-inductance of the branch DAC , and R_2, L_2 the corresponding values in the branch DBC ; then, if z_G be the impedance operator for the galvanometer branch CD , the two mesh equations are

$$\begin{aligned} -z_G \mathbf{i} + (R_1 + j\omega L_1) \mathbf{w} &= -j\omega M_1 \mathbf{u}, \\ (R_2 + j\omega L_2 + z_G) \mathbf{i} + (R_2 + j\omega L_2) \mathbf{w} &= -j\omega M_2 \mathbf{u}; \end{aligned}$$

so that for $\mathbf{i} = 0$

$$\begin{vmatrix} M_1 & R_1 + j\omega L_1 \\ M_2 & R_2 + j\omega L_2 \end{vmatrix} = 0.$$

Evaluating and comparing components,

$$\frac{M_1}{M_2} = \frac{R_1}{R_2} = \frac{L_1}{L_2}.$$

are the balance conditions. Hence, the rheostats and the mutual inductance standard must be adjusted to secure balance, while at the same time l is altered to keep the ratio $L_1/L_2 = M_1/M_2$ (see also page 53).

When the current \mathbf{i} is zero, the mesh equations given above admit of simple graphical interpretation as shown in Fig. 63 (*d*). In this the vector diagram of currents and electromotive forces in the balanced network is drawn, the impedance drop round each mesh being equated to the electromotive force of mutual induction in the mesh. Two similar triangles are the result, a comparison of their corresponding sides giving at once the relations proved above.

Since it is immaterial which winding of a pair of coils be referred to as primary or secondary, it is clear that there are four possible groupings of the windings of M_1 and M_2 . That arrangement should be chosen which makes L_1/L_2 most nearly equal to M_1/M_2 . However, if the resistances R_1 and R_2 be made large compared with the reactances ωL_1 and ωL_2 , then the angle ϕ will be small, and $R_1/R_2 \approx M_1/M_2$. Hence, balance can be very nearly secured without making any provision for adjustment of the self-inductances, provided the resistances be made sufficiently large. In order that R_1 and R_2 shall not be unduly great, and thereby diminish the sensitivity, it is as well to take as the secondary coil of each pair that having the smaller self-inductance. A further advantage of increasing the resistances is that the effect of temperature change of resistance in the copper secondaries is swamped by the inclusion

of resistance coils of manganin or other material of low temperature resistance coefficient, and greater permanence of the balance is thereby secured.

Experimental Example. The mutual inductance tested by Felici's and by Taylor's methods was compared with an 11 mH. Campbell mutual inductometer at 407.1 cycles per second by Maxwell's method, using a bridge of ratio about 2. The inductance l was included with the secondary (low inductance coil) of M_1 , and was about 18 mH. in value to balance the inductance L_2 of the fixed winding of the Campbell standard, the latter being about 9 mH. The resistance boxes in the two meshes were set in the ratio of 2 to 1, the true value of R_1/R_2 being measured on a Wheatstone bridge after the a.c. tests were completed. Balance was found by adjustment of M_2 and l , the following results being obtained—

R_1 ohms.	R_2 ohms.	M_2 $\mu\text{H.}$	M_1 henry.
211.2 ₀	106.0 ₂	494 ₃	0.00984 ₇
411.1 ₄	206.0 ₄	493 ₅	0.00984 ₇
610.7 ₉	306.0 ₀	493 ₃	0.00984 ₆
		Average	0.00984, henry

29. Campbell's Modification of Maxwell's Method. In the preceding method the coils M_1 , M_2 were supposed to be separate pairs. A. Campbell* has shown, however, that the method can be applied to the rather more general case of a primary with two secondaries, such as would be encountered in the process of adjusting to equality the various sections of a stranded mutual inductance standard.

Let there be mutual inductance M_{12} between the secondaries AD and DB in Fig. 63. Then the mesh equations are

$$(j\omega M_{12} - z_0)i + [R_1 + j\omega(L_1 + M_{12})]w = -j\omega M_1 u,$$

$$(R_2 + j\omega L_2 + z_0)i + [R_2 + j\omega(L_2 + M_{12})]w = -j\omega M_2 u,$$

and the balance determinant

$$\begin{vmatrix} M_1 & R_1 + j\omega(L_1 + M_{12}) \\ M_2 & R_2 + j\omega(L_2 + M_{12}) \end{vmatrix} = 0.$$

From this

$$\frac{M_1}{M_2} = \frac{R_1}{R_2} = \frac{L_1 \pm M_{12}}{L_2 \pm M_{12}}$$

are the balance conditions, it being observed that M_{12} may be either positive or negative.

In particular, as in the practical example cited above, let $M_1 = M_2$; then the above equations become independent of M_{12} and give

* A. Campbell, *Proc. Phys. Soc., loc. cit.*, pp. 73-74; also *Elec.*, Vol. 60, p. 626 (1908).

$M_1/M_2 = R_1/R_2 = L_1/L_2 = 1$, as if M_{12} were not present. Hence, in adjusting the mutual inductances of two secondaries to equal values with respect to a common primary, the mutual inductance between the secondaries does not affect the balance of the network. Mr. Campbell's paper should be consulted for a discussion of several other practical applications of the principle of this method.

30. Campbell's Method for Comparison of Two Mutual Inductances. A further method for the comparison of two mutual inductances of any value has been introduced A. Campbell,* and is illustrated in Fig. 64.

In this method the primary of the unknown mutual inductance and that of the standard are connected in the branches AC and AD .† The secondaries are arranged so that they can be connected at will in circuit with the detector. Let it be assumed that M_1 be greater than the largest value of the standard M_2 .

With the two-way switches arranged so that Cb and Dd are joined, let the self-inductances L_1 , L_2 of the primaries be compared by Maxwell's method (see p. 180). The branch AC should contain an auxiliary adjustable self-inductance so that L_1 can be made greater than L_2 , and also a rheostat so that P is greater than R . Let a balance be obtained by adjustment of Q and S and the rheostats or auxiliary inductance in AC , then

$$L_1/L_2 = P/R = Q/S. \quad (a)$$

Now, without altering the branches $ACBDA$, connect Ca and Dc , so that the secondaries of the two mutual inductances are in series with the detector and act in opposition. Then let balance be restored by adjustment of M_2 .

The condition for no current in the detector can easily be found from p. 52. Putting all mutual operators zero except $m_{15} = j\omega M_1$ and $m_{45} = -j\omega M_2$, $\alpha = j\omega M_2$, $\beta = -j\omega(M_1 - M_2)$, $\gamma = -j\omega M_2$, $\delta = j\omega(M_1 - M_2)$, since the secondaries are in opposition. Remembering

* A. Campbell, *Proc. Phys. Soc., loc. cit.*, pp. 79-80.

† The winding of the Campbell mutual inductometer which forms the branch AD should be the one which is not subdivided.

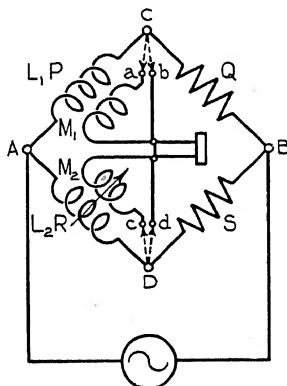


FIG. 64.—CAMPBELL'S METHOD FOR COMPARING TWO MUTUAL INDUCTANCES

that the branch impedance operators are $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S$, $z_4 = R + j\omega L_2$, the balance equation reduces to

$$[SP - QR - \omega^2(L_1M_2 - L_2M_1)] + j\omega[SL_1 - QL_2 + M_2(P + Q) + M_1(S + R)] = 0.$$

Inserting the conditions given in Equation (a), the balance relations are, numerically,

$$M_1/M_2 = L_1/L_2 = (P + Q)/(R + S) = Q/S,$$

which gives M_1 in terms of M_2 and resistances Q and S , neither of which contains copper coils.

Experimental Example. In illustration of this procedure the following experiment was carried out at 407.1 cycles per second, using the mutual inductance tested by previous methods. The branch AC contained the low inductance coil of the mutual, an Ayrton-Perry variable standard of self-inductance and an adjustable resistance. Branch AD contained the fixed winding of a Campbell 11 mH. mutual inductometer, together with a resistance box. Q and S were resistances in the ratio of 2 to 1. Balance was found by adjustment of P , R , and the variable self inductance. The second balance was secured by adjustment of M_2 .

Q ohms.	S ohms.	P ohms.	R ohms.	L mH.	M_2 μ H.	M_1 henry.
50	25	49.4 ₃	24.7 ₁	18.1	492 ₃	0.00984 ₈
100	50	99.4 ₃	49.7 ₁	18.1	492 ₄	0.00984 ₈
200	100	199.4 ₃	99.7 ₀	18.1	492 ₂	0.00984 ₄
Average						0.00984 ₆ henry

31. Mutual Inductance Measured as Self-inductance. When two coils are given with their terminals completely accessible, the mutual inductance, M , between them can be found by a simple expedient. Let the coils be connected together in series or in parallel, the apparent self-inductance of the combination being measured by any suitable method. Then, re-arranging the connections so that the effect of mutual inductance is reversed, let a second observation of apparent self-inductance be taken. From these two measurements, M can now be found.

The simplest and most obvious way of arranging the coils is to join them in series, as shown in Fig. 65. When connected as in the left-hand diagram the apparent self-inductance between the terminals A and C will be

$$\lambda_1 = L_1 + L_2 + 2M;$$

and when the connections are changed so that M is reversed the apparent inductance is

$$\lambda_2 = L_1 + L_2 - 2M,$$

whence

$$M = \frac{1}{4}(\lambda_1 - \lambda_2).$$

Experimental Example. The values of λ_1 and λ_2 were found at a frequency of 407.1 cycles per second for the 10.0 henry mutual tested by previous methods, the Heaviside-Campbell bridge of page 251 being used. The results were $\lambda_1 = 0.17005_6$ henry and $\lambda_2 = 0.13066_4$ henry, whence $M = 0.00984_8$ henry, in agreement with values previously determined.

This method appears to have been known and used some time before it was described in the technical press.

Dr. Alexander Russell,* in 1894, drew attention to it as a possible ballistic method; A.

Trowbridge,† in 1904, re-introduced it as a new method and pointed out a simple modification. Assuming that λ_1 and λ_2 are to be found by Maxwell's method, and that a suitable variable self-inductance standard is not available, Trowbridge suggests that λ_1 and λ_2 should be compared with a coil of inductance L , the value of which lies between λ_1 and λ_2 . Then, if $\lambda_1/L = p$ and $\lambda_2/L = q$, the value of M is easily seen to be

$$M = \frac{1}{4}(p - q)L.$$

Dr. Russell, in his paper, suggested that the coils might be arranged in parallel, measurements being taken as before, with M successively positive and negative. Though the resulting relationships from which to determine M are simple when a ballistic bridge is used, the equations become unwieldy when applied to alternating currents. Fig. 65 shows the two arrangements, λ_3 and λ_4 being their apparent inductances; it is easy to show that‡

$$\lambda_3, \lambda_4 = \frac{L_1 R_2^2 + L_2 R_1^2 \pm 2 M R_1 R_2 + (L_1 L_2 - M^2)(L_1 + L_2 \mp 2M)\omega^2}{(R_1 + R_2)^2 + \omega^2(L_1 + L_2 \mp 2M)^2}$$

from which M is to be found. The method is inconvenient in practice.

* A. Russell, "Measuring coefficients of induction," *Elec.*, Vol. 33, pp. 5-6 (1894).

† A. Trowbridge, "A method for the determination of coefficients of mutual induction," *Phys. Rev.*, Vol. 18, pp. 184-186 (1904).

‡ See, for example, Alexander Russell, *A Treatise on the Theory of Alternating Currents*, Vol. 1, 1st edition, p. 170 (1904).

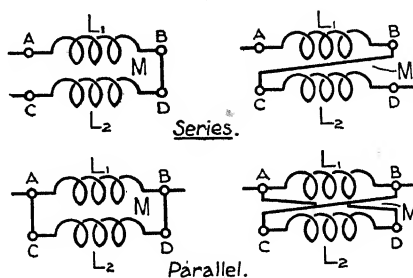


FIG. 65.—CONNECTIONS FOR THE MEASUREMENT OF MUTUAL INDUCTANCE AS SELF-INDUCTANCE

32. Maxwell's Method for Comparison of Self and Mutual Inductance. Maxwell* has described a bridge in which the mutual inductance between two coils is compared with the self-inductance of one of them by a ballistic method. Wien,† in 1891, published his experiments with the method when used with an alternating current, Fig. 66 (a) showing the network adopted.

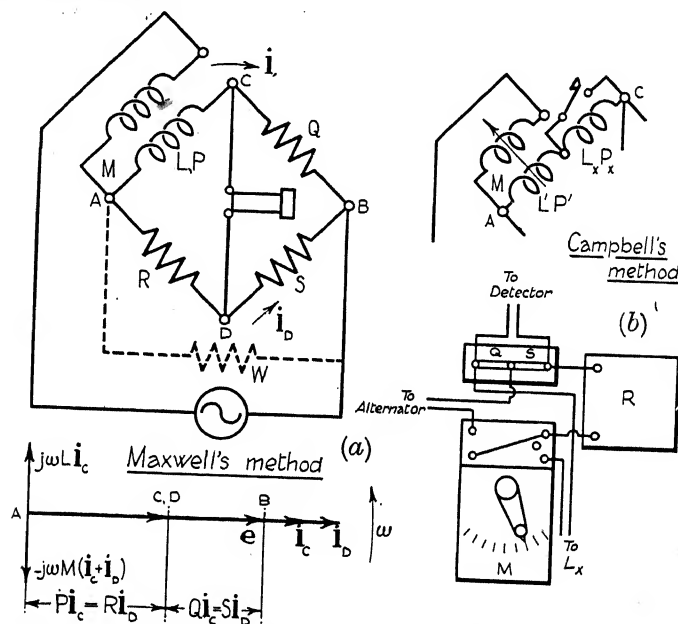


FIG. 66.—MAXWELL'S METHOD FOR COMPARING THE MUTUAL INDUCTANCE BETWEEN A PAIR OF COILS WITH THE SELF-INDUCTANCE OF ONE OF THEM. CAMPBELL'S MODIFICATION FOR THE MEASUREMENT OF LARGE INDUCTANCES

Making use of Heaviside's theory of the generalized Wheatstone network,‡ on page 52, put all mutual inductance operators equal to zero, except $m_{16} = j\omega M$. Then $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$, and the balance equation becomes

$$S(P + j\omega L) - QR + j\omega M(Q + S) = 0.$$

* *Treatise*, 1st Edn., Vol. 2, Sec. 756, p. 356 (1873).

† Max Wien, "Messung der Induktionskonstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Bd. 44, pp. 689-712 (1891).

‡ The shortest method is adopted here. This network has, however, been worked out by two other methods on pp. 49 and 51, to which the student is referred.

Separating components, the two balance conditions are

$$SP = QR$$

$$L = -\left(1 + \frac{Q}{S}\right)M. \quad (a)$$

From this it is seen that M must be arranged to be negative (since L is positive), and that balance is only possible if $L > M$.

The vector diagram for the network is drawn in Fig. 66 (a). The vector \mathbf{e} is the potential difference between A and B . Since Q , R , and S are pure resistances and the points C , D are at the same potential, the currents \mathbf{i}_c and \mathbf{i}_d must be in phase with \mathbf{e} . The inductive drop $j\omega L\mathbf{i}_c$ must then be exactly balanced by the induced electromotive force $-j\omega M(\mathbf{i}_c + \mathbf{i}_d)$. From the geometry of the diagram, the balance conditions follow at once.

Assuming L/M to be fixed, as, for example, in the case of the windings of a transformer, balance must be secured by the successive adjustment of Q/S and P/R until the balance equations are satisfied. These two adjustments obviously interfere with one another, so that the process of balancing becomes tedious. In order to avoid this trouble, Maxwell* has described a simple modification, adapted to alternating current by H. Rowland,† in 1898. Referring to Fig. 66 (a), let a resistance W be connected across the branch points AB ; it is then easy to prove that there will be no current in the detector when

$$SP = QR,$$

and

$$L = -\left\{1 + \frac{Q}{S} + \frac{Q(S+R)}{WS}\right\}M.$$

Balance is now attained by fixing Q/S and adjusting R and W successively. These adjustments do not interfere, so that balance is quickly found. As before, L must be greater than M , the latter being negative (see also page 292).

Experimental Example. To illustrate these methods, the following experiment was performed. M was a 111 mH. Campbell mutual inductometer, L being the inductance of its fixed winding. Q and S were equal ratio coils, R being an adjustable resistance. Balance was

* *Loc. cit.*, p. 357.

† H. Rowland, "Electrical measurement with alternating currents," *Phil. Mag.*, 5th series, Vol. 45, pp. 66-85 (1898); *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429-448 (1897).

secured at 407.1 cycles per second by adjustment of R and M , using a Duddell vibration galvanometer.

Q ohms.	S ohms.	$R = P$ ohms.	M μ H.	L henry.
10	10	39.1 ₆	5029 ₀	0.10058 ₀
100	100	39.1 ₆	5029 ₀	0.10058 ₀
			Average	0.10058 ₀ henry

In a second trial M was fixed, balance being found by adjustment of R and of the auxiliary resistance W .

Q ohms.	S ohms.	$R = P$ ohms.	W ohms.	M μ H.	L henry.
10	10	39.2 ₂	95.8 ₀	40,000	0.10055 ₂
10	10	39.2 ₅	36.3 ₀	30,000	0.10060 ₂
				Average	0.10057 ₂ henry

In order to make use of Maxwell's method when L is less than M , M. Brillouin,* in 1882, and C. H. Lees,† in 1916, have suggested modifications to the ballistic bridge. Unfortunately, these bridges can only be balanced for aggregate zero quantity in the galvanometer, and the modifications are not applicable to alternating current. In the case of an alternating current bridge in which L is less than M , balance can be attained by loading the branch AC with known self-inductances until the total inductance of the branch exceeds the value of M . Balance is then found in the usual way.

If a variable standard of mutual inductance is available, A. Campbell‡ has shown how Maxwell's method can be used to measure self-inductances of a value greater than twice the highest reading of the standard. The fixed coils of the standard are included in the branch AC (Fig. 66 (b)) in series with the coil to be tested, L_x , P_x . The ratio Q/S is fixed and the coil L_s

* M. Brillouin, "Comparison des coefficients d'induction," *Ann. de l'Ecole normale*, tome 11, pp. 339-424 (1882).

† C. H. Lees, "On a general bridge method for comparing the mutual inductance between two coils with the self-inductance of one of them," *Proc. Phys. Soc.*, Vol. 28, pp. 89-93 (1916).

‡ A. Campbell, "Inductance measurements," *Electr.*, Vol. 60, pp. 626-627 (1908).

short circuited, balance being secured by adjustment of M and the rheostat R . Let M_o , R_o be the readings, then from Equation (a), page 245,

$$P' = \frac{Q}{S} R_o \text{ and } L' = -\left(1 + \frac{Q}{S}\right) M_o.$$

Now unshort the coil and re-balance by adjustment of M and R to values M_1 and R_1 , then

$$P' + P_x = \frac{Q}{S} R_1 \text{ and } L' + L_x = -\left(1 + \frac{Q}{S}\right) M_1.$$

Subtracting these two relations gives numerically

$$P_x = \frac{Q}{S} (R_1 - R_o),$$

and
$$L_x = \left(1 + \frac{Q}{S}\right) (M_1 - M_o).$$

In practice, Q/S is made equal to 9, 99, etc., so that L_x becomes 10 ($M_1 - M_o$), 100 ($M_1 - M_o$), etc. The method is very convenient but, for precise work, suffers from the usual disadvantages attending the use of unequal ratio branches. If Q be made equal to S , then numerically

$$P_x = R_1 - R_o$$

$$L_x = 2(M_1 - M_o)$$

Experimental Example. M was a 111 mH. Campbell mutual inductometer, Q and S coils in a Paul ratio box (Fig. 27), R an adjustable resistance. The frequency was 407.1 cycles per second, the detector being a Duddell vibration galvanometer. The observed values were $Q = 900$ ohms, $S = 100$ ohms, $M_o = 1006_2 \mu\text{H.}$, $R_o = 4.3_6$ ohms; $M_1 = 7064_5 \mu\text{H.}$, $R_1 = 14.9_3$ ohms. Hence, $P_x = 9(14.9_3 - 4.3_6) = 95.1_8$ ohms, and $L_x = 10(7064_5 - 1006_2)10^{-6} = 0.6058_3$ henry.

33. Campbell's Method for Measuring Self-inductance. In using Campbell's modification of Maxwell's method to measure self inductance (Fig. 66 (b)), the result is given in terms of the difference of readings on the inductometer. The initial balance value M_o accounts for the self inductance L' of the inductometer winding in the branch AC , and may be fairly considerable in value. In order that the range of the bridge be not unduly restricted by M_o being too large, and, further, to guard against inaccuracy in the difference $M_1 - M_o$ when M_1 is not very different from M_o , Campbell* has introduced a

* A. Campbell, "On the use of variable mutual inductances," *Proc. Phys. Soc.*, Vol. 21, pp. 75-76 (1910).

"balancing coil" into the branch AD (Fig. 67). The object of this is to reduce the value of M_o to be zero, or, at the worst, very small.

On page 52 put $m_{16} = j\omega M$, so that $a = 0, \beta = 0, \gamma = -j\omega M$,

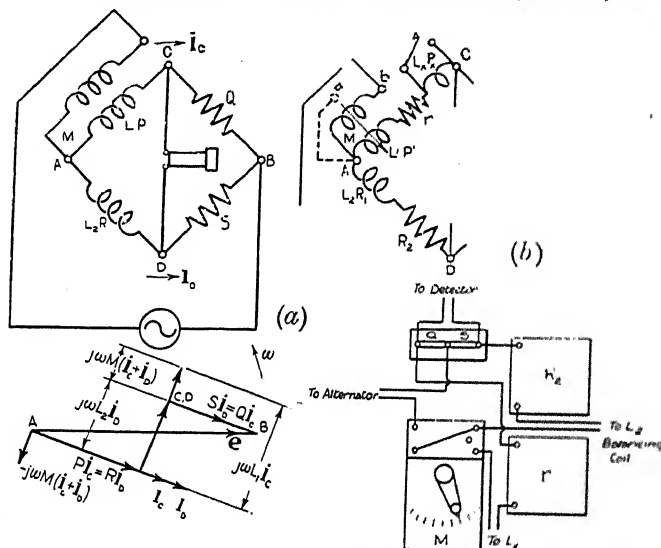


FIG. 67.—CAMPBELL'S METHOD FOR MEASURING SELF-INDUCTANCE, USING A "BALANCING COIL."

$\delta = 0$. Since $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S$, $z_4 = R + j\omega L_2$, balance occurs when

$$S(P + j\omega L_1) - Q(R + j\omega L_2) + j\omega M(S + Q) = 0;$$

from which

$$SP = QR,$$

$$L_1 - L_2 \frac{Q}{S} = -\left(1 + \frac{Q}{S}\right) M. \quad (b)$$

The vector diagram is drawn in Fig. 67 (a) and is self-explanatory.

Fig. 67 (b) shows the arrangement of the network for the measurement of a large inductance. The coil to be measured, L_x , and a constant inductance rheostat r are included in the branch AC in series with the fixed winding of the inductometer, L' . In the branch AD , L_2 is a balancing coil having an inductance equal to $L'S/Q$; R_2 is a resistance such that the total resistance of AD is about equal to S/Q times that of AC .

Let L_x be short-circuited, balance being obtained by adjustment of M to a value M_0 and r to a value r_0 ; M_0 is now a very small quantity, accounting for the inductance of the leads to the coil L_x and for any slight differences between the inductances of AC and AD . Then, from Equation (b),

$$S(P' + r_0) = QR$$

$$\text{and } l + L' - \frac{Q}{S} \cdot \frac{S}{Q} L' = -\left(1 + \frac{Q}{S}\right) M_0 = l$$

if l represents the inductance excess of AC over AD .

Inserting the coil L_x , the reading of r must be reduced to a value r_1 while M is increased to M_1 . Then

$$S(P' + P_x + r_1) = QR$$

$$\text{and } L_x + l = -\left(1 + \frac{Q}{S}\right) M_1.$$

Subtracting these relations from the former, gives numerically

$$P_x = r_0 - r_1$$

$$L_x = \left(1 + \frac{Q}{S}\right) (M_1 - M_0) \quad (c)$$

In practice, the ratio Q/S is made equal to 9, 99, etc., giving $L_x = 10$ or 100 times the difference of readings of the inductometer. The balancing coil has a value $L_2 = 1/9$ or $1/99$ of L' .

Experimental Example. Using the apparatus described on page 247, a balancing coil about $1/9$ of L' was inserted in AD , together with a resistance box; r was now an adjustable resistance. At 407.1 cycles per second the balancing values were $Q = 900$ ohms, $S = 100$ ohms, $M_0 = -5 \mu\text{H.}$, $r_0 = 239.4_9$ ohms; $M = 6058_0 \mu\text{H.}$, $r = 144.2_0$ ohms. Hence, $P_x = 239.4_9 - 144.2_0 = 95.2_9$ ohms, and $L_x = 10 (6058_0 + 5) 10^{-6} = 0.6058_5$ henry.

34. Residual Errors in Campbell's Method. In using the bridge of Fig. 67, large errors in effective resistance measurements may arise on account of the small residual reactances of the unequal ratio branches Q , S . The general nature of the effect is similar to that discussed by Giebe in connection with Maxwell's method for comparison of two self-inductances (see p. 185), and the theory has been given by Campbell* in a similar manner.

* A. Campbell, "On the use of mutual inductometers," *Proc. Phys. Soc.*, Vol. 22, pp. 207-219 (1910).

Following the method on p. 185, put $Q + j\omega L$ for Q and $S + j\omega\mu$ for S in the analysis on p. 248 preceding Equation (b). The balance equations are then

$$\begin{aligned} SP - QR &= \omega^2[(L_1 - M)\mu - (L_2 + M)\lambda] \quad \dots \quad (d) \\ SL_1 - QL_2 &= (Q + S)M - P\mu + R\lambda \end{aligned}$$

when M is taken as negative, as is necessary to secure balance.

$$(i) \text{ If } Q = S, \text{ then } (P - R)Q = \omega^2[(L_1 - M)\mu - (L_2 + M)\lambda],$$

$$\text{and} \quad (L_1 - L_2 - 2M)Q = R\lambda - P\mu.$$

$$\text{If, in addition, } \lambda = \mu, (P - R)Q = \omega^2\lambda(L_1 - L_2 - 2M)$$

$$\text{and} \quad -(P - R)\lambda = Q(L_1 - L_2 - 2M),$$

so that either $Q^2 = -\omega^2\lambda^2$, which is absurd, or

$$L_1 - L_2 = 2M$$

and

$$P = R,$$

which is in agreement with Equation (b), p. 248, when $Q = S$. Thus, if the ratio branches have equal resistances and also equal residuals, no error will be introduced into the bridge. This can be checked by balancing the bridge, and then interchanging the ratios. They should be adjusted until the interchange does not upset the balance.

(ii) If $Q \neq S$, consider the bridge shown in Fig. 67(b). Suppose the balancing coil L_x to be adjusted so that with L_x short-circuited, balance occurs when $M = M_0$ and $r = r_0$, M_0 being a very small reading. Let $M = M_1$ and $r = r_1$ when L_x is introduced and balance is again secured. Putting these conditions in Equation (d) and applying the method of differences gives

$$(r_1 - r_0 + P_x)S = \omega^2[L_x\mu - (M_1 - M_0)(\lambda + \mu)]$$

$$SL_x = (Q + S)(M_1 - M_0) - (r_1 - r_0 + P_x)\mu.$$

Now λ and μ are usually very small compared with L_x and M_1 , so that, approximately,

$$P_x \approx r_0 - r_1 + \omega^2\left(\frac{Q}{S}\mu - \lambda\right)(M_1 - M_0)$$

$$L_x \approx \left(1 + \frac{Q}{S}\right)(M_1 - M_0).$$

From these equations, L_x and P_x are found in terms of changes of r and M and the residuals λ, μ . The effect of the latter is negligible in L_x , but may be considerable in P_x . For example, suppose $Q = 99$ ohms, $S = 1$ ohm, $r_0 - r_1 = 20$ ohms, $M_1 - M_0 = 1$ millihenry, $\lambda = 10$ microhenrys, $\mu = 1$ microhenry; then at 1,000 \sim per second,

$$P_x \approx 20 + 3.6 \approx 23.6 \text{ ohms}$$

$$L_x \approx 0.1 - 3.6 \times 10^{-6} \text{ henry.}$$

Hence the error in P_x due to neglect of residual effects is about 18 per cent, whereas in L_x the error would only be 3.6 in 100,000.

(iii) If $Q \neq S$, but in addition, $Q\mu = S\lambda$; i.e. if the residuals of

Q and S are in the same ratio as their resistances, then the balance equations reduces to

$$P_x = r_o - r_1$$

$$L_x = \left(1 + \frac{Q}{S}\right) (M_1 - M_o)$$

which is identical with Equation (c), p. 249. Hence, if the residuals of the ratio branches are in the same ratio as their resistances, errors due to residual effects are entirely eliminated. This result can be very nearly attained by the use of a ratio box made up of Duddell-Mather gauze,* in which capacitance is negligible, the resistance and residual inductance being each proportional to the length of gauze used. The same end can also be attained by the following artifice. Let a coil be constructed of carefully stranded wire, and its self capacitance be measured. Then, from the formulae given on page 90, calculate the effective inductance and resistance of the coil at a given frequency, making use of the measured value of self capacitance and of its inductance and resistance at low frequency. Now put the coil in the bridge and supply current at the frequency for which the calculations are made. Then, if the measured values do not agree with the calculated values, add inductance to Q or S until agreement is obtained. The residual effects are then eliminated and another coil will be measured correctly with the same Q/S and frequency.

35. The Heaviside-Campbell Equal Ratio Bridge. O. Heaviside† has given the general theory of a large number of induction balances, including, in particular, certain important bridges now to be described in which equal ratio branches are used. A. Campbell‡ has shown that the adoption of the method of differences described above renders these bridges of great value for the measurement of small self-inductances.

A convenient practical arrangement is shown in Fig. 68(a). The branch AC now contains the inductometer fixed coil L' , and the coil to be tested, L_x . The branch AD contains a balancing coil equal to L' and a constant inductance rheostat r . By taking balances with L_x in and out of the bridge, the balance conditions give numerically

$$P_x = r_1 - r_o$$

$$L_x = 2(M_1 - M_o),$$

when the branches CB , DB are equal.

The balancing coil in AD in the equal ratio bridge introduces

* See p. 61.

† O. Heaviside, *Electrical Papers*, Vol. 2, pp. 33-38, pp. 106-115, pp. 284-286 (1892).

‡ A. Campbell, "On the use of variable mutual inductances," *Proc. Phys. Soc.*, Vol. 21, pp. 76-78 (1910). "On the use of mutual inductometers" *Proc. Phys. Soc.*, Vol. 22, pp. 207-219 (1910).

a loss of sensitivity and range; a simple modification of the inductometer renders the use of a balancing coil unnecessary and permits of increased sensitivity. Let the primary of the inductometer be composed of two equal coils, (marked L'' in Fig. 68 (b)), one being put in the branch AC and the other

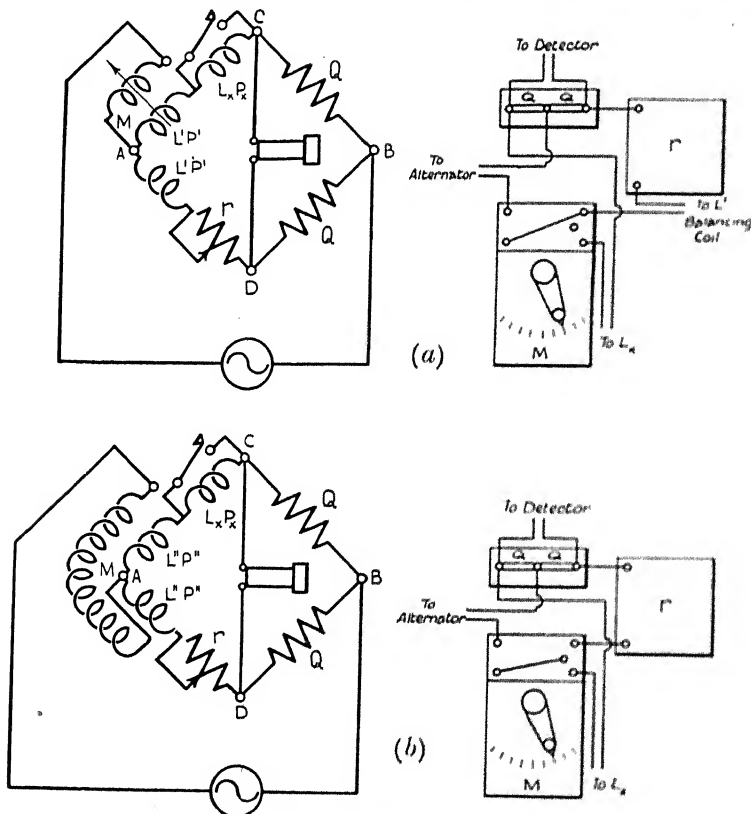


FIG. 68.—THE HEAVISIDE-CAMPBELL EQUAL RATIO BRIDGE FOR THE MEASUREMENT OF SMALL INDUCTANCES

in AD . Mutual induction then acts on each of these branches from the secondary coil, and there will also be mutual inductance between the two halves of the primary winding.

First suppose that L_1, P are the constants of AC , Q of CB , S of BD , and L_2, R of AD . Then, on p. 52, put $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S$, $z_4 = R + j\omega L_2$, $m_{12} = j\omega M_{12}$, $m_{14} = j\omega M_{14}$, $m_{23} = -j\omega M_{23}$.

the remaining mutual operators being zero. This results in $\alpha = 0$, $\beta = 0$, $\gamma = j\omega(M_{14} - M_{16} - M_{46})$, $\delta = 2j\omega M_{14}$, and therefore,

$$S(P + j\omega L_1) - Q(R + j\omega L_2) - j\omega(M_{14} - M_{16} - M_{46})(Q + S) + 2j\omega S M_{14} = 0.$$

Separating components gives

$$SP = QR$$

$$L_1 - L_2 \frac{Q}{S} = (M_{16} + M_{46}) \left(1 + \frac{Q}{S}\right) + M_{14} \left(\frac{Q}{S} - 1\right),$$

as shown by Campbell.

If the bridge has equal ratio branches, as in Fig. 68(b), the above equation shows that the effect of M_{14} is zero. Also $M_{16} + M_{46}$ is the total mutual inductance between primary and secondary, i.e. the reading of the standard. Then, applying the method of differences, and remembering that the two primary coils are identical,

$$P_x = r_1 - r_0$$

$$L_x = 2(M_1 - M_0).$$

The equal ratio method is subject to the following errors—

Residual Effects. The effects of the leads joining L_x to the bridge, of slight inequalities in the two primary coils, and of the inductance of r , are eliminated by the method of differences. Slight residuals in the branches Q, Q can be eliminated by repeating observations with these branches reversed and taking the mean. Since r is of constant inductance, residuals in AC and AD are cancelled by the procedure described.

Impurity. Owing to the effects of self and inter-capacities and of eddy currents, the secondary electromotive force in the inductometer will not be truly $\pi/2$ ahead of the primary current; the mutual inductance operator becomes $\sigma + j\omega M$, σ being called the "impurity." S. Butterworth* has shown how this enters into the balance of an equal ratio bridge. Consider Fig. 67(b) and suppose $Q = S$; with the source connected to a and L_x removed balance by adjustment of r and L_2 . Introduce L_x , connect the source to b , and re-balance by adjustment of r and M . Then $L_x = 2M$, and $P_x = 2\sigma + r_0 - r_1$. Transference of the ratios removes their residuals as described above.

Thus, impurity has no effect on the inductance measurement, but may considerably influence a measurement of effective resistance. For example, in a 10 millihenry standard at 2,000 cycles per second, $\sigma = 0.1$ ohm; hence neglect of σ may lead to an error of 2 per cent in the measurement of effective resistance of a 20 millihenry inductance having a resistance of 10 ohms.

Experimental Examples. The following experiment was made at 407.1 cycles per second using a 111 mH. Campbell mutual inductometer for M . With the arrangement of apparatus shown in Fig. 68(a),

* S. Butterworth, "Capacity and eddy current effects in inductometers," *Proc. Phys. Soc.*, Vol. 33, pp. 312-354 (1921). See also page 102.

the equal ratios were 100 ohms each. The balancing coil was an Ayrton-Perry variable self-inductance. With $M = 0$, and L_x short circuited, the variable self-inductance and r were adjusted to give balance, r_0 being then 0.4 ohm. With the self-inductance of the mutual inductometer fixed winding thus balanced, L_x was unshorted, balance being restored by adjustment of M to $2033_s \mu\text{H.}$, and r to 5.6 ohms. Hence, $P_x = 5.6_s - 0.4_s = 5.1_s$ -ohm and $L_x = 2 \times 2033_s \times 10^{-3} = 40.67_0 \text{ mH.}$

Removing the balancing coil and re-arranging the connections to the mutual inductometer, as shown in Fig. 68 (b), the test was repeated with the following results—

Q ohms.	M_0 $\mu\text{H.}$	r_0 ohms.	M $\mu\text{H.}$	r_1 ohms.	L_x mH.	P_x ohms.
10	5	0.0 ₃	2034 ₁	5.2 ₂	40.67 ₂	5.1 ₉
90	5	0.0 ₃	2034 ₀	5.2 ₂	40.67 ₀	5.1 ₉
100	5	0.0 ₃	2034 ₂	5.2 ₂	40.67 ₄	5.1 ₉
			Average		40.67 ₂ mH.	5.1 ₉ ohms

To show the applicability of the method for the measurement of small inductances, a 1.1 mH. Campbell mutual inductance was used to measure at 407.1 cycles per second a small coil tested by Butterworth's method (see p. 223). Using the arrangement of Fig. 68(b), $Q = 10$ ohms, $r_0 = 0.0_3$ ohm, $M_0 = 0.0_0 \mu\text{H.}$, $r_1 = 0.6_s$ ohm, $M_1 = 25.7_0 \mu\text{H.}$, were found to give balance. Hence, $P_x = 0.6_s - 0.0_3 = 0.65$ ohm, and $L_x = 2(25.7_0 - 0.0_0) = 51.4_0 \mu\text{H.}$

36. Campbell's Methods for the Measurement of the Self-inductance of Four Terminal Resistances. Closely related to the above methods are certain bridges suggested by Campbell for the measurement of the small inductance of a low resistance shunt. A shunt is a four-terminal resistance, i.e. it has two current terminals and two potential terminals, and cannot be directly measured in an inductance bridge owing to this fact. Various devices* have been suggested by different experimenters to enable the inductance of a four-terminal resistance to be determined. One of the most generally applicable is that due to Campbell,† described on page 266 for another purpose.

* S. G. Barnett, *Phys. Rev.*, Vol. 34, p. 74 (1912). C. H. Sharp and W. W. Crawford, *Trans. Amer. I.E.E.*, Vol. 29, p. 1540 (1910). F. Wenner, *Bull. Bur. Stds.*, Vol. 8, pp. 559-610 (1913).

† A. Campbell, "On the measurement of small inductances and on power losses in condensers," *Proc. Phys. Soc.*, Vol. 29, pp. 347-349 (1917).

Referring to Fig. 71 (b) on page 265, let the condenser C and resistance ρ be replaced by the shunt whose inductance l and resistance r are to be determined. The current terminals of the shunt are joined to the alternator side of the network, its potential terminals being connected to the detector branch. M_1 and M_2 are adjustable mutual inductances whose value need not necessarily be known. The secondary of the former is joined to the primary of the latter to form a loop of inductance L and resistance R . The coils M_1 , M_2 should be at some distance from each other and from the low reading inductometer M , the only direct mutual inductance between the alternator and the detector circuits being *via* the latter. Balance is obtained by adjustment of M and M_1 or M_2 , when

$$(M - l)/r = L/R$$

and

$$Rr = [M_1 M_2 - (M - l)L]\omega^2.$$

From these,

$$l = M - (Lr/R)$$

so that, if r is known, l is found without the need to know M_1 and M_2 .

In carrying out the test, M must be greater than l and also $M_1 M_2 > L^2 r/R$.

37. Butterworth's Method for Frequency Measurement. Butterworth* has suggested a modification of Maxwell's method which is useful for the measurement of frequency and also as a wave-filter. In Fig. 66 (a) on page 244 imagine the resistance S to have inductance L_2 , then the conditions for balance become

$$\omega^2 L_2 (M - L) = QR - SP,$$

and

$$LS + L_2 P = M(S + Q).$$

In practice, it is simpler to make $L = L_2$ and to fix M , P , Q , and S . The second condition is then a constant and the first can be satisfied by adjustments of R alone; the range of frequency is infinite, and M should be greater than L .

Experimental Example. Butterworth's method was used to calibrate a triode valve oscillator used for a.c. bridge measurements and illustrated in Fig. 41, page 143. The frequency could be varied by changing the anode condenser. In the bridge, M was an 11 mH. Campbell mutual inductometer whose fixed winding had an inductance L of $958_2 \mu\text{H}$. With $P = 12$ ohms, $Q = 9$ ohms, $S = 10.6_9$ ohms, $L_2 = 955_0 \mu\text{H}$,

* S. Butterworth, *Proc. Phys. Soc.*, Vol. 24, p. 86 (1912).

balance was secured by adjustment of R and small changes of M , with the following results (a Duddell vibration galvanometer was used) —

Anode Condenser $\mu\text{F.}$	R ohms.	M $\mu\text{H.}$	Frequency cycles/second.
0.3	29.4 ₅	1087 ₈	529.1
0.4	25.8 ₅	1088 ₇	459.6
0.5	23.3 ₉	1089 ₇	407.3

38. The Hughes Balance. This bridge is of considerable historical interest, since it was employed by Professor Hughes* in some of the earliest measurements of inductance effects in thin and thick wires of different materials. He, however, interpreted his results by a totally incorrect theory of the bridge, his astonishing conclusions leading the late Lord Rayleigh, Oliver Heaviside, and others to investigate the problem and to deduce the correct balance conditions for the network.†

In the original arrangement, interrupted current and a telephone were employed, H. Rowland,‡ in 1898, applying alternating current and an electro-dynamometer to the method. A. Campbell,§ in 1907, replaced the latter instrument by the superior vibration galvanometer. The network is shown in Fig. 69, from which it is seen that mutual inductance between the detector and battery branches is compared with a self-inductance and resistances. To find the balance conditions, on page 52 put $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$, and

* D. E. Hughes, "Induction balance and experimental researches therewith," *Proc. Phys. Soc.*, Vol. 3, pp. 81-89 (1880). "The self-induction of an electric current in relation to the nature and form of its conductors," *Journal, S.T.E.*, Vol. 15, pp. 6-25 (1886).

† O. J. Lodge, "On intermittent currents and the theory of the induction balance," *Proc. Phys. Soc.*, Vol. 3, pp. 187-212 (1880). Lord Rayleigh, *Journal, S.T.E.*, Vol. 15, pp. 28-40, 54-55 (1886); also "Notes on Electricity and Magnetism; II. The self-induction and resistance of compound conductors," *Phil. Mag.*, 5th series, Vol. 22, pp. 469-500 (1886). O. Heaviside, "On the use of the bridge as an induction balance," *Electr.*, Vol. 16, pp. 489-491 (1886); *Electrical Papers*, Vol. 2, pp. 33-38 (1892). The student interested in the historical development of electrical science will find the perusal of this classic dispute of considerable interest.

‡ H. Rowland, "Electrical measurement by alternating currents," *Phil. Mag.*, 5th series, Vol. 45, pp. 66-85 (1898). See also L. Graetz, "Eine neue Methode zur Messung von selbstpotential und Inductionscoefficienten," *Ann. der Phys.*, Bd. 50, pp. 766-771 (1893).

§ A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626-638 (1907).

all mutual operators equal to zero except $m_{56} = j\omega M$. Then, $\alpha = -j\omega M$, $\beta = 0$, $\gamma = j\omega M$, $\delta = 0$, so that

$$S(P + j\omega L) - QR - j\omega M(P + R + j\omega L) - j\omega M(Q + S) = 0$$

Separation of the components gives

$$\omega^2 ML = QR - SP$$

$$SL = M(P + Q + R + S)$$

for balance.

A. Campbell has shown that the method is useful as a means of measuring frequency.* For this purpose M is a variable mutual inductance; the resistance S is a portion of a slide wire, B being the contact thereon. The resistance Q consists of the remainder of the slide wire and a non-inductive resistance box; by this means $Q + S$ can be kept constant although Q/S is varied. The resistance R is a fixed non-inductive resistance. The resistance of P will usually require a correction for temperature, since it is wholly or partly composed of a copper coil; this correction is more important at low frequencies. Two different methods can be adopted in practice—

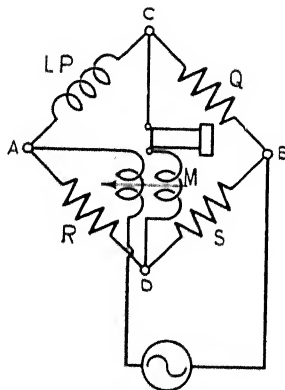


FIG. 69.—THE HUGHES BALANCE

If P be constant and known, then $P + Q + R + S = a$ is constant, since $Q + S = b$ is constant. If L be fixed and balance is secured by varying M and the position of the slider B , eliminate M from the above equations. Then

$$\omega^2 = \frac{a}{L^2} \left[\frac{R(b-S)}{S} - P \right],$$

so that the frequency can be found in terms of a single variable S . The slide wire can thus be marked directly in frequency values. The range of the bridge can be extended to n times the frequency, either by reducing L to L/n , or by changing P and Q so that $(P + R)/(Q + S) = 1/n$.

If P be not exactly known, it will be necessary either to apply

* C. E. Hay has also recommended the method for the measurement of inductance and effective resistance of telephonic loading coils; see *Journal P.O.E.E.*, Vol. 5, p. 451 (1912-13).

a correction to the above formula to allow for the change of P with temperature; or it may be simpler, if the standard inductometer M is accurately known, to eliminate P from the equations. The resulting equation for ω^2 is then

$$\omega^2 = \frac{(R+S)(Q+S)}{ML} - \frac{S^2}{M^2}.$$

Campbell* gives the following values as suitable for a range of frequency from 10 up to 120 cycles per second; $L = 0.1$ henry, $P = 25$ ohms; $R = 5$ ohms; $Q + S = 4$ ohms; M from 1.7 to 0.28 millihenry, and S from 0.6 to 0.1 ohm.

Experimental Example. The Hughes balance was used in a frequency calibration of the valve oscillator tested previously by Butterworth's method (p. 255). The branch AC contained a coil for which $L = 10$ mH. and $P = 25$ ohms. $Q + S$ was a slide wire of 4.08₁ ohms resistance, and $R = 5$ ohms. Using the first method, $b = 4.08_1$ ohms and $a = 34.08_1$ ohms. M was a 1.1 mH. Campbell inductometer, balance being found by adjustment of M and S . For the two highest frequencies the detector was a 150 ohm telephone; for the remaining observations a Duddell vibration galvanometer was employed.

Anode Condenser $\mu\text{F.}$	S ohms.	M $\mu\text{H.}$	Frequency cycles/second.
0.1	0.16 ₀	46 ₉	929.6
0.2	0.26 ₀	76 ₃	647.5
0.3	0.32 ₇	96 ₀	529.4
0.4	0.37 ₅	110 ₀	459.2
0.5	0.41 ₅	121 ₇	406.9

NETWORKS CONTAINING RESISTANCE, SELF-INDUCTANCE, MUTUAL INDUCTANCE AND CAPACITANCE

39. Modified Carey Foster's Method. This method is adapted from a ballistic bridge, introduced by G. Carey Foster,† in 1887, for the comparison of a condenser with a mutual inductance. The ballistic bridge in its original form has a quantity or aggregate balance, but by a simple modification, continuous balance is obtained and the bridge can be used with alternating current. This modification, originally

* *Dictionary of Applied Physics*, Vol. 2, p. 434.

† G. Carey Foster, "Note on a method of determining coefficients of mutual induction," *Proc. Phys. Soc.*, Vol. 8, pp. 137-146 (1887). Also see A. Ròiti, "Misure assolute di alcuni condensatori," *Mem. Acc. Tor.*, 2nd series, tome 38, pp. 57-77 (1888).

introduced by A. Heydweiller,* in 1894, consists in the inclusion of a resistance S in series with the condenser C , as shown in Fig. 70 (a).

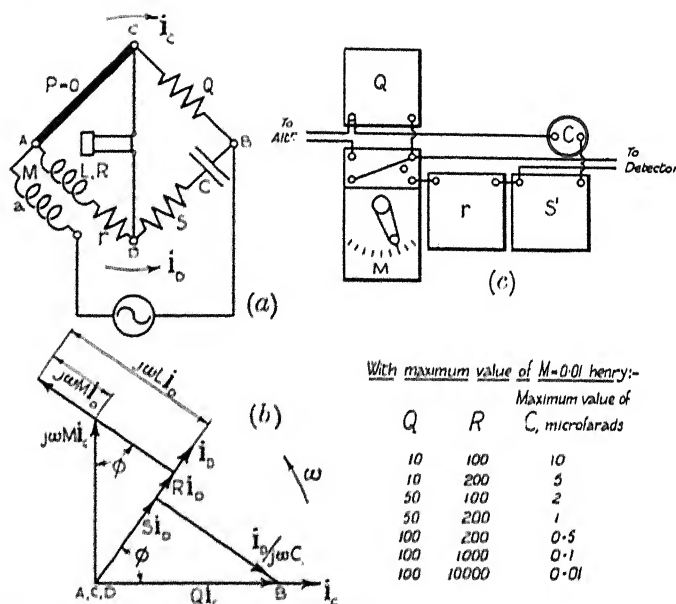


FIG. 70.—MODIFIED CAREY FOSTER'S METHOD FOR COMPARING CAPACITANCE WITH MUTUAL INDUCTANCE

Assuming first that the resistance of the branch AC is not zero, substitute on page 52 the operators $z_1 = P$, $z_2 = Q$, $z_3 = S - \frac{j}{\omega C}$, $z_4 = R + j\omega L$, and $m_{46} = j\omega M$. Then $\alpha = 0$, $\beta = 0$, $\gamma = j\omega M$, $\delta = 0$, so that

$$P\left(S - \frac{j}{\omega C}\right) - Q(R + j\omega L) - j\omega M\left(Q + S - \frac{j}{\omega C}\right) = 0.$$

* A. Heydweiller, "Ueber die Bestimmung von Inductionscoefficienten mit dem Telephon," *Ann. der Phys.*, Bd. 53, pp. 499-504 (1894). The modification has also been independently introduced by H. Rowland, "Electrical measurement by alternating currents," *Phil. Mag.*, 5th series, Vol. 45, pp. 66-85 (1898), using an electro-dynamometer; A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626-638 (1907), and also "Inductance measurements," *Elec.*, Vol. 60, pp. 626-627 (1908).

Separating the components, gives

$$M = C(SP - QR)$$

$$L = -M \left(1 + \frac{S}{Q} \right) - \frac{P}{\omega^2 CQ}$$

when balance is secured. By the simple expedient of making $P = 0$, the balance can be made independent of frequency, as shown by Heydweiller. Putting $P = 0$ and noting that M must be negative to secure balance, numerically

$$M = CQR,$$

$$L = M \left(1 + \frac{S}{Q} \right).$$

Fig. 70 (b) shows the vector diagram for the bridge. The vector AB is the applied potential difference across the points A and B ; and, since $P = 0$, AB must also equal the drop of potential Qi_c in the branch CB . Since C and D are at the same potential, the sum of the potential drops Si_p and $i_p/j\omega C$ must balance Qi_c . Since the drop of potential down the branch AC is zero, the total e.m.f. of mutual induction in AD , namely, $j\omega M(i_c + i_p)$, must exactly balance the impedance drop $(R + j\omega L)i_p$. From the geometry of the two similar triangles the above equations may be verified at once.

In practical working, the second balance condition implies that L be greater than M . Hence, if necessary, an auxiliary self-inductance should be included in the branch AD until $L \geq M$. The resistance Q , being connected across the source—neglecting the impedance drop in the coil a —should be capable of carrying a fair current. It is preferably an oil-cooled standard, (so that temperature changes can be allowed for), of low residual inductance.

When the method is used to measure a mutual inductance in terms of a standard condenser, Q is fixed and L arranged to be equal to or greater than M . Balance is secured by successive adjustment of R or C and S .

A. Campbell has shown (*loc. cit.*) that Carey Foster's method is one of the best for the measurement of the capacitance and equivalent series resistance of a condenser, using a standard mutual inductometer for M . Fig. 70 (c) shows a convenient arrangement of apparatus for this test. The resistance Q is set at a definite fixed value. The branch AD contains the fixed winding of the inductometer, a resistance box r and, if

necessary, an auxiliary inductance to ensure that the inductance L of the branch exceeds M . The value of r is chosen so that QR is a convenient multiplier, e.g. some power of 10, R being the total resistance of AD . A convenient set of values for Q and R are shown in the diagram. The branch DB contains the condenser whose capacitance C and series resistance ρ are to be found, together with an adjustable resistance box S' . Balance is secured by successive adjustment of S' and M ; then, since $S = S' + \rho$, the above balance condition gives

$$C = M/QR \text{ and } \rho = Q\left(\frac{L}{M} - 1\right) - S'.$$

In accurate work, the capacitance of the leads to C should be allowed for by repeating the test with the leads disconnected from the terminals of the condenser, and deducting the resulting capacitance from the first balance.

The values of Q and R should be measured after the test, and L should be accurately known. S' should not be too large, i.e. if L is much greater than M , a smaller value of Q should be chosen. A Campbell inductometer reading up to 10 millihenrys is convenient for M , and gives a range from a small fraction of a microfarad up to 10 microfarads.

Experimental Example. Carey Foster's method was used to test a paper condenser at 407.1 cycles per second, the detector being a Duddell vibration galvanometer. M was an 11 mH. Campbell inductometer, the following values being found at balance. $Q = 100$ ohms, $R = 200$ ohms, $L = 1658_2 \mu\text{H.}$, $S' = 100.4_4$ ohms, $M = 8008_0 \mu\text{H.}$ With the leads removed from C , $M_0 = 0.4 \mu\text{H.}$, and $C = 8007_6/20,000 = 0.4003_8 \mu\text{F.}$, and $\rho = 100\left(\frac{1658_2}{800_8} - 1\right) - 100.4_4 = 5.20$ ohms.

In a second experiment on a good mica condenser, $Q = 100$ ohms, $R = 200$ ohms, $L = 1922_2 \mu\text{H.}$, $S' = 186.9_5$ ohms, $M = 6693.0 \mu\text{H.}$, and $M_0 = 1 \mu\text{H.}$ Thus, $C = 6692_0/20,000 = 0.3346_0 \mu\text{F}$ and also $\rho = 100\left(\frac{1922_2}{669_2} - 1\right) - 186.9_5 = 0.2_9$ ohm.

40. Experimental Troubles in the Modified Carey Foster Method. In practical work, especially in measurements of high precision on small condensers, or on condensers with low power-factors, certain important sources of error enter into the method.

(i) *Residual Errors and Impurity Effects.* At high frequencies residual inductance* in Q and S' , self capacitance in

* See A. Campbell, *Proc. Roy. Soc., A*, Vol. 87, pp. 402-406 (1912).

the windings of the mutual inductance and the effects of impurity become important. Indeed, when measuring small air condensers by this method, the measurements serve rather as a drastic test of the imperfections of the apparatus than as a means of testing the condenser. Butterworth* has shown how these various sources of error may be allowed for, and the reader is advised to consult the original paper for full details of the procedure. The corrections are particularly important in power-factor measurements, especially at the upper audio frequencies.

Since the effect of residual inductance in Q and S' may become important, even at frequencies of moderate value in tests on low power factors, it is useful to see how the balance conditions become altered by the residuals. In the analysis above, p. 259, put $Q + j\omega\lambda$ for Q , and $S + j\omega\mu$ for S then

$$\frac{1}{C} = \frac{QR}{M} \left[1 - \frac{\omega^2}{QR} \left\{ \lambda L - (\lambda + \mu)M \right\} \right],$$

$$\text{and } S' + \rho = Q \left(\frac{L}{M} - 1 \right) + \frac{\lambda R}{M},$$

are the balance conditions. The correction in the value of $1/C$ is, except at very high frequencies, usually small. Hence

$$C = \frac{M}{QR} \left[1 + \frac{\omega^2}{QR} \left\{ \lambda L - (\lambda + \mu)M \right\} \right],$$

$$\text{and } \rho = Q \left(\frac{L}{M} - 1 \right) - S' + \frac{\lambda R}{M}.$$

In tests on large condensers the correction to be applied to C is usually negligible, except at the upper telephonic frequencies. The correction in the expression for ρ may become important in accurate tests of the power-factor of good condensers. When small condensers are tested, it is generally necessary to apply corrections for residuals both to the capacitance and the loss resistance. In measurements at high frequencies made upon good condensers (e.g. a well-made mica standard or an air condenser), ρ calculated from the above formula is often an apparently negative quantity. This absurd result is the consequence of neglecting the effects of impurity in the mutual inductance; Butterworth has shown in the paper cited how this quantity can be corrected for in such cases.

* S. Butterworth, *Proc. Phys. Soc.*, Vol. 33, p. 313, pp. 334-337 (1921).

In the experiments on page 261, if the value of λ be $0.5 \mu\text{H.}$, the correction in the first case is 0.0_{12} ohm, or 0.2 per cent, so that $\rho = 5.2_1$ ohms. In the second test the correction is 0.0_{15} ohm, so that ρ for the mica condenser is 0.3_{05} ohm; this is an important correction, being about 5 per cent.

(ii) *Earth Capacities.* With small condensers trouble is experienced due to earth capacities. It is sometimes a good plan to earth the terminal of the inductometer which is joined to the source. At the higher audio frequencies, when telephones are used to detect balance, it becomes very difficult to secure sharp balance, unless it can be ensured that the points C and D at balance are at the potential of the observer. To a first approximation this can be attained by earthing the point D instead of the source terminal of the inductometer. Both these devices only provide rough allowance for earth capacity effects. To remove such troubles entirely Mr. Dye has shown how to find a proper earth point which is free from this defect. His device is described on page 287.

41. Campbell's Method. A. Campbell* has given a method which serves very conveniently to check a mutual inductance standard against a known condenser. The network is arranged as in Fig. 67 (a), a condenser of capacitance C being put in parallel with the resistance S .

In the analysis on page 248, put $S/(1 + j\omega CS)$ in place of S ; then the balance equation is

$$\frac{S(P + j\omega L_1)}{(1 + j\omega CS)} - Q(R + j\omega L_2) + j\omega M \left(Q + \frac{S}{(1 + j\omega CS)} \right) = 0;$$

from which

$$(SP - QR) = \omega^2 QSC(M - L_2),$$

and $S(L_1 + M) - Q(L_2 - M) = QRSC$.

When the bridge is used with equal ratio branches, $Q = S$, and $L_2 = L_1$, then

$$P - R = \omega^2 QC(M - L)$$

and

$$2M = QRC.$$

From this, M is found without the need for measurement of ω , which must, however, be steady.

42. Haworth's Method. Haworth† has given a simple modification of the Heaviside-Campbell bridge of Fig. 68, by means of which the effective capacitance and series resistance of an imperfect condenser may be measured.

* A. Campbell, *Proc. Phys. Soc.*, Vol. 21, pp. 78-79 (1910).

† H. F. Haworth, "The measurement of electrolyte resistance using alternating currents," *Trans. F. Soc.*, Vol. 16, pp. 365-391 (1921) The second balance condition is incorrectly quoted.

In the bridge illustrated in Fig. 68 (b) on page 252, let the coil L_x, P_x be replaced by an imperfect condenser C, ρ . Now the impedance operator for a condenser is $-j/\omega C = -j\omega/\omega^2 C$, so that a condenser may be thought of as possessing a negative inductance of amount $1/\omega^2 C$. Hence, on page 253, write $1/\omega^2 C$ for L_x , giving for balance

$$\begin{aligned}\rho &= r_1 - r_o, \\ C &= 1/2(M_1 - M_o)\omega^2,\end{aligned}$$

numerically, r_1, r_o and M_1, M_o being the readings of the rheostat and inductometer with the condenser in and out of the bridge.

43. Hay's Method. C. E. Hay* has suggested a modified form of the Hughes balance to measure the equivalent capacitance and shunt resistance of an imperfect condenser by means of a mutual inductometer. In Fig. 69, let the coil $P + j\omega L$ be replaced by $P/(1 + j\omega CP)$; then from page 257 the balance equation is

$$\frac{SP}{(1 + j\omega CP)} - QR - j\omega M \left\{ Q + R + S + \frac{P}{(1 + j\omega CP)} \right\} = 0,$$

from which

$$\begin{aligned}\omega^2 M(Q + R + S)CP &= QR - SP \\ -QRCP &= M(P + Q + R + S)\end{aligned}$$

are the balance conditions.

From the second it should be noted that M must be negative, since C is essentially positive. Inserting this condition and solving for C and P gives the capacitance and shunt loss resistance,

$$C = \frac{M(Q + S)(R + S)}{Q^2 R^2 + \omega^2 M^2 (Q + R + S)^2}$$

and

$$P = \frac{Q^2 R^2 + \omega^2 M^2 (Q + R + S)^2}{QRS - \omega^2 M^2 (Q + R + S)}$$

44. Campbell's Frequency Bridge. The circuit for this method† is shown in Fig. 71 (a). Suppose the condenser to be perfect and let the self-inductance and resistance of the coil of the variable mutual inductance in the detector circuit be L_2, R_2 . Then the equation for i in terms of u is

$$\left\{ R_2 + z_c + j\left(\omega L_2 - \frac{1}{\omega C}\right) \right\} i = -j\left(\omega M + \frac{1}{\omega C}\right) u,$$

where $z_c = R_c + jX_c$ is the operator for the detector. Hence, $i = 0$ when $\omega^2 = -1/MC$. Since ω^2 is essentially positive, it

* C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal, P.O.E.E.*, Vol. 5, pp. 451-454 (1913); also *Professional Papers*, No. 53, pp. 25-26 and 44-45.

† A. Campbell, "On the use of variable mutual inductances," *Proc. Phys. Soc.*, Vol. 21, pp. 80-82 (1910). See also p. 53.

follows from this equation that the primary and secondary coils of the inductometer must be in opposition, so that M is negative. Assuming this adjustment to be made, the balance condition is

$$\omega^2 = 1/MC$$

The method serves to measure at ordinary frequencies fairly large capacitances in terms of mutual inductance and frequency.

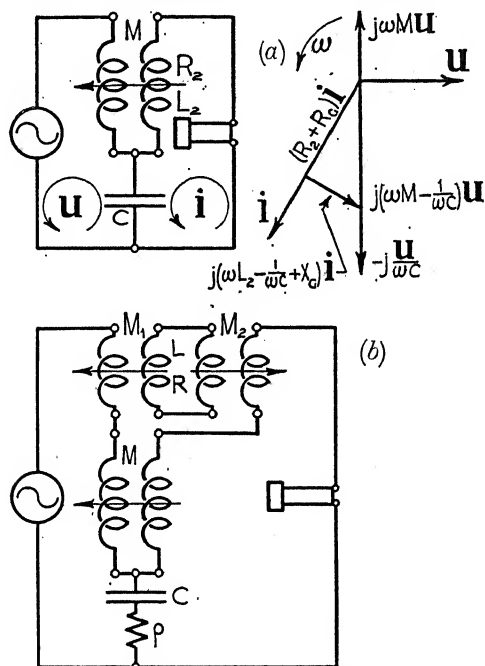


FIG. 71.—CAMPBELL'S FREQUENCY BRIDGE

It is, however, chiefly used in the inverse sense for the measurement of frequency in terms of M and C . The method is most suitable for the determination of high frequencies,* say 1,000 cycles per second upwards, since MC is inconveniently great unless ω is large.

When used as a frequency bridge, it is only possible to obtain sharp balance if certain important conditions are observed, as follows.

* For a number of ways of adapting the method for low frequencies, see S. Chiba, *Journal I.E.E. Japan*, No. 405, pp. 294-300 (1922).

The Frequency must be Constant and the Wave Form Pure. The need for constancy of frequency is obvious. The wave form must be pure when, as is usual at the higher audio frequencies, telephones are used to detect balance, since two or more coexistent frequencies cannot be balanced by the same value of MC . This property is sometimes employed when the bridge is used as a wave filter, see page 144.

The Condenser must be as Perfect as Possible. In order to see the necessity for this condition, suppose the condenser to have losses represented by a resistance ρ in series with it. Then, for $-j/\omega C$ write $\rho - (j/\omega C)$ in the above analysis.

Balance will occur when $\rho - j\left(\omega M - \frac{1}{\omega C}\right) = 0$; i.e. $\rho = 0$, necessitating a perfect condenser, and $\omega^2 = 1/MC$. In practice, it will be found that a mica condenser is usually sufficiently good to allow of reasonably sharp balance. It is even possible to attain fair success with a good paper condenser, but, as a rule, a minimum indication is all that can be secured with the average condenser.

To obtain a sharp balance, it is necessary, therefore, to compensate for the imperfection of the condenser. Campbell* has suggested several devices which enable this to be carried out, one of the best being shown in Fig. 71 (b). In this, M is the inductometer; C, ρ the imperfect condenser; M_1, M_2 are auxiliary mutual inductances, preferable variable, the secondary of one being linked to the primary of the other to form a loop, L, R . M, M_1, M_2 should be at a distance from one another so that they do not have any mutual influence. Balance is obtained when M and M_1 or M_2 are adjusted so that

$$1/\omega^2 C = M + \rho L/R,$$

$$\text{and} \quad \rho R = \omega \left[M_1 M_2 + L \left(M - \frac{1}{\omega^2 C} \right) \right].$$

Solving for ρ and $\omega^2 C$ gives,

$$\rho = \omega^2 R M_1 M_2 / (R^2 + \omega^2 L^2)$$

$$\text{and} \quad 1/\omega^2 C = M + \omega^2 L M_1 M_2 / (R^2 + \omega^2 L^2).$$

If ω be known, ρ and C can be determined; conversely, if the method is intended for frequency measurements, let L/R

* A. Campbell, "On the measurement of small inductances and on power losses in condensers," *Proc. Phys. Soc.*, Vol. 29, pp. 350-353 (1917). For another use of the network, see p. 254.

l, then $\omega^2 \equiv 1/MC$. The auxiliary circuit need not, be known, but by its aid sharp balance may be l. When using telephones, trouble due to harmonics largely overcome if the primary of the inductometer rly high inductance.

mental Examples. The simple Campbell bridge (Fig. 71(a)) to test the frequency calibration of a valve oscillator previously other methods (see p. 255 and 258). In the first two observa- 50 ohm telephone was used; in the remainder the detector idell vibration galvanometer. As paper condensers, having ble losses, were employed, only a minimum indication could d. M was a 111 mH. Campbell mutual inductometer.

Condenser F.	C $\mu F.$	M $\mu H.$	Frequency cycles/second.
0.1	1.016 ₁	30,030	911.1
0.2	1.016 ₁	59,790	645.7
0.3	1.016 ₁	89,650	527.3
0.4	2.023 ₄	59,690	458.0
0.5	2.023 ₄	75,610	406.9

second test, with the anode condenser set at 0.5 $\mu F.$, C was C . and $M = 72720 \mu H.$ gave a minimum indication; hence 0 cycles per second as found above. The balance was then xp by the use of the arrangement shown in Fig. 71 (b). M_2 was H. Campbell mutual inductance, M_1 a fixed value mutual of L contained the fixed coil of M_2 and the low inductance coil talling 10,757 $\mu H.$ A resistance box made R up to 193 ohms. adjusting M and M_2 successively, true balance was obtained = 7257, $\mu H.$ and $M_2 = 2500 \mu H.$ Inserting these values in ace equations and solving the resulting quadratic in ω^2 gives 0.4 cycles per second. Using the approximate formula gives ycles per second, showing the smallness of the correction M_1 and M_2 , these serving to give exact balance with little on the calculated value of frequency. It should also be noted simple Campbell arrangement, giving 406.9, as against the 0.4, is only in error by about 3 parts in 10,000.

Mutual Inductometer should be Free from Impurity. At quencies, the electromotive force induced in the second- mutual inductance is slightly out of quadrature with ary current (see p. 102), the mutual inductance being be impure.

the Campbell frequency bridge is peculiarly adapted at high frequencies, the effects of impurity become of nce. Butterworth,* in an elaborate paper, has shown tterworth, *loc. cit.*, p. 337 (1921).

how the Campbell bridge must be modified in order that impurity may be compensated, the reader interested in precise measurements at the higher acoustic frequencies being referred to this paper for a detailed discussion.

Referring to Fig. 71(a), let the condenser have a series loss resistance ρ , and let the mutual have an impurity σ . Then Butterworth's modification of the Campbell bridge consists in connecting a small condenser of capacitance C' across the upper open ends of the mutual inductometer and also including a small adjustable resistance S in series with the condenser C (see also Fig. 76, p. 292). Balance can then be found by independent successive adjustments of S and M . When the condenser loss and the mutual impurity are thus compensated, if R_1 , L_1 are the resistance and inductance of the inductometer winding joined to the alternator, Butterworth shows that very nearly,

$$\omega^2 MC = 1 - \omega^2 CC' [R_1 R_2 - \omega^2 (L_1 - M)(L_2 - M)],$$

$$\text{and } S + \rho + \sigma = \omega^2 C' [R_1(L_2 - M) + R_2(L_1 - M)],$$

from the first of which ω can be calculated (see p. 293).

Experimental Example. In a test on a valve oscillator by the simple bridge of Fig. 71(a), mica condensers were used for C , totalling 1.451_μF . Minimum indication occurred when $M = 10555_\mu\text{H.}$, so that the frequency is approximately 406.6_s . A small mica condenser of 0.0110_μF. was then connected across the open ends of the mutual inductance, and a small resistance S was joined in series with C . Adjusting M and S , true balance occurred when $M = 10550_\mu\text{H.}$, and $S = 0.02$ ohm; then since $R_1 = 56.6$ ohms, $L_1 = 244,200_\mu\text{H.}$, $R_2 = 39.4$ ohms, $L_2 = 100,570_\mu\text{H.}$, $C' = 0.0110_\mu\text{F.}$, the above equation gives $f = 406.0_1$ cycles per second.

Campbell's method is very quick in practice. Even when uncompensated for condenser losses and impurity the minimum point can be located within a few parts in 10,000 when using a good condenser. Compensated by Butterworth's method to give a true balance, it forms an extremely sensitive means of detecting very slight changes of frequency. By connecting an adjustable air condenser in parallel with C , variations in f as small as a few parts in a million can be measured by the small adjustments of this condenser necessary to restore balance.

45. Kennelly* and Velandar's Frequency Bridge. On

* A. E. Kennelly and E. Velandar, "A rectangular component two-dimensional alternating current potentiometer," *Journal, F. Inst.*, Vol. 188, pp. 1-26 (1919). E. Velandar, "A frequency bridge," *Journal Amer. I.E.E.*, Vol. 40, pp. 835-839 (1921).

page 264 Campbell's method of measuring frequency has been described, and it has been shown, moreover, that perfect balance is impossible unless the condenser be quite free from losses. Now, in practice, condensers with solid dielectrics are never free from imperfection, so that some device is necessary in order that the imperfection may be compensated. Campbell's bridge, even when corrected for condenser losses by one of the methods described above is unsuitable for low frequency measurements, below 250 cycles per second for example, owing to the large values of M and C necessarily involved. To overcome both these difficulties, Kennelly and Velandier have devised a wide-range frequency bridge shown in Fig. 72.

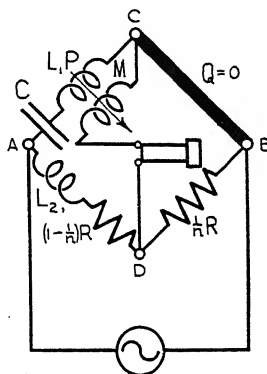


FIG. 72.—KENNELLY
AND VELANDIER'S
FREQUENCY BRIDGE

The bridge contains the following elements: in the branch AC the condenser is connected in series with the fixed coils of the inductometer, the secondary winding of which is put into the detector circuit. A small variable self-inductance L_2 is included in the branch AD . Then, if R be the total resistance of the branches ADB (inclusive of the resistance of L_2), the branch DB consists of the n th part of R .

To find when no current flows in the detector, put

$$z_1 = P + j\left(\omega L_1 - \frac{1}{\omega C}\right), z_2 = 0, z_3 = R/n, z_4 = \left(1 - \frac{1}{n}\right)R + j\omega L_2,$$

and $m_{15} = j\omega M$ on page 52. Then $\alpha = 0$, $\beta = j\omega M$, $\gamma = 0$, $\delta = j\omega M$, and the balance equation is

$$\frac{R}{n} \left\{ P + j\left(\omega L_1 - \frac{1}{\omega C}\right) \right\} + j\omega M \frac{R}{n} + j\omega M \left\{ \left(1 - \frac{1}{n}\right)R + j\omega L_2 \right\} = 0.$$

Separating components gives

$$\omega = \frac{1}{\sqrt{C(L_1 + nM)}}$$

$$L_2 = CRP \left(1 + \frac{L_1}{nM} \right)$$

for balance.

From this it is seen that the resistance P , which includes the equivalent series resistance of the condenser, does not enter into the expression for ω , so that an imperfect condenser may be used. Moreover, the effect of the potential divider arrangement of the resistance R is to multiply the range of the inductometer n times. Hence, by a suitable choice of n , low frequencies can be measured by means of a condenser and an inductometer of reasonably small value.

The most convenient practical arrangement of the bridge is to use a fixed condenser C and to fix the multiplier n . The branch AC should contain a small rheostat, P' , say. Then balance by varying M and P' or L_2 . Provided that the primary of the inductometer has a low resistance, very precise adjustment of L_2 will not be necessary.

For sensitivity the balance detector should have a low impedance, about equal to that of the secondary of the inductometer together with R/n . A low impedance vibration galvanometer or telephone is most suitable, though high impedance detectors can be used if connected to the bridge through a transformer (see p. 151).

It is very convenient to make up the capacitance C from two equal condensers which can be connected in series or parallel as desired. Since their capacitance in series will be one-quarter of the value when in parallel, the range of the bridge can be doubled, without altering any other constituent of the bridge. Moreover, it is not necessary that the two condensers be very accurately adjusted. Velandar has shown that a difference of 2 per cent between the two condensers will only produce an error of 5 in 100,000 in the doubling of the frequency range.

Velandar has described a portable, self-contained frequency bridge having a range from 400 to 3,200 cycles per second. The resistance R/n is fixed at 100 ohms, R being 500, 2,000, or 8,000 ohms. This gives $n = 5, 20$, and 80, corresponding roughly to frequencies in the ratio 4 : 2 : 1. The inductometer could be varied up to 10 millihenrys; and C was made of two mica condensers of 0.4 microfarad each, so that the working capacitance is either 0.2 or 0.8 microfarad. The whole apparatus is fitted up in a box with terminals for attachment to the source and to the telephones. A system of dial switches provides for the selection of a suitable multiplier n , for the insertion of the necessary compensating inductance L_2 , and for the adjustment of M . A four-point plug enables the condensers to be changed from the series to the parallel position.

At frequencies below 500 a vibration galvanometer will give high sensitivity; for acoustic frequencies a telephone is

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convenient. Since, by suitable choice of n , small coils can be used and impurity effects reduced thereby, the method appears to be applicable at high frequencies above the telephonic range. A crystal detector and galvanometer is then suitable, and the point C may be earthed to avoid earth capacitance troubles (Fig. 72).

Experimental Example. The valve oscillator tested by other methods (see pp. 255, 258 and 267) was calibrated by Kennelly and Velandar's bridge. M was an 11 mH. Campbell mutual inductometer, its fixed winding forming L_1 . C was a mica condenser of $0.334_6 \mu\text{F}$. P was about 6 ohms. R/n was a decade resistance box set at 100 ohms, the total value of R being given the values tabulated. L_2 was an Ayrton-Perry variable inductance.

Anode Condenser μF .	R ohms.	M μH .	L_2 mH.	Frequency cycles/second.
0.1	1510.7	532 ₈	5.0 ₀	918.3
0.2	3510.7	488 ₂	10.9 ₅	647.0
0.3	8510.7	306 ₀	24.1 ₀	530.1
0.4	8510.7	411 ₀	24.1 ₀	459.3
0.5	8510.7	524 ₀	23.3 ₀	407.1

The detector was a telephone for the two highest frequencies and a tuned Duddell vibration galvanometer for the remainder.

CHAPTER V

ON THE CHOICE OF A BRIDGE METHOD AND THE PRECAUTIONS TO BE OBSERVED WHEN USING IT

THE object of this chapter is to summarize for use in the laboratory the theoretical and practical information contained in preceding chapters. The subject will be treated under two headings, the first dealing with the choice of suitable apparatus and methods, the second with the precautions which must be observed in practice in order to obtain exact results.

THE CHOICE OF A BRIDGE METHOD

The choice of a method for a given measurement is affected by a number of circumstances, e.g. the available source and detector, the available standards, and the nature of the quantity to be measured. Each of these factors will be considered in turn.

1. The Source of Current. The most suitable source depends upon the nature of the test, the frequency at which it is to be carried out, and on the amount of power required. In general, there is little difficulty in obtaining an adequate amount of power, since only a small number of watts is required in most bridges.

For ordinary rough tests at frequencies of a few hundred cycles per second, the buzzer forms a very convenient source. The frequency is, however, somewhat unsteady and the wave form impure. If a more steady source be desired, several types are available. For a range between about 20 and 1,000 cycles per second, a maintained tuning fork can be used. Up to about 500 cycles/second the string interrupter is suitable. The microphone hummer in one of its many forms can also be used; the vibrating bar type extending the range to about 3,000 cycles/second or higher.

All these devices when in action emit a loud note which, when telephones are used to detect balance, becomes a great nuisance to the experimenter listening for the feeble note in the telephones when balance is nearly secured. Such sources should, therefore, be placed at a considerable distance from the bridge, and are best enclosed in a sound-proof box so that the direct note cannot be heard.

Again, all these sources of current have rather impure wave forms. When telephones are used in the bridge, it becomes very desirable to choose methods which balance independently of frequency, so that silence occurs for all harmonics of the wave simultaneously. Otherwise the balance point will be obscured by the persistence of noise due to overtones.

For the lowest frequencies, up to about 200 cycles/second, when a fair amount of power is required, the most convenient source is an ordinary motor-driven alternator of pure wave form. If means are provided for maintaining constant speed, it becomes possible to use low frequency vibration galvanometers in the bridge. Though small alternators are still in use for work at higher frequencies up to the limit of the telephonic range, they are now being gradually superseded by other devices, notably by the valve oscillator.

For all high-class work at frequencies above about 200 or 300 cycles/second the valve oscillator is the source *par excellence*. It provides an adequate output at a perfectly steady frequency and with a nearly pure wave form. In combination with a vibration galvanometer, or, for the higher acoustic values, a good telephone, it is the source most suitable for precise tests. It is cheap, easy to set up and to maintain. For lower frequencies it possesses the same advantages, but the large coils and condensers and the larger valves which are necessary rather increase the expense.

Whichever source be chosen, it is advisable always to connect it to the bridge *via* a small transformer. By providing an earthed screen between the two windings of the transformer, capacity effects between the bridge and the source are greatly minimized. By the choice of a suitable ratio of transformation the internal impedance of the source may be adapted to the impedance of the network and an increase in sensitiveness secured. It is well to arrange the source and its transformer at a distance from the bridge, so that any stray magnetic field therefrom may not upset the balance.

2. The Detector. For low frequencies, below 150 cycles-second, the Campbell type of vibration galvanometer forms the most sensitive detector. For frequencies lying between about 200 and 900 cycles per second, a vibration galvanometer of the Duddell pattern is best. For the higher acoustic values from above 800 cycles/second, the telephone is by far the best

detector. The telephone should be chosen to suit the frequency of the test, the type which has a tunable diaphragm being preferable to the fixed pattern.

With all detectors the impedance should be chosen to suit that of the bridge. An interbridge transformer is in this case, as with the source, a great advantage, since by adapting the effective impedance of the detector to the network, a marked increase of sensitivity can be secured. Again, when telephones are used, precautions should be taken to eliminate electrostatic effects between the observer and the telephones by the use of one of the devices described on page 286. For practical details connected with the use of telephones and vibration galvanometers, *see* pages 151-152, 161.

3. Standards. The choice of a method will be largely guided by the standards which are available in the laboratory. A few remarks on standards in general will, however, assist the experimenter to determine which are the most suitable for a given purpose.

The resistances used in the bridge network should, for work of the highest accuracy at high frequencies, be wound in some way which shall ensure that they are free from residual effects, i.e. they should be as nearly non-reactive as possible. Dial decade boxes reading down to 1 ohm are most convenient; the contacts should be good and located by a click, a great advantage when working in a darkened room with a vibration galvanometer, loose plugs being, in such circumstances, a great nuisance. Below 1 ohm a mercury rheostat or some form of slide wire rheostat is useful for fine adjustments. In the case of the latter a little paraffin oil on the wire greatly improves the contact and the ease of setting. For rougher tests at low frequencies, ordinary resistance boxes with bifilar wound coils are frequently quite useful.

Speaking in a general way, the choice of other standards lies between self-inductances, mutual inductances, and condensers. Of these, undoubtedly the most convenient to use is a mutual inductometer. It is the most permanent, is easily and continuously adjusted from zero over a large range of positive or negative values, and is the one most free from corrections, except at high frequencies, when air condensers have the advantage in this respect. Self inductometers are of great service in certain methods, but do not possess the advantages of a good mutual inductometer. Fixed value self-inductance

standards are useful for comparison and substitution methods. Of condensers, a good mica standard is convenient in many cases, but, for accurate work, its frequency and temperature variation must be known and also its phase angle. For work at high frequencies and especially at high voltages, air condensers, which can be taken as loss-free, form the most suitable standard. Their capacitance is, however, usually small, which renders them inconvenient standards for low frequencies, owing to their high impedance.

4. Self-inductance Measurements. Methods for the measurement of self-inductance are chosen with reference not only to the value of the inductance but to the resistance which accompanies it. For example, an inductance of a few microhenrys may be easily measured when the resistance is of moderate value, but quite special treatment is necessary if the resistance be very large—as in a standard resistance coil—or very small, as in a shunt. Accordingly, the time-constant, L/R , of the coil to be measured must be taken into account in deciding upon the bridge to be used.

5. Measurement of Average Inductances. The number of ways of measuring the inductances of average value found in general laboratory testing is very large, but certain methods have outstanding features which recommend them for general use.

Probably the most convenient inductance bridge is the Heaviside-Campbell arrangement, in which a standard mutual inductometer is used (p. 251). The bridge is used preferably with equal ratios and the adjustments are very quickly and conveniently made. The method is particularly free from errors, and it is easy to allow for the leads connecting the test coil to the bridge by taking a preliminary balance with the coil removed. With a mutual inductometer reading up to 0.01 henry, inductances lying between a few microhenrys and 0.02 henry can be measured with considerable precision. The method is particularly valuable, therefore, for tests on coils of low inductance. By the use of a larger inductometer the range can be increased; for example, to 0.2 henry, while keeping all the advantages of an equal ratio bridge.

For larger inductances, the Campbell modifications on pages 247 and 248 can be used. In these the ratios are not equal, hence the advantages of the Heaviside-Campbell arrangement—freedom from residual errors—are, in general,

lost. These methods will give the value of inductance with fair accuracy, but measurements of effective resistance of the coil under test may be considerably in error. (See page 249.)

When a suitable self inductometer is available, Maxwell's method, page 180, is often very useful. Here, again, an equal ratio bridge should be arranged; hence the coil to be tested should not exceed the value of the available self-inductance standard. With proper arrangement, the method can be made very sensitive and is capable of precise settings.

When the laboratory contains a suitable set of good condensers, ranging, say, from 0.1 to 1 microfarad, Anderson's method forms a very useful and accurate method of measuring inductances over a wide range of values (page 215). Rosa and Grover have shown that measurements can be made with high precision, since the errors to which the method is liable are, with proper arrangement, very small and easily allowed for. Inductances from a few millihenrys to 1 or 2 henrys are easily measured.

For very low inductances, Anderson's method is not quite so convenient. For coils of a few microhenrys, Butterworth's modification, page 223, proves very useful and serves as a check on measurements made by the Heaviside-Campbell bridge with a low range mutual inductometer.

Another useful bridge in which standard condensers are used is that of Owen, (p. 223). This is particularly quick for routine laboratory tests over a large range of inductance. For exact work, however, the method requires some correction for residual effects and for imperfections in the condensers.

6. Measurement of Large Inductances. It is rather a difficult matter to measure a large inductance, say, of several henrys, especially if the time-constant is large and simultaneous measurement of the effective resistance of the coil is required. In most ordinary bridge methods, e.g. Campbell's methods of pages 247 and 248, or Maxwell's method of p. 180, it is necessary to use unequal ratios to obtain a suitable range of inductance with available standards of reasonable value. Enormous errors are thereby frequently occasioned in the value of the effective resistance, though the value of the inductance may be obtained with fair accuracy.

In telephonic work, coils having a time-constant of $1/40$ second or higher are much used, the inductances ranging from

140 millihenrys upwards. The effective resistance is thus very small when compared with the large value of inductance, and special methods are necessary. If a good subdivided condenser can be procured, Hay's method of p. 225 may be used with accurate results. The resonance bridge of page 227 is also available. In both these methods, the frequency must be constant and known. If tests are made at acoustic frequencies, with telephones to detect balance, a pure wave form is essential. It is important to screen the coil to make the electrostatic capacity effects definite in the bridge.

7. Measurement of the Residual Inductance of Resistance Coils. The resistance coils used in a.c. bridge work are not entirely free from reactance, coils up to about 1,000 ohms having a slight residual inductance which must be taken into account in accurate work. It is necessary, therefore, to be able to measure the very small residual inductance in coils whose time-constants are of the order of 10^{-7} second or less.

The principle of all methods of measuring such small inductances is the same. The coil to be tested is compared against a standard resistance of approximately equal value, the inductance of which can be calculated. Such standard resistances of calculable inductance usually consist of a pair of parallel wires, or, for the low resistance units, of a wire bent into circular form, as described on page 79. In all the methods, full allowance must be made for any small inductances in the network, and earth capacities must be corrected for. For this reason the bridges are usually worked on the "dummy" balance or substitution method.

Prerauer* and Wien† used Maxwell's bridge on page 180 to measure an inductance of 500×10^{-9} henry, comparing it with a 1 millihenry calculated standard by means of various auxiliary intermediate standards. The time-constant was 10^{-5} seconds and an accuracy of 1 per cent was secured. Giebe, by using a bifilar bridge network in which all residuals can be calculated, modified Maxwell's method to measure an inductance of 500×10^{-9} henry with a time constant of 10^{-8} second, as described on page 186. He showed that by introducing relatively large inductances into the ratio branches the corrections due to residuals in the network can be made very small and are capable of experimental determination.

* O. Prerauer, *Ann. der Phys.*, Bd. 53, pp. 772-784 (1894).

† Max Wien, *idem*, pp. 928-947 (1894).

Curtis and Grover, in an extensive investigation of the residuals of resistance coils, have measured inductances of very low time constant by a substitution method with a calculable standard, using the bridges of pages 186, 212, and 223. Other methods have been developed by Wagner and Wertheimer* for the same purpose, based on the condenser bridges of pages 196, 201, and 203.

8. Measurement of the Inductance of Shunts. A shunt to carry large alternating currents is a low resistance with a correspondingly small self-inductance, the measurement of which is, consequently, a matter of some difficulty. Moreover, such a shunt is a four-terminal resistance, having two current and two potential terminals. The inductance cannot, therefore, be measured by the ordinary bridge methods, which are applicable only to two-terminal inductances. Special methods have been devised to enable the null principle to be applied to the measurement of four-terminal resistances.

One of the best ways of measuring the resistance of a shunt to direct current is the well-known Kelvin double bridge. Sharp and Crawford, Barnett, and Wenner have adapted this principle to alternating current so that inductances of shunts can be measured by the double bridge method. Campbell has devised other methods, involving the use of a mutual inductometer, described on page 254. Other methods, not essentially on the null or bridge principle, have been introduced, the interested reader being referred to *Dictionary of Applied Physics*, Vol. 2, pp. 441-442, for further information.

9. Special Case of Inductance Measurement. An important special application of self-inductance occurs in the determination of the International ohm in absolute measure. The inductance of a carefully constructed and measured coil can be calculated in absolute units from its dimensions. The inductance of the coil in International units can be found by comparing it in a bridge with a standard resistance at a known frequency. The ratio of the C.G.S. calculated value of the coil to its measured International value is the same as the ratio of the C.G.S. ohm (10^9 cm. per second) to the International ohm.

Wien's method on page 190 for the measurement of inductance in terms of frequency and resistance is not sufficiently

* See *Elekt. Zeits.*, 34 Jahrgang, pp. 613-616, 649-652, 1913, and 36 Jahrgang, pp. 606-609, 621-624 (1915).

accurate for very precise measurement. Grüneisen and Giebe,[†] who have determined the value of the ohm with high precision by the inductance method at the Physikalische Technische Reichsanstalt, first used the resonance bridge of page 227. They later used Maxwell's method (p. 211) to compare the inductance with an air condenser and resistances, full allowance being made for residual errors. The condenser was then compared with resistance and frequency by the usual Maxwell interrupter bridge. Very precise determinations were secured.

10. Effective Resistance Measurements. When an inductive coil is measured in a bridge network, the balance conditions involve simultaneously the effective self-inductance and resistance of the coil at the frequency of the test. Hence, if the effective inductance and resistance of all the other branches of the network be known that of the coil will be determined.

In practice, the accurate determination of the inductance of a coil presents no serious complication, but it is often very difficult to get an exact measure of the effective resistance, especially at high frequencies, due to the presence of residual inductances and capacitances in the bridge network. For this reason, when effective resistance is to be measured, the utmost care must be taken to make allowance for residuals, since they have more effect on the resistance measurement than on the inductance. The network must be very carefully set up, the standards used having known or calculable residuals. This is very conveniently secured in practice by constructing the bridge on Giebe's bifilar principle, as described on page 186.

Alternatively, if a coil of known effective resistance is procurable, effective resistance measurements are easily made by measuring this standard coil in the bridge and adjusting the branch residuals until the measured and known values agree at a given frequency. Any other coil can then be accurately measured by the bridge at the same frequency. This principle is very useful in connection with the bridges in which a standard mutual inductometer is used, and has been discussed on page 251.

For coils of about 0.1 henry the resonance bridge of p. 227 or Hay's method of p. 225 are very useful, the former in particular being capable of considerable sensitiveness.

11. Mutual Inductance Measurements. Mutual inductances

[†] *Zeits. f. Inst.*, 30 Jahrgang, p. 147 (1910); *ibid.*, 34 Jahrgang, p. 160 (1914); *Ann. der Phys.*, Bd. 63, p. 179 (1920).

are most conveniently measured by comparison with a standard mutual inductometer. Provided that the unknown does not exceed the standard, the comparison may be made directly by Felici's method (p. 235). The settings can be made with great precision, and the method is remarkably free from errors.

If the unknown is greater than the maximum reading of the inductometer, Maxwell's method on page 238 is applicable, but the double adjustment for balance is often rather troublesome to effect in practice. Campbell's bridge on page 241 avoids this difficulty and is applicable to a wide range of measurement.

Mutual inductance between two coils can always be measured as a self-inductance by measuring the self-inductance of the coils in series with the mutual effect assisting and then opposing, as on page 242. Any suitable method for the measurement of self inductance is then available.

If a standard condenser is obtainable, mutual inductance can be determined with precision by the modified Carey-Foster bridge (p. 258).

12. Capacitance and Condenser Power-factor Measurements.

The choice of method for the measurement of capacitance is governed by the magnitude of the condenser to be tested. With capacitances exceeding about $0.1 \mu F$, the measurements are easily to carry out without special precautions. In testing small condensers, especially at high frequencies, full allowance must be made (i) for the earth capacitances of the electrodes, using Orlich's method of page 112; and (ii) for the influence of small earth capacities between the bridge and earth, using the Wagner device described on page 286.

The determination of a power-factor of a condenser involves the measurement of its equivalent shunt or series resistance in addition to its capacitance. Since the power-factor is usually low, the measurement of the equivalent resistance is a matter of some difficulty, owing to the importance of the errors introduced by residuals in the bridge network. As in the measurement of the effective resistance of a coil, considerable care is necessary when setting up the network to ensure that all residuals can be allowed for. If this is not done, the power factor may be widely in error, although the capacitance may be found with considerable precision.

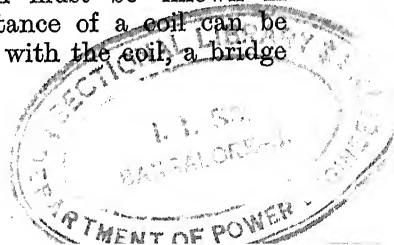
Probably the best way to test the capacitance and equivalent

series resistance of a condenser is by the modified Carey Foster method on page 258, in which a standard mutual inductometer is used. This method is adopted at the National Physical Laboratory for condenser tests, and can be used over a wide range of measurements from a few micro-microfarads upwards. On small condensers and at high frequencies, allowance must be made for residual effects, as shown on page 261, and it may become important to correct for earth capacity effects, as discussed on page 263.

In most other methods of testing condensers, the unknown is compared with a standard condenser whose capacitance and equivalent resistance have been accurately determined by comparison with an air condenser and resistances. Curtis and Grover have made an extensive research on the properties of paper and mica condensers by the methods on pages 196 and 201, the reader being referred to these pages for a summary of their work. When the condenser to be tested has a small capacitance, less than 0.01 microfarad, Grover's series inductance method on page 231 has important advantages.

Apart from the investigation of standard condensers, an important practical application of capacitance and power-factor measurement is found in the testing of samples of high voltage cables, porcelain insulators, samples of dielectrics, and similar work. In such tests, the capacitances are not very large and have to be measured at low frequencies, e.g. 50 cycles per second, and at high voltages. The losses in such technical condensers may be considerable, and should be measured at the working voltage. For such work, Wien's bridge on page 203 is widely used, and has been applied up to several thousand volts by Monasch and others. Owing to the difficulty of producing non-reactive high resistances required in a bridge with small capacitances at low frequency, the Wien bridge is best set up on the four-condenser plan recommended by Fleming and Dyke (p. 208). Alternatively, the high voltage bridge due to Semm (p. 207) may be used. In all high tension bridge tests the utmost care must be taken to correct for earth capacity errors.

13. Measurement of the Residual Capacitance of Resistance Coils. Resistance coils over 1,000 ohms have usually a very small residual self capacitance, which must be known in accurate work. Since the self capacitance of a coil can be represented by a condenser in parallel with the coil, a bridge



which is suitable for the measurement of a small condenser in parallel with a relatively high resistance will serve also to measure the self capacitance of a resistance coil.

The procedure is the same as for the measurement of residual inductance of low resistance coils; the coil is compared by a method of substitution with a parallel wire standard whose residual can be calculated. Allowance must be made for earth capacities, especially between the parallel wire standard and earth, the reader being referred to the papers of Wagner and Wertheimer, and of Curtis and Grover, for further details. (*See* p. 186 and 200.) Hay has also suggested a modification of Anderson's method, in which the coil is compared with a condenser (*see* p. 206).

14. Measurement of Electrolyte Resistance. The resistance of an electrolytic cell is conveniently measured by an alternating current method. In the earliest experiments of Kohlrausch and others, the cell is treated as a resistance and is measured in a simple slide-wire bridge, using an induction coil to minimize polarization effects and a telephone to detect balance (*see* p. 3, for example).

Now an electrolytic cell is not a simple ohmic resistance, since it conducts by electrolytic dissociation. Moreover, the electrodes have an appreciable capacitance. Hence, it is impossible to balance an electrolytic cell against resistances alone. The cell behaves as if it were a resistance in series with a small capacitance, and should be measured as such. By connecting it in a suitable a.c. bridge network and making the proper adjustments, it is possible to secure exact balance of the cell against standard resistances in combination with inductances or condensers.

Taylor has used a series condenser method for electrolyte tests (p. 228). Haworth has made an extensive research on the properties of electrolytic cells by means of the resonance bridge (p. 227) and a modified Heaviside-Campbell bridge (p. 263).

15. Frequency Measurements. In theory, any alternating current bridge which depends for balance upon the frequency of the source, may be used for the measurement of frequency. In practice, it is found that some of these bridges are more useful than others, owing to the greater ease with which they can be set up and balanced, and also to the range of frequency which can be measured.

Probably the most used frequency bridge is that of Campbell (p. 264), in which a mutual inductometer M and a standard condenser C are arranged so that $\omega^2 = 1/MC$. The method is best suited to the measurement of high frequencies, since for the lower values the product MC becomes inordinately large, necessitating the use of a large inductometer and condensers. The principal defect of the bridge is, however, the fact that sharp balance is not possible unless the condenser and the mutual inductance are perfectly free from losses. The main advantage of the method is its extreme simplicity and the ease with which readings may be taken.

By the use of two auxiliary mutual inductances, or by means of an additional condenser, the imperfections of Campbell's method can be compensated, as described on pages 266 and 268. The same end can be advantageously attained with simpler apparatus by the Kennelly-Velander bridge of page 268. This method can be arranged to measure any frequency, low or high, in terms of a mutual inductance of reasonable value and an ordinary laboratory condenser, and for most work is probably the simplest and best method to employ.

Other wide-range frequency bridges make use of a mutual inductometer and a standard self inductance. Of this type are the Hughes balance (p. 256) and Butterworth's method (p. 255), both of which are easy to adjust, of wide range, and can very simply be made direct-reading.

For the measurement* of high frequencies, a two condenser bridge is often convenient. The condensers should be air condensers, and one, at least, should be adjustable. Let a series resistance bridge be set up, as on page 197, and balanced; then $P/R = Q/S = C_2/C_1$. Now, without alteration of the rest of the bridge, let the resistance P be put in parallel with C_1 as on page 204. Re-balancing by alteration of these to values P' and C'_1 gives $(C'_1/C_2) = (S/Q) - (R/P')$ and $C'_1 C_2 = 1/\omega^2 P'R$. From these and the results of the former balance is found $\omega^2 = 1/C_1 C'_1 P P'$. In practice, some trouble can be avoided if one sets $P = 1/\omega C_1$ approximately; then $P' = 2P$ and $C'_1 = \frac{1}{2}C_1$. Hence, if balance be obtained for the series arrangement with this value of P , it will be possible to obtain balance in the parallel arrangement on the same adjustable condenser C_1 .

* For other frequency bridges, see footnote on p. 265, and also Schering and Engelhardt, *Zeits. f. Inst.*, 40 Jahrgang, p. 123 (1920).

When a frequency bridge is balanced, no current of the frequency under test can flow in the detector circuit. Hence, if the wave-form of the source be impure, a harmonic of the chosen frequency is totally suppressed from the detector current. A frequency bridge acts, therefore, as a wave form sifter, suppressing any desired frequency; such a use of a frequency bridge is often of service in dealing with one troublesome harmonic in the wave form of a source (see p. 144).

PRECAUTIONS

Assuming that a suitable method of measurement has been chosen and that the source and detector are given, there are several practical precautions which must be observed in obtaining accurate results. The general principle is to reduce the possibility of experimental error to a minimum, and, in cases where this is not possible, to make allowance for errors unavoidably introduced.

General. Before connecting up the network, the necessary apparatus should be collected together and a diagram made to show the various instruments in their correct relative positions. By this means an orderly arrangement of the connecting wire can be made, and it is only by some such systematic procedure that residual errors can be reduced.

Leads. The leads should be carefully laid out in such a way that no loops or long lengths enclosing magnetic flux are produced, with consequent stray inductance errors. Twisted leads, such as bifilar flex, is often useful for bridge connections. In measuring small condensers, it may be necessary to remove capacity effects of the leads connected to the bridge by enclosing them in earthed metal tubes.

In measuring an inductance, the leads connecting it to the bridge should be a metre or more in length so that the coil is well removed from the bridge. With a small inductance the leads should be close together so that their inductance is very small. With a large inductance the self capacitance of the leads is more important than their inductance, so they should be spaced apart.

The apparatus should also be arranged so that direct inductive action between the source and the bridge or the source and detector is reduced to the minimum. The apparatus,

especially in high impedance bridges, should be well insulated on blocks of paraffin wax.

Procedure. Having satisfactorily connected up the bridge network so that the leads are properly disposed, all contacts are good, and mutual action between the separate branches is eliminated, the following systematic procedure may be advantageously adopted—

(i) When possible, choose an equal ratio bridge. Residual errors are thereby rendered very much less important and can be allowed for with ease by taking two balances, the second with the ratios interchanged in the network. The average is the corrected value.

(ii) Make the two balancing adjustments successively, altering first one and then the other until balance is secured. Read all scales and observe temperatures and frequency.

(iii) Reverse connections to the source to detect and, if necessary, to allow for capacity effects and direct inductive action between the source and the bridge. Re-balance as in (ii).

(iv) Repeat the balances with the ratio branches reversed in the case when they are equal.

(v) When measuring an inductance balancing should be repeated when the coil is turned through 180° so that inductive action of stray magnetic fields can be eliminated.

(vi) Make allowance for the leads connecting the apparatus to be measured into the bridge. In the case of inductance measurements, the leads should be short-circuited at the terminals of the coil and their inductance measured. When measuring a condenser the leads should be disconnected from the condenser terminals, taking care to disturb their position as little as possible, and a measurement of their capacitance obtained. In condenser tests the leads are preferably of fine wire to reduce their capacitance.

(vii) Most bridge errors can be eliminated by the process of substitution or dummy balance. The bridge is first balanced in the usual way. The apparatus under test is then removed, and is replaced by a standard as nearly equal to it as possible, so that balance can be restored by *slight* adjustments of one branch of the network.

16. Earth Capacities. In all the above discussion it is assumed that the adjustments which can be made in the branches of the network are adequate to produce balance in the detector. In practice, especially in tests on small

condensers, this is often not the case, since the network consists not only of the visible branches which have been allowed for in the balance conditions but also of invisible branches formed by the intercapacities between the branches and by their capacities to earth. As a result, even though the branch balance condition be satisfied, current may still flow in the detector *via* the capacity paths (*see also* page 188).

To render definite the intercapacities of the branches, the effects of which are important at high frequencies, Campbell* has proposed a system of electrostatic screening. Each branch is surrounded by a metallic screen, the potential of which is determined by connection to one or other of the branch points. The detector is screened from the bridge, and the source is connected *via* a transformer with an earthed screen between its windings.

The effects of earth capacity are usually taken to be adequately represented by condensers joining the four branch points to earth. Their action can be easily eliminated in bridges of the Wheatstone type by an ingenious device due to Wagner† (Fig. 73). The principle to be aimed at in removing the effect of any earth capacitance is to bring both its terminals to zero potential. In the diagram, if $z_1 z_3 = z_2 z_4$ be satisfied this merely implies that the branch points CD are instantaneously at the *same* potential and not necessarily at *zero* potential. If the detector be a telephone on the head of an earthed observer, it follows that current will still flow in the instrument, even when the balance condition is fulfilled, *via* the earth capacities from C and D to the observer's head. To eliminate the effect, two auxiliary branches, z_A, z_B , imitating in characteristics the impedances z_1, z_2 , are joined across the alternator and earthed at their common point E . The earth capacities from A and B to E merely shunt z_A, z_B . To eliminate the capacities between C, D , and E , let the bridge be balanced as nearly as possible so that minimum sound occurs in the telephone when joined between C and D . Now transfer the telephone into the auxiliary network between C and E , balancing by means of z_A, z_B . - Then C and E are nearly at the

* G. Campbell, "The shielded balance," *Elec. World*, Vol. 43, pp. 647-649 (1904). For simple screened bridges, *see* pp. 189 and 211.

† K. W. Wagner, "Zur Messung dielektrischer Verluste mit der Wechselstrombrücke," *Elekt. Zeits.*, 32 Jahrgang, pp. 1001-1002 (1911). Also *see idem*, 33 Jahrgang, pp. 635-637 (1912).

same potential, the latter being earthed. Reverting to the original bridge, balance can now be more nearly secured, and repetition of the process finally results in exact balance. Then C, D, E are at the same potential, namely, zero, and the "head effect" to the earthed observer is eliminated.

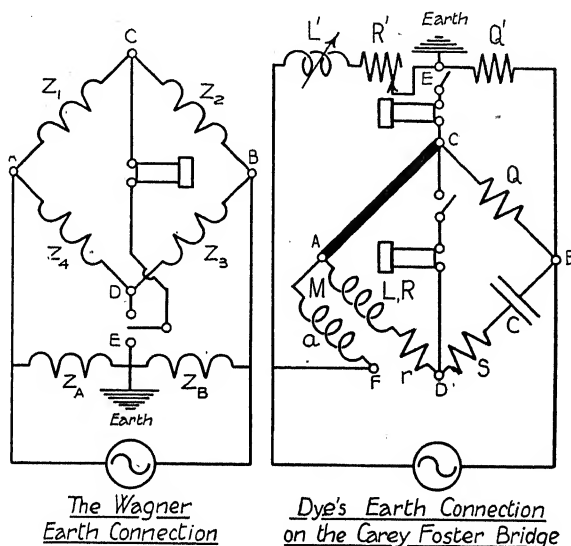


FIG. 73.—EARTH CONNECTIONS FOR THE ELIMINATION OF EARTH CAPACITANCE EFFECTS AT THE DETECTOR BRANCH POINTS IN ALTERNATING CURRENT BRIDGES

An instance in which the head effect is of great importance is the Carey Foster bridge (see p. 258). Mr. Dye* has shown how the Wagner earth may be adapted to this case, as in Fig. 73. The main bridge is first approximately balanced, with the auxiliary telephone switch open, by successive adjustments of M and S . The main telephone is then disconnected from the branch point C , while the second telephone is joined to E . Balance in the auxiliary bridge is then obtained approximately by adjustments of L' (which is about equal to the inductance of a), R' , and, if necessary, of Q' also. The telephone connection in the main bridge is then replaced on the point C , the auxiliary being removed from E , and the

* D. W. Dye, "The 'Wagner' earth connection," *Elecn.*, Vol. 87, pp. 55-56 (1921); also G. E. Moore, *Elecn.*, Vol. 86, pp. 744-745 (1921).

main bridge balance is corrected finally to zero by adjustments of M and S . Thus the points C , D , and E are brought to zero potential. (Alternatively, a single telephone and change-over switch can be used, as in the Wagner earth connection.)

For a full discussion of earth capacities, and an additional method of allowing for them, the reader is referred to a paper by Mr. S. Butterworth.* The procedure with regard to capacity troubles is, in general: (i) to render the effects definite by careful arrangement of the apparatus or by suitable screening; (ii) to remove the "head effect" by means of the Wagner earth; and (iii) to allow for capacitance errors in the branch under test by use of a substitution method.

* S. Butterworth, "Notes on earth capacity effects in alternating current bridges," *Proc. Phys. Soc.*, Vol. 34, pp. 8-16 (1922).

APPENDIX

ON THE APPLICATION OF STAR-MESH TRANSFORMATIONS IN THE THEORY OF A.C. BRIDGE METHODS

THE methods given in Chapter II for dealing with problems connected with a.c. bridge networks are usually sufficient for most purposes. In certain cases, particularly when a large number of meshes become involved, the algebraical work can be much reduced by transformation of the network into one of a simpler type. For this purpose the well-known star-mesh transformations introduced by Kennelly* are frequently of service and will be briefly dealt with here in their application to bridge networks.

In the first instance, consider three terminals in a network, A, B, C , joined by a simple mesh of impedances whose operators are z_a, z_b, z_c , as in Fig. 74(a). It is required to replace this mesh by a simple star-connected system of three impedances, z_A, z_B, z_C , which shall be equivalent, with respect to the external circuit joined to A, B, C , to the original mesh (see Fig. 74(b)). Let it be supposed that measurements are made of the impedance existing between successive pairs of terminals both for the star and for the mesh arrangements. Then, since the two are to be equivalent,

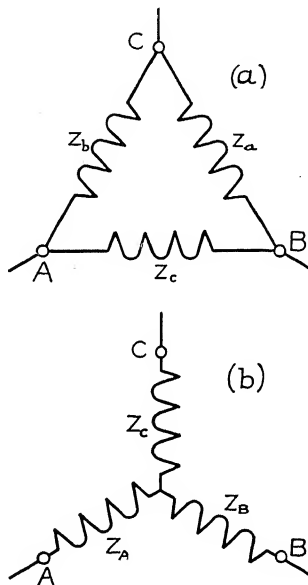


FIG. 74.—EQUIVALENT STAR
AND MESH CONNECTIONS
OF IMPEDANCES

$$\text{across } A \text{ and } C, \quad z_A + z_C = \frac{z_b(z_a + z_c)}{z_a + z_b + z_c},$$

$$\text{across } C \text{ and } B, \quad z_C + z_B = \frac{z_a(z_b + z_c)}{z_a + z_b + z_c},$$

$$\text{across } B \text{ and } A, \quad z_B + z_A = \frac{z_c(z_a + z_b)}{z_a + z_b + z_c}.$$

* A. E. Kennelly, "The equivalence of triangles and three-pointed stars in conducting networks," *Elec. World*, Vol. 34, pp. 413-414 (1899).

Solving for the impedance operators of the star in terms of those of the mesh,

$$\left. \begin{aligned} z_A &= \frac{z_b z_c}{z_a + z_b + z_c}, \\ z_B &= \frac{z_a z_c}{z_a + z_b + z_c}, \\ z_C &= \frac{z_a z_b}{z_a + z_b + z_c}. \end{aligned} \right\} \quad \cdot \quad \cdot \quad \cdot \quad (a)$$

Conversely, suppose that it is desired to replace a star-connected system of impedances by an equivalent mesh. Let admittance operators be $y_A, y_B, y_C, y_a, y_b, y_c$, corresponding to $1/z_A, 1/z_B, 1/z_C, 1/z_a, 1/z_b, 1/z_c$ respectively. Then by a process similar to the above it is easy to show that the mesh equivalent to the star is given by

$$\left. \begin{aligned} y_a &= \frac{y_B y_C}{y_A + y_B + y_C}, \\ y_b &= \frac{y_A y_C}{y_A + y_B + y_C}, \\ y_c &= \frac{y_A y_B}{y_A + y_B + y_C}. \end{aligned} \right\} \quad \cdot \quad \cdot \quad \cdot \quad (b)$$

Three examples will now be considered to show the utility of the mesh to star transformation in finding the balance conditions for certain bridge networks. Other applications of this transformation and of the converse represented by Equations (b) will occur to the student.

Anderson's Network. Consider the Anderson network of Fig. 15, page 46, which is re-drawn in Fig. 75(a). This network differs from a simple four-branch or Wheatstone network by the additional mesh formed by the impedances z_3, z_6, z_7 joining the points A, B, C as shown in the diagram. Transform this mesh into its equivalent star by means of Equation (a), then,

$$\begin{aligned} z_A &= \frac{z_3 z_6}{z_3 + z_6 + z_7}, \\ z_B &= \frac{z_6 z_7}{z_3 + z_6 + z_7}, \\ z_C &= \frac{z_3 z_7}{z_3 + z_6 + z_7}. \end{aligned}$$

Referring to the transformed network shown in Fig. 75(b),

z_B lies in the detector circuit and does not, therefore, enter into the balance equation. The remaining impedances form a four-branch network $z_1, z_2, z_0, z_4 + z_A$, so that, for balance,

$$z_1 z_0 = z_2 (z_4 + z_A);$$

that is $z_1 z_3 z_7 = z_2 [z_4 (z_3 + z_6 + z_7) + z_3 z_6]$,

on substituting the values of z_A and z_0 . Re-arranging terms,

$$z_7 (z_1 z_3 - z_2 z_4) = z_2 \{ z_6 (z_3 + z_4) + z_3 z_4 \}$$

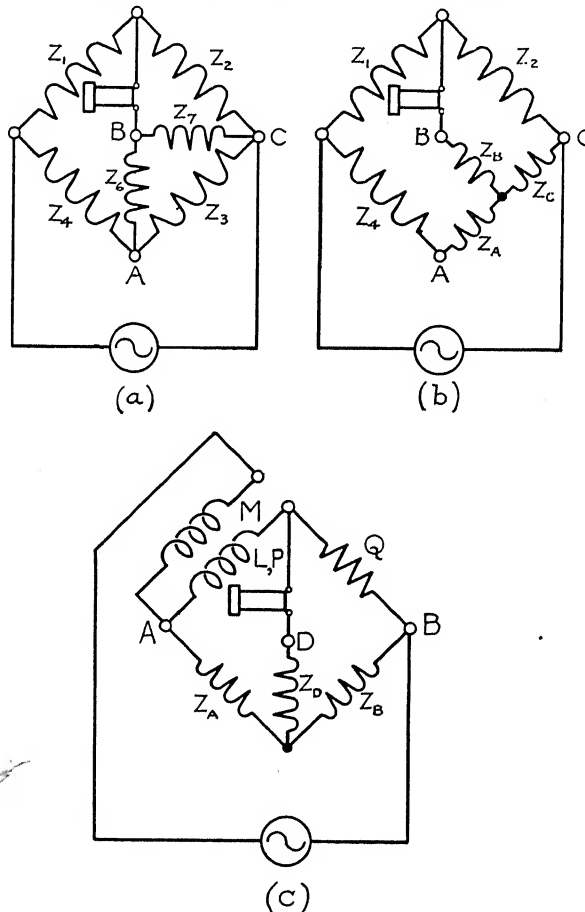


FIG. 75 (a) AND (b).—MESH-STAR TRANSFORMATION OF THE ANDERSON NETWORK. (c) MESH-STAR TRANSFORMATION OF MAXWELL'S BRIDGE (see Fig. 66 (a))

is the balance condition, in agreement with the result proved on page 46.

Maxwell's Bridge for Comparison of Self with Mutual Inductance. Examine next the case of Maxwell's bridge shown in Fig. 66(a) on page 244, assuming the resistance W to be used to adjust for balance. Convert the mesh formed of R , S , and W into a star connected set of impedances z_A , z_B , z_D joined to the points A , B , D . The transformed network is shown in Fig. 75(c), which is seen to be of the four branch type,

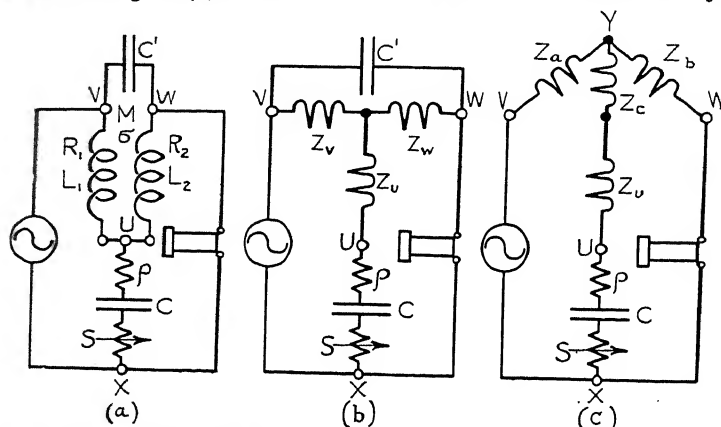


FIG. 76.—TRANSFORMATION OF BUTTERWORTH'S MODIFICATION OF THE CAMPBELL FREQUENCY BRIDGE

in which $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = z_B$, $z_4 = z_A$, and $m_{16} = j\omega M$. Inserting these values in the equations given on page 52, $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$, so that for balance

$$z_B(P + j\omega L) - z_A Q + j\omega M(Q + z_B) = 0.$$

Applying Equations (a),

$$z_A = \frac{RW}{S + R + W} \text{ and } z_B = \frac{SW}{S + R + W},$$

hence,

$$SW(P + j\omega L) - RWQ + j\omega M[SW + Q(S + R + W)] = 0;$$

whence, by equating components,

$$SP = QR$$

$$\text{and } L = -\left\{1 + \frac{Q}{S} + \frac{Q(S + R)}{WS}\right\}M,$$

as quoted on page 245.

Butterworth's Modification of the Campbell Frequency Bridge. The details of this bridge have been discussed on page 268, the network being shown in Fig. 76(a). This example is rendered more complex by the fact that the mesh joining the points U, V, W contains an impure mutual inductance. Reduction of the network to a simpler type can be made in two stages, as follows—

(i) By means of the transformation given on page 51, substitute a star system of impedances z_u, z_v, z_w for the two coils of the mutual inductance, the mutual operator for which is $m = -(\sigma + j\omega M)$, since the mutual inductance is impure and must be negative for balance (see p. 265); then,

$$\begin{aligned} z_u &= (\sigma + j\omega M), \\ z_v &= R_1 - \sigma + j\omega(L_1 - M), \\ z_w &= R_2 - \sigma + j\omega(L_2 - M), \end{aligned}$$

as in Fig. 76(b).

(ii) Now by means of Equation (a), replace the mesh of impedances $z_v, z_w, 1/j\omega C'$ by their equivalent star z_a, z_b, z_c , as in Fig. 76(c). Since z_a and z_b lie in circuit with the source and detector respectively, for balance the impedance of the common branch XY must be zero, that is,

$$z_c + z_u + \rho + S + \frac{1}{j\omega C} = 0.$$

Substituting for $z_c = \frac{z_w z_v}{z_w + z_v + \frac{1}{j\omega C'}}$ and for z_u ,

$$\frac{z_w z_v}{z_w + z_v + \frac{1}{j\omega C'}} + \left[\sigma + \rho + S + j\left(\omega M - \frac{1}{\omega C}\right) \right] = 0$$

Since σ and ρ are small and at balance $\omega M - \frac{1}{\omega C}$ is nearly zero, the bracketed expression is small compared with z_w and z_v , so that to the first order of small quantities

$$j\omega C' z_w z_v + \left[\sigma + \rho + S + j\left(\omega M - \frac{1}{\omega C}\right) \right] = 0,$$

since $1/j\omega C'$ is large. Inserting the values for z_v and z_w , separating the two components, and neglecting the product $\sigma C'$ gives

$$\omega^2 MC = 1 - \omega^2 CC' [R_1 R_2 - \omega^2 (L_1 - M)(L_2 - M)],$$

$$\text{and } \sigma + \rho + S = \omega^2 C' [R_1 (L_2 - M) + R_2 (L_1 - M)],$$

as quoted on page 268.



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